ECON4271 - Distributive Justice and Economic Inequality

Lecture 3 - 16-03-16

- A short summary of the previous lectures.
- Interpretation of Arrow's impossibility result:
  - "It is the great merit of Bergson's 1938 paper to have carried the same [ordinalist] principle into the analysis of social welfare. The social welfare function [or welfare criterion] was to depend only on indifference maps; in other words, welfare judgments were to be based only on interpersonally observable behavior." (Arrow, 1963, p.109)
  - "If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial." (Arrow, 1963 , p.59)
  - "The potential usefulness of irrelevant alternatives is that they may permit empirically meaningful interpersonal comparisons," (Arrow, 1967, p.19)
  - The conclusion: IIA leads to truncating preference information and, if done, requires aggregating preferences based on utility information.

Sen's social welfare functional approach: introducing utility information in the social ranking.

- The framework
  - X is the set of all possible social states with $|X| \geq 3$.
  - The society consists of a finite set of individuals $N \equiv \{1, ..., n\}$ with preferences over X represented by a utility function U_i.
  - The n-tuple of utilities, a utility profile, is denoted by U. Its domain is $\mathcal{U}$.
  - For each $x \in X$, we denote by U_x the levels of utility achieved by each individual at alternative x.
  - Social preferences are denoted by R. Strict social preference relation is denoted P; indifference relation is denoted I. The set of all social preferences on X is $\mathcal{R}$.
  - Sen's problem is to define a mapping $F : \mathcal{D} \to \mathcal{R}$ that assigns social preferences R_U for each **UTILITY** profile in some subdomain $\mathcal{D} \in \mathcal{U}$, i.e. $R_U = F(U)$. The mapping F is called a social welfare function**AL**.

- One can study in detail the consequence of adding utility information.
- This framework includes the "Arrowian" one as a special case (apart from technicalities), i.e. when the utility information only allows to identify indifference curves.

- Let each preference relation R_i be represented by a utility function U_i. Possible informational assumptions:
  - **Ordinality and non-comparability**. Invariance to individual increasing transformations $V_i = \varphi_i \circ U_i$
  - **co-ordinality (common ordinal scale)**. Invariance to common increasing transformations $V_i = \varphi \circ U_i$
  - **co-cardinality (cardinal scale and full comparability).** Invariance to common positive affine transformation $V_i = a + bU_i$
  - **(cardinal scale and no comparability)**. Invariance to individual positive affine transformations $V_i = a_i + b_i U_i$
  - **(cardinal scale and unit comparability).** Invariance to common rescaling and individual change of origin $V_i = a_i + bU_i$
  - **(ratio-scale and full comparability)**. Invariance to common rescaling $V_i = bU_i$
  - **(ratio-scale and no comparability).** Invariance to individual rescaling $V_i = b_i U_i$
  - GRAPH with implications

- Formal welfarism
  - A **social welfare ordering** R^* is a ranking of profiles of utility levels.
  - A social welfare functional F satisfies **formal welfarism** if there exists a SWO R^* such that:

$$\forall u, v \in \mathcal{H}(X, \mathcal{D}), \forall x, y \in X, \forall U \in \mathcal{D},$$

$$\langle u = U_x \text{ and } v = U_y \rangle \Rightarrow \langle u R^* v \text{ iff } x R_U y \rangle .$$

  - **Pareto indifference**:

$$\forall U \in \mathcal{D}, \forall x, y \in X, x I_U y \text{ if } U_x = U_y.$$

  - **Binary independence:**

$$\forall V \in \mathcal{D}, \forall x, y \in X, x R_V y \text{ if } \exists U \in \mathcal{D} \text{ such that } V_x = U_x, V_y = U_y \text{ and } x R_U y.$$

  - **Theorem:** When $\mathcal{D} = \mathcal{U}$, formal welfarism is equivalent to jointly binary independence and Pareto indifference.