ECON3220/4220 – Exam (autumn 2023)

Question 1 (10%)

Define the following terms and explain how they are related: Nash equilibrium, Subgame Perfect equilibrium and Perfect Bayesian equilibrium.

Quest on 2 (10%)

Two firms, *I* and *E*, simultaneously set prices, where p_I denotes the price of *I* and p_E the price of *E*. Demand facing the two firms are given by $q_I = 1 - p_I + p_E$ and $q_E = 1 - p_E + p_I$, respectively, while unit costs are constant and given by c_I and c_E.

Find the Nash equilibrium of this game and demonstrate that, at equilibrium, prices are $p_I = \frac{1}{3}(3 + 2c_I + c_E)$ and $p_E = \frac{1}{3}(3 + 2c_E + c_I)$ while profits are $\pi_I(c_I, c_E) = \frac{1}{9}(3 - c_I + c_E)^2$ and $\pi_E(c_E, c_I) = \frac{1}{9}(3 - c_E + c_I)^2$.

Question 3 (10%)

Suppose $c_E = 2$. Suppose moreover that firm *E* incurs a fixed cost $f = \frac{7}{9}$ when it operates (so its profit becomes $\pi_E(c_E, c_I) - f = \pi_E(2, c_I) - \frac{7}{9}$).

Explain that firm *E* runs a surplus if $c_I = 2$, and hence would like to operate in the market, but not if $c_I = 1$.

Question 4 (20%)

Suppose firm *I*, by incurring an investment cost of $k = 1$, may reduce unit costs from $c_I = 2$ to $c_I = 1$ before the market opens and that, subsequent to firm *I*'s investment – which is assumed to be observed by firm *E* – firm *E* decides whether to enter the market or not. If firm *E* enters, firms choose prices simultaneously as above, while if firm *E* does not enter, firm *E* receives its reservation payoff of 0 and firm *I* operates alone and receives monopoly profits $\pi^M(c_I) = \frac{1}{4}(4 - c_I)^2$ (less any investment cost).

A.

Characterise Nash equilibria of this game.

B.

Explain that in the Subgame Perfect equilibrium firm *E* does not enter.

Question 5 (10%)

Consider again the setting above, but assume now that firm *E* cannot observe whether or not firm *I* invested before making its entry decision (firm *E* does become aware of firm *I*'s decision before prices are set).

How does this affect the equilibrium analysis? In particular, explain that an equilibrium in which firm *E* enters cannot be ruled out.

Question 6 (30%)

We return to the setting in Question 4, in which firm *E* can observe any investment by firm *I*. However, we now assume that firm *I* can be of two types: a high-cost type with (initial) cost $c_E^H = 3$, and a low-cost type with cost $c_E^L = 1$. The type of firm *I* is decided before firms make any decisions, with the probability of high-cost type equal to $\frac{1}{5}$ and the probability of a low-cost type equal to $\frac{4}{5}$. Firm *E* does not observe firm *I*'s type but firm *I* knows its type. Independently of type, firm *I* may, at an investment cost of $k = 1$, reduce its cost by 1 (from 3 to 2 if it is a high-cost type and from 1 to 0 if it is a low-cost type). After firm *I* has made its investment decision, firm *E* makes its entry decision; if it enters, the two firms set prices simultaneously; if it does not enter, firm *I* acts as a monopolist.

A.

Explain that there cannot be a Perfect Bayesian equilibrium in which the high-cost type invests.

B.

Explain that in any separating Perfect Bayesian equilibrium firm *E* enters if firm *I* is of the high-cost type, but not otherwise.

C.

Explain that in any pooling Perfect Bayesian equilibrium firm *E* does not enter.

Question 7 (10%)

In light of the results above in Questions 4, 5 and 6, discuss under what conditions the informed player (i.e. firm *I*) would want to share its information with the uninformed player (i.e. firm *E*).