

Problem 1: Decision theory (35%)

Consider the lottery

$$\begin{array}{ll} -1 & \text{with probability 99\%} \\ +100 & \text{with probability 1\%} \end{array}$$

Consider first a risk averse expected utility maximizer that is indifferent between this lottery and no gambling. The utility function is calibrated to $u(W_0 - 1) = -1$, and $u(W_0) = 0$, where W_0 is initial wealth.

a) Compute $u(W_0 + 100)$.

Solution: As the person is indifferent, the expected utility of both alternatives are equal. This yields an expression with only one unknown, $u(W_0 + 100)$.

$$\begin{aligned} 0.99u(W_0 - 1) + 0.01u(W_0 + 100) &= 0 \\ u(W_0 + 100) &= 99 \end{aligned}$$

Now consider a person behaving in accordance with prospect theory with reference point 0, and with decision weights $\pi(p) = p$ and value function

$$v(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2x & \text{for } x < 0 \end{cases}$$

b) Will this person choose the lottery?

Solution: No. Calculating the value of the lottery, we see that it is lower than the value of not gambling.

$$0.99(2v(-1)) + 0.01v(100) = -198 + 100 = -98 < 0 = v(0)$$

In the specification above we assumed $\pi(p) = p$, while Prospect theory suggest using decision weights that differs from the probabilities.

c) Explain the intuition for the use of decision weights in Prospect theory. Will small probability be over weighted or under weighted? To what extent will probability weights reinforce or potentially alter the conclusion in c).

Solution: The intuition of probability weights is that it is easier to note the difference between a small probability and full certainty ($p = 0.01$ versus $p = 0.00$) versus a small change in probability ($p = 0.48$ versus $p = 0.49$) For one thing, the certainty is qualitatively different from uncertainty, and we would have to envision the possible outcome. Thus subjects will put a weight higher than the probability on outcomes with low probability, and opposite with high probability. In other words: $\pi(0.01) > 0.01$ while $\pi(0.99) < 0.99$. The positive outcome will thus be given more weight with such decision weights than when $\pi(p) = p$, which may cause the person to choose the lottery, that is alter the conclusion.

In Köszegi and Rabin's version of Prospect theory, the utility is of the form $u(x|r) = m(x) + n(x|r)$ where m is consumption utility, which we set to $m(x) = x$ here, and $n(x|r) = v(x-r)$ which is gain-loss utility with $v(x)$ as above and where r is the reference point. Decision weights are $\pi(p) = p$ as above. Köszegi and Rabin argue that the reference point is determined by expectations.

- d) Show that zero could be a reference point according to Köszegi and Rabin's theory. I.e. show that $r = 0$ is a personal equilibrium.

Solution: *With weights=probabilities, as is the case in Köszegi and Rabin, the subject will not choose the lottery with zero as reference point. The calculation is as in the case of prospect theory above, with the only difference that the expected value of the consumption utility is added, but that equals one, while the calculation in c) showed a gain-loss utility of -98 from the lottery. Thus if the person expect not to choose the lottery, the expected payoff is zero, and with this reference point it is rational not to choose the lottery. Not choosing the lottery is a personal equilibrium.*

Consider finally a customer who enters a shop where a pair of shoes is for sale for 1100 kroner, while the consumption utility is 1000 kroner. The consumer thought the price was 900 kroner and expected to buy a pair at that price. The gain-loss utility is the same as above.

- e) Show that the consumer will buy the shoes according to Köszegi and Rabin's theory, with separate reference points for money and shoes.

Solution *Here the consumption value is 0 if the consumer does not buy the shoes, and -100 if the consumer buys the shoes.*

*There is no gain/loss from shoes if they are bought as expected, but as the customer expected to pay 900, there is a loss of 200, relative to the reference point. Buying the shoes thus yields a total utility $-500 = -100 + 2 * (-200)$.*

Not buying the shoes is a +900 gain in money, but a 1000 loss in the shoe-dimension. The net loss/gain is thus $900 - 2(1000) = -1100$.

Thus the utility of buying is -500, the utility of not buying is -1100. Thus the consumer buys the shoes.