

Postponed exam

Question 1 (20%)

Define and explain the following terms: hidden action, moral hazard, incentive compatibility constraint and participation constraint.

Solution: See Watson or Varian & Melitz.

Question 2 (40%)

A government considers contracting the running of a certain service to a private party (think of a train line). The service may be of high, q^H , or low, q^L , quality (eg. measured by interruptions of the service), depending on the amount of resources spent by the private party, x ; if resources spent equal or exceed the threshold $\bar{x}$, i.e. $x \geq \bar{x}$, quality will be high, otherwise, i.e. if $x < \bar{x}$, it will be low. The contract, which is determined by the government, provides a fixed payment, π , for running the service and a penalty, p , in case the quality of service is low. If a contract is agreed upon and the quality of service is high, payoffs are $q^H - \pi$ to the government and $\pi - x$ to the private party; if a contract is agreed upon and the quality of service is low, payoffs are $q^L - \pi + p$ to the government and $\pi - x - p$ to the private party; if a contract is not agreed upon, (reservation) payoffs are zero.

A

What amount of resources will the private party spend if it decides to supply low quality? And what if it decides to supply high quality? Explain that the payoff to the private party in these two cases will be $\pi - p$ and $\pi - \bar{x}$, respectively.

Solution: If q^L , resources spent are $x = 0$ and payoff $\pi - p$; if q^H , resources spent are $x = \bar{x}$ and payoff $\pi - \bar{x}$.

B

Suppose the government wants the private party to run the service with high quality. Define the relevant incentive compatibility constraint and explain why the penalty for low service must satisfy $p \geq \bar{x}$.

Solution: The incentive compatibility constraint becomes $\pi - \bar{x} \geq \pi - p$, which implies $p \geq \bar{x}$.

C

Define the participation constraints of the private party. What is the optimal contract if the government wants the private party to run the service with high quality? What if it wants the service with low quality?

Solution: The participation constraints are $\pi - \bar{x} \geq 0$ if the private party offers high quality and $\pi - p \geq 0$ if the private party offers low quality. The optimal contract is $\{\pi = \bar{x}, p \geq \bar{x}\}$ in the former case and $\{\pi = p\}$ in the latter case.

D

Under what conditions on quality and required resources is high quality of service efficient? Explain why the government wants a high quality of service only if that is efficient.

Solution: High quality is optimal if the cost of the required resources is smaller than the increase in quality, i.e. $q^H - q^L \geq \bar{x}$. In this case the payoff to the government from high quality is greater than that from low quality, i.e. $q^H - \pi = q^H - \bar{x} \geq q^L = q^L - \pi + p$.

Question 3 (40%)

Suppose now that supplied quality not only depends on resources spent, but also on circumstances that are outside the control of both the government and the private party. Specifically, quality will certainly be low if insufficient resources are spent, i.e. $x < \bar{x}$, but will be high only with probability, r , $0 < r < 1$ if enough resources are spent, i.e. $x \geq \bar{x}$. The relevant circumstances are not verifiable and hence not contractable.

A

Explain that (expected) payoffs to the government and private party now become $q^L - \pi + p$ and $\pi - x - p$ if $x < \bar{x}$ if $x < \bar{x}$ and $r q^H + (1 - r)(q^L + p) - \pi$ and $\pi - x - (1 - r)p$ if $x \geq \bar{x}$.

Solution: Straightforward.

B

Explain why, the lower is r , the higher the penalty needs to be in order to induce the private party to offer a service of high quality. Derive the optimal contract if the government wants high quality of the service and explain why the cost is decreasing in r .

Solution: The incentive compatibility constraint becomes $\pi - \bar{x} - (1 - r)p \geq \pi - p$, which implies $p \geq \frac{\bar{x}}{r}$, with the left-hand side decreasing in r . The participation constraint becomes $\pi - \bar{x} - (1 - r)p = \pi - \frac{\bar{x}}{r} \geq 0$. The optimal contract is therefore $\left\{ \pi = \frac{\bar{x}}{r}, p \geq \frac{\bar{x}}{r} \right\}$.

C

Above, it is implicitly assumed that both the government and the private party do not care about risk. Discuss how risk aversion on behalf of the private party would affect the optimal contract, including the fixed payment and the penalty, as well as whether inducing high quality is optimal.

Solution: The private party is only exposed to risk when expending resources to achieve high quality. Hence, it becomes more costly to induce the private party to expend such resources when it is risk averse. Both the fixed payment and the penalty for low quality must therefore be higher for the private party to accept a contract that induces high quality.