nhf, 17.11.23

ECON 3220-4220

Exam

Question 1 (10%)

Define the following terms and explain how they are related: Nash equilibrium, Subgame Perfect equilibrium and Perfect Bayesian equilibrium.

Solution: See Watson or Varian.

Question 2 (10%)

Two firms, *I* and *E*, simultaneously set prices, where p_I denotes the price of *I* and p_E the price of *E*. Demand facing the two firms are given by $q_I = 1 - p_I + p_E$ and $q_E = 1 - p_E + p_I$, respectively, while unit costs are constant and given by c_I and c_E.

Find the Nash equilibrium of this game and demonstrate that, at equilibrium, prices are $p_I = \frac{1}{3}(3 + 2c_I + c_E)$ and $p_E = \frac{1}{3}(3 + 2c_E + c_I)$ while profits are $\pi_I(c_I, c_E) = \frac{1}{9}(3 - c_I + c_E)^2$ and $\pi_E(c_E, c_I) = \frac{1}{9}(3 - c_E + c_I)^2$.

Solution: Straightforward.

Question 3 (10%)

Suppose $c_E = 2$. Suppose moreover that firm *E* incurs a fixed cost $f = \frac{7}{9}$ when it operates (so its profit becomes $\pi_E(c_E, c_I) - f = \pi_E(2, c_I) - \frac{7}{9}$).

Explain that firm *E* runs a surplus if $c_I = 2$, and hence would like to operate in the market, but not if $c_I = 1$.

Solution: Equilibrium profit of firm *E* is $\pi_E(2,2) - \frac{7}{9} = \frac{1}{9}(3 - 2 + 2)^2 - \frac{7}{9} = \frac{2}{9}$ when $c_I = 2$ and $\pi_E(2,1) - \frac{7}{9} = \frac{1}{9}(3 - 2 + 1)^2 - \frac{7}{9} = -\frac{1}{3}$ when $c_I = 1$.

Question 4 (20%)

Suppose firm *I*, by incurring an investment cost of $k = 1$, may reduce unit cost from $c_I = 2$ to $c_I = 1$ before the market opens and that, subsequent to firm *I*'s investment – which is assumed to be observed by firm *E* – firm *E* decides whether to enter the market or not. If firm *E* enters, firms choose prices simultaneously as above, while if firm *E* does not enter, firm *E* receives its reservation payoff of 0 and firm *I* operates alone and receives monopoly profits $\pi^M(c_I) = \frac{1}{4}(4 - c_I)^2$ (less any investment cost).

A.

Characterise Nash equilibria of this game.

B.

Explain that in the Subgame Perfect equilibrium firm *E* does not enter.

Solution: There is one Nash equilibrium in which the strategy of firm *I* is to invest and set price $p_I = \frac{7}{3}$ if firm *E* enters and the monopoly price otherwise, while the strategy of firm *E* is not to enter (which is a best response, given the result above in Question 3.). There is also a Nash equilibrium in which the strategy of firm *I* is not to invest and set price $p_I = 3$, while the strategy of firm *E* is to always enter and set price $p_E = 3$ (the strategy of firm *I* is a best response, since, given firm *E*'s strategy, investing would lead to negative profits). Only the latter is a Subgame Perfect equilibrium.

Question 5 (10%)

Consider again the setting above, but assume now that firm *E* cannot observe whether or not firm *I* invested before making its entry decision (firm *E* does become aware of firm *I*'s decision before prices are set).

How does this affect the equilibrium analysis? In particular, explain that an equilibrium in which firm *E* enters cannot be ruled out.

Solution: The two Nash equilibria exist in this formulation also. However, since what follows from the the point at which firm *E* makes it entry decision is not a proper subgames (unlike in the formulation above), none of the Nash equilibria can be ruled out by applying subgame perfection.

Question 6 (30%)

We return to the setting in Question 4, in which firm *E* can observe any investment by firm *I*. However, we now assume that firm *I* can be of two types: a high-cost type with (initial) cost $c_I^H = 3$, and a low-cost type with cost $c_I^H = 1$. The type of firm *I* is decided before firms make any decisions, with the probability of high-cost type equal to $\frac{1}{5}$ and the probability of a low-cost type equal to $\frac{4}{5}$. Firm *E* does not observe firm *I*'s type but firm *I* knows its type. Independently of type, firm *I* may, at an investment cost of $k = 1$, reduce its cost by 1 (from 3 to 2 if it is a high-cost type and from 1 to 0 if it is a low-cost type). After firm *I* has made its investment decision, firm *E* makes its entry decision; if it enters, the two firms set prices simultaneously; if it does not enter, firm *I* acts as a monopolist.

A.

Explain that there cannot be a Perfect Bayesian equilibrium in which the high-cost type invests.

Solution: If the high-cost type of firm *I* invests, it earns $\pi_I(2,2) - 1 = 0$ if firm *E* enters and $\pi^M(2) - 1 = 0$ if firm *E* does not. If the high-cost type of firm *I* does not invest, it earns $\pi_I(3,2) = \frac{4}{9}$ if firm *E* enters and $\pi^M(3) = \frac{1}{4}$ if firm *E* does not. In other words, the high-cost type of firm *I* is better off not investing independently of what firm *E* does.

B.

Explain that in any separating Perfect Bayesian equilibrium firm *E* enters if firm *I* is of the high-cost type, but not otherwise.

Solution: Since the high-cost type does not invest at equilibrium, in a separating equilibrium the low-cost type does invest. Firm *E*'s consistent belief is that firm *I* is a low-cost type if it invests and a high-cost type if it does not. Given these beliefs, the best response of firm *E* is to enter if it believes it is facing a high-cost type (entering gives $\pi_E(2,3) - f = 1$, while not entering gives 0) and not to enter if it is facing a low-cost type (not entering gives 0, while entering gives $\pi_E(2,0) - f = -\frac{2}{3}$). Given the beliefs and strategies of firm *E*, investing is a best response for the low-cost type of firm *I* (investing gives $\pi^M(0) - f = 3$, while not investing (and inviting entry), gives $\pi_I(1,2) = \frac{16}{9}$). From A., the strategy of the high-cost type of firm *I* is optimal.

C.

Explain that in any pooling Perfect Bayesian equilibrium of this game firm *E* does not enter.

Solution: Since the high-cost type does not invest at equilibrium, in a pooling equilibrium the low-cost type also does not invest. Firm *E*'s consistent belief if observing no investment is equal to its initial belief that the the probability of high-cost type equals $\frac{1}{5}$ and the probability of a low-cost type equals $\frac{4}{5}$. Given these beliefs, the best response of firm *E* is not to enter (this gives an expected profit of 0, while entering gives an expected profit of $\frac{1}{5}\pi_E(2,3) + \frac{4}{5}\pi_E(2,1) - f = \frac{1}{5} \cdot \frac{16}{9} + \frac{4}{5} \cdot \frac{4}{9} - \frac{7}{9} = -\frac{1}{15}$). For this to be an equilibrium, it must be the case that firm *I* does not want to deviate by investing; this will never be the case for the high-cost type, but it could be the case for the low-cost type. To ensure that this is not the case, the out-of-equilibrium beliefs of firm *E* must be such that it decides to enter if firm *I* invests (if firm *E* decided not to enter after investment, the low-cost type would deviate): this is optimal for firm *E* if its out-of-equilibrium belief of firm *I* being high cost, p, is such that entering is profitable, i.e. if $p\pi_E(2,2) + (1-p)\pi_E(2,0) - f = p \cdot 1 + (1-p) \cdot \frac{1}{9} - \frac{7}{9} > 0$, or $p > \frac{3}{4}$.

Question 7 (10%)

In light of the results above in Questions 4, 5 and 6, discuss under what conditions the informed player (i.e. firm *I*) would want to share its information with the uninformed player (i.e. firm *E*).

Solution: Comparing the games in Questions 4 and 5 (the latter a game of hidden action), since the outcome when firm *E* does not enter is superior to firm *I*, firm *I* benefits from making its investment known to firm *E* since this would lead to the unique Subgame Perfect equilibrium in which firm *E* does not enter. On the other hand, in Question 6 (a game of hidden information), firm *I* is better off in the pooling equilibrium, where no information is revealed and firm *E* stays out, than in the separating equilibrium where firm *E* enters if firm *I* is of the high-cost type. Intuitively, when an action affects an opponent in a desired direction, it is better if it is clear that the action is taken; on the other hand, when revealing information may lead an opponent to an undesired response, it is better not to reveal information.