ECON4260 Behavioral Economics

2nd lecture

Cumulative Prospect Theory

Expected utility

- This is a theory for ranking lotteries
  - Can be seen as normative: This is how I wish my preferences looked like
  - Or descriptive: This is how people actually choose between lotteries
- A little note showing some basic ideas of a proof will be provided, but I will here only:
  - Explain what expected utility is
  - Discuss the basic axiom – the independence axiom
  - The note try to present the basic intuition on why expected utility follows from this axiom

What is a lottery?

- A list of possible outcome: x1,x2,x3…,xn
- Associated probabilities p1,p2,…pn
  - Probabilities add to one.
- Example1: 100 kroner with 40% probability and -20 koner with 60% probability
- A lottery can have only one outcome:
  - 70 kroner with 100% probability – that is 70 kroner for sure.

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Notation

- $(x_1,p_1;\dots;x_n,p_n)$ means
  - x_1 with probability p_1;
  - … and
  - x_n with probability p_n
- Null outcomes not listed:
  - (x_1,p_1) means x_1 with probability p_1 and 0 with probability $1-p_1$
- (x) means x with certainty.

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As usual – a utility function can represent reasonable preferences

- Consider lotteries with only three outcomes
  - x_1, x_2, x_3
- Now we may simplify
  - Write $(x_1, p_1; x_2, p_2; x_3, p_3)$, as (p_1, p_2, p_3)
  - Since $p_3 = 1 - p_1 - p_2$ we only need to state (p_1, p_2)
- Preferences over these lotteries can be represented by the utility function $\mathrm{U}(p_1, p_2)$
- Expected utility claim that the utility function has a particular form, it is **linear in probabilities**
- $U = \sum_{i=1}^{n} p_i u(x_i)$

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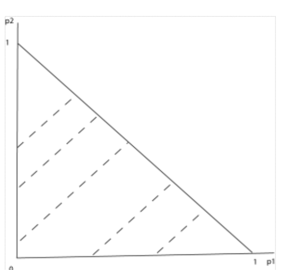
Expected utility:

Linear & parallel indifference curves



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Positive linear transforms - we may choose u(0)=0

- Consider two utility functions u and v such that
  - $v(x)=au(x)+b, \quad a>0$
- They yield the same ranking of lotteries:

$E\, v(x) = \sum p_i v(x_i)$

$= \sum p_i a u(x_i) + \sum p_i b = a\, Eu(x) + b$

- Maximizing Ev isequivalent to maximizing Eu

- Start with any $u(x)$ and use $v(x)=u(x)-u(0)$
  - Note that $v(0)=0$

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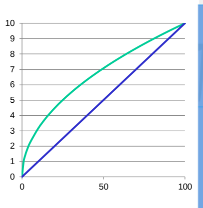
Risk aversion

- $u(x) = \sqrt{x}$

- Two lotteries with the same expectation
  - Lottery A: 0 or 100 Kr equal probability
  - Lottery B: 50 Kr

- Expected utility – read off from 50
  - A: the blue line
  - B: The green
- Risk aversion: Concave utility function.



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Independence Axiom

- Consider a lottery, L_X, where you get something, X, with probability p and 0 otherwise (probability 1-p)

- Suppose that there are two lotteries, call them A and B that are equally good: A ~ B
  - Now it will not matter if X is lottery A or B
  - That is $L_A \sim L_B$

- Why is this called independence?
  - The ranking of A and B is independent of context. If they are equally good when they stand alone they are equally good inside a lottery.

The independence axiom in action

- Consider the lotteries
  - A: 3000 for sure
  - B: 4000 with 80% probability
  - C: 3000 with 25% probability
  - D: 4000 with 20% probability
- If A is better than B, then C is better than D

- Why?
  - Let L be the lottery X with 25% probability and 0 otherwise
  - If X=A we get C
  - If X=B we get D

A theorem proven by von Neuman and Morgenstern (1944)

- Take the independence axiom
- Add continuity:
  if B(est) > x > W(orst) then there is a probability p such that (B,p;W,1-p) ~ (x)
- Standard assumptions like complete and transitive.

- It follows that lotteries should be ranked according to Expected utility
  Max $\sum p_i u(x_i)$

- In the following we will focus on alternative theories
  - And the evidence for these

Prospect theory

- Loss and gains
  - Value v(x-r) rather than utility u(x) where r is a reference point.
- Decisions weights replace probabilities
  Max $\sum \pi_i v(x_i - r)$
  (Replaces Max $\sum p_i u(x_i)$)

Evidence; Decision weights

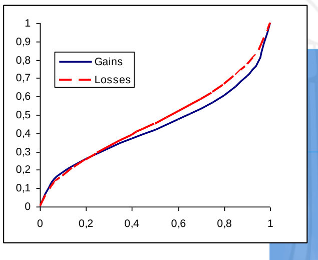
- Problem 3
  - A: (4 000, 0.80) or B: (3 000)
  - N=95 [20] [80]*
- Problem 4
  - C: (4 000, 0.20) or D: (3 000, 0.25)
  - N=95 [65]* [35]
- Violates expected utility
  - B better than A : u(3000) > 0.8 u(4000)
  - C better than D: 0.25u(3000) > 0.20 u(4000)
- Perception is relative:
  - 100% is more different from 95% than 25% is from 20%

Suggested approximation (See Benartzi and Thaler, 1995)

$$w(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{1/\gamma}}$$

$\gamma = 0.61$ for gains

$\gamma = 0.69$ for losses



Lotto

- 50% of the money that people spend on Lotto is paid out as winning prices
- Stylized:
  - Spend 10 kroner
  - Win 1 million kroner with probability 1 to 200 000
- Would a risk avers expected utility maximizer play Lotto?
  - Is Lotto participation a challenge to expected utility?
- Can prospect theory explain why people participate in Lotto?
- What is maximum willingness to pay for this winning prospect, for an
  - Risk avers expected utility maximizer?
  - A person acting acording to prospect theory?

Suggested answer

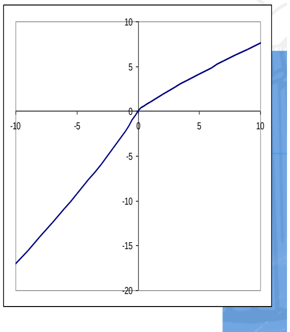
- A risk neutral expected utility maximizer will value the winning prospect to the expected value
  - 1 million kroner* (1/200 000) = 5 kroner
  - WTP for a risk avers person < 5 kroner
- Prospect theory
  - π = w(1/200 000) ≈ 0.0002
  - v(1 million) = (1 000 000)^0.88 ≈ 190 000
  - WTP = x where: 2.25(x)^0.88 ≈ 190 000 * 0,0002
  - Solution: WTP ≈ 27.5 kroner
  - Would buy Lotto even with only 2 kroner expected value for each 10 kroner spent. (5 kroner/2.7)
- Some people do NOT buy Lotto tickets
  - Is that a challenge to CPT?

The reference point

- Problem 11: In addition to whatever you own, you have been given 1 000. You are now asked to choose between:
  - A: (1 000, 0.50) or B: (500)
  - N=95 [16] [84]*
- Problem 12: In addition to whatever you own, you have been given 2 000. You are now asked to choose between:
  - A: (-1 000, 0.50) or B: (-500)
  - N=95 [69]* [31]
- Both equivalent according to EU, but the initial instruction affect the reference point.

The value function (see Benartzi and Thaler, 1995)

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$



- $\alpha = \beta = 0.88$
- $\lambda = 2.25$

Why not make the distinction of losses and gains in expected utility?

- A person participate in a lottery (-1000,50%)
  - If he loses his budget set will be
    - All consumption bundles such that $\sum_{i=1}^{n} x_i p_i \leq W - 1000$
    - W-1000 > 0
  - If he not lose his budget set will be
    - All consumption bundles such that $\sum_{i=1}^{n} x_i p_i \leq W$
  - Indirect utility u(W) or u(W-1000)
- Standard economics see lotteries as adding uncertainty to overall income/wealth
- We derive utility from commodities not money
- This is not an implication of the independence axiom

Value function
Reflection effect

- Problem 3
  - A: (4 000, 0.80) or B: (3 000)
  - N=95 [20] [80]*
- Problem 3'
  - A: (-4 000, 0.80) or B: (-3 000)
  - N=95 [92]* [8]
- Ranking reverses with different sign (Table 1)
- Concave (risk aversion) for gains and
- Convex (risk lover) for losses

Isolation Effect
(recall the independence axiom)

"In order to simplify the choice between alternatives, people often disregard components that the alternatives share and focus on the components that distinguishes them"

- Problem 10:

  Consider the two-stage game. The first stage is (2. stage, 0.25; 0, 0.75) (proceed to stage to with 25% probability. If you reach the second stage you have the choice between A: (4000, 0.80) and B (3000) [78%]. Your choice must be made before the game starts.
- The choice in 10 is equivalent to.

  A': (4000, 0.20) [65%] and B': (3000,0.25)

The editing phase
Finding the reference point

- Combination
  (200,0.25,200,0.25) =(200,0.5)
- Segregation
  (300,0.8;200,0.2)=200 + (100,0.8)
- Cancellation
  (200,0.2;100,0.5;-50,0.3) vv (200,0.2;150,0.5;-100,0.3)
  Can be seen as a choice between
  (100,0.5;-50,0.3) vv (150,0.5;-100,0.3)
- Simplifications
  - (500,0.2 ; 99,0.49) dominates (500,0.15; 101,0.51) if the last part is simplified to (… ; 100,0.50)

Stochastic dominance

Lottery A (58%)	White	red	green	yellow
Probability %	90	6	1	3
Price	0	45	30	-15

Lottery B (42%)	White	red	green	yellow
Probability %	90	7	1	2
Price	0	45	-10	-15

	White	red	green	blue	yellow
Prob. %	90	6	1	1	2
Lottery C (A)	0	45	30	-15	-15
Lottery D (B)	0	45	45	-10	-15

The status of cumulative prospect theory (See Starmer 2000)

- Explains data much better than alternative theories.
  - Rank dependent utility, does a fair job but not as good
- Starmer claim a limited impact on economic theory
  - But loss aversion is increasingly referred to
- But, some very interesting application
  - We will use it to understand equity return
- See Camerer 2000 for other examples.

Rabin's Theorem
A teaser

- How many of you would participate in the following lotteries (the alternative is (0)).
  - A: (-100, 33 %, +100, 67 %)
  - B: (-100, 45 %, +100, 55 %)
  - C: (-100, 85 %, +10 billions, 15%)
  - D: (-100, 50 %, +10 billions, 50%)
- Would changes in wealth (±10 000 kroner) affect your preferences?

Main difference: CPT - EU

- Loss aversion
  - Marginal utility twice as large for losses compared to gains
  - Requires an editing phase
- Decision weights
  - 100% is distinctively different from 99%
  - 0.1% is also distinctively different from 0%
  - 49% is about the same as 50% (also: *Simplifications*)
- Reflection
  - Risk seeking for losses
  - Risk aversion form gains.
  - Most risk avers when both losses and gains.

Decision weights

- Suggested by Allais (1953).
- Originally a function of probability
  $\pi_i = f(p_i)$
- This formulation violates stochastic dominance and are difficult to generalize to lotteries with many outcomes ($p_i \rightarrow 0$)
- The standard is thus to use cumulative prospect theory

Violation of Stochastic dominance

- An urn contains 500 balls, numbered: 1,2,3,…
- Which of the following lottery do you prefer?
  - A: You win 1000 kroner for sure
  - B: You win (1000 –0.01 x) kroner, where x is the ball number.
- Prospect theory yields:
  - A: v(1000)
  - B: V(999,99) w(0.002)+ V(999.98) w(0.002)+ …
  - \> V(995) (500 w(0.002))
  - 500 w(0.002) is much larger than 1
- Prospect theory will violate stochastic dominance in some such cases.

Rank dependent weights

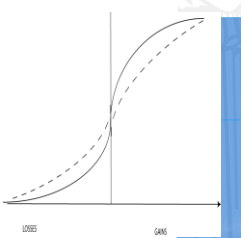
- Order the outcome such that

  $x_1 > x_2 > \ldots > x_k > 0 > x_{k+1} > \ldots > x_n$
- Decision weights for gains

$$\pi_j = w^+\left(\sum_{i=1}^{j} p_i\right) - w^+\left(\sum_{i=1}^{j-1} p_i\right) \text{for all } j \le k$$

- Decision weights for losses

$$\pi_j = w^-\left(\sum_{i=j}^{n} p_i\right) - w^-\left(\sum_{i=j+1}^{n} p_i\right) \text{for all } j > k$$



Why rank dependence avoids problems of stochastic dominance

- Reconsider the urn with numbered balls
- Prospect theory yields:
  - A: v(1000)
  - B: V(999.99) w(0.002)

    \+ V(999.98) [w(0.004)- w(0.002)]

    \+ V(999.97) [w(0.006)- w(0.004)]

    …

    < V(1000)
  - All weights now adds to one
- A person acting consistent with cumulative prospect theory will choose A over B.