**Seminar 2:**

In the seminar we will cover exercises 1 and 2(iv)-(viii) from the first of the attached problem sets.

We will also cover exercises 1, 2, 3 and 4 from the second of the attached problem sets.

Seminar
ECON4271
Spring 2016

# Exercise 1: Primal and dual social welfare functions

(i) Give an account of the major difference between the primal and the dual social welfare functions, and present and discuss the basic conditions that have been used to justify these two families of social welfare functions.
(ii) What conditions ensure that respectively the primal and dual social welfare functions exhibit inequality aversion? What is the underlying normative principle that is used to justify inequality aversion?
(iii) Provide a sketch of the proof that justifies the condition associated with inequality aversion for the primal social welfare functions.

# Exercise 2: Primal and dual inequality measures

(i) A summary measure of (relative) inequality has to satisfy two basic conditions. What are these conditions? Explain their impact on our perception and interpretation of inequality.

(ii) The Lorenz curve is normally considered as the basic device for measuring inequality in a distribution function. Explain the content of the inequality concept when the Lorenz curve is used as a basis for comparing inequality between distribution functions.

(iii) Explain why poverty measures (for example the proportion of poor), top income shares or ratios of quantiles (for example the ratio between the 10 per cent richest and poorest person) do not justify the conditions of being measures of inequality.

(iv) Let F be a cumulative distribution function with mean $\mu$. Demonstrate why

$$(1) \qquad \frac{\int_0^\infty u(x)dF(x)}{u(\mu)}$$

cannot be considered as an appropriate measure of inequality.

(v) Demonstrate how the primal social welfare functions can be used as a basis for obtaining a family of inequality measures.

(vi) Provide a justification for why the dual social welfare functions, as opposed to the primal social welfare functions, can directly be used as a basis for defining a family of inequality measures.

(vii) Demonstrate whether or not primal and dual measures of inequality can be given an explicit expression in terms of the Lorenz curve.  When this not the case show that there is an implicit relationship between the measures of inequality and the Lorenz curve.

(viii) Demonstrate why and how social welfare functions can be decomposed with respect to the mean and the inequality of a distribution function (the size and the division of the cake).

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# Exercise 1: The relationship between Lorenz dominance criteria and measures of inequality

(i) Explain why first-degree Lorenz dominance is a valid criterion for ranking income distributions with respect to inequality.
(ii) What conditions does first-degree Lorenz dominance impose on the general families of primal and dual measures of inequality?
(iii) What conditions do respectively second-degree upward and downward Lorenz dominance impose on the general family of dual inequality measures?
(iv) What is the normative interpretation of second-degree upward and downward Lorenz dominance?

# Exercise 2: Axiomatic characterization of the Gini coefficient

(i) The family of dual inequality measures (also called rank-dependent measures of inequality) can either by justified as an axiomatic characterizations of an ordering of cumulative distribution functions (of income) or as an ordering of Lorenz curves. By adding an additional axiom to the Lorenz ordering axioms that characterize the family of dual inequality measures a complete axiomatic characterization of the Gini coefficient is obtained. Explain this axiom and discuss how it differs from the standard independence axiom on the orderings of Lorenz curves.
(ii) Assume that the five axioms that characterize the Gini coefficient as an ordering of Lorenz curves are imposed on an ordering of cumulative distribution functions (of income). In this case we will obtain a complete axiomatic characterization of one specific social welfare function. Specify this social welfare function and explain its basic normative property.

# Exercise 3: Alternative presentation of the Lorenz curve

(i) Explain how the information content of the Lorenz curve can be given an alternative presentation and interpretation which is consistent with the notion of inequality captured by the Lorenz curve.

(ii) The alternative representation of the Lorenz curve, called the scaled conditional mean curve (the M-curve), shows to possess several attractive properties. Explain these properties.

# Exercise 4: Choosing measures of inequality for empirical applications

(i) Provide a justification for why the moments of the M-curve can be considered to form a family of inequality measures that uniquely determines the M-curve and the Lorenz curve.

(ii) The standard approach in empirical work is to employ the Gini coefficient in combination with measures from the Atkinson family and/or the Theil family. Explain why it is hard to justify such an approach.

(iii) Provide arguments for why it might be attractive to employ the three first moments of the M-curve to summarize inequality exhibited by an income distribution.

(iv) Let $C_1$, $C_2$ and $C_3$ denote the three first moments of the M-curve. Show that these three measures can be given the following expressions in terms of the distribution function F:

$$C_1 = \frac{1}{\mu}\int_0^\infty F(x)\log F(x)dx\,,\quad C_2 = G = \frac{1}{\mu}\int_0^\infty F(x)(1-F(x))dx\,,\quad C_3 = \frac{1}{2\mu}\int F(x)(1-F^2(x))dx\,.$$

(v) Show that $C_1$, $C_2$ and $C_3$ can be given explicit expressions in terms of social welfare and that the associated social welfare functions can be decomposed with regard to the mean and (in)equality.