Consumption and saving

Optimization is forward-looking

Saving today is future consumption

Saving and borrowing is used to smooth the path of consumption

Consumption does not follow (annual) income (departure from the Keynesian model where $C_t = cY_t$ and $S_t = (1 - c)Y_t$).
Lifetime utility

Finite horizon (Romer)

\[
\sum_{t=1}^{T} u(c_t)
\]

Infinite horizon (Williamson)

\[
\sum_{t=0}^{\infty} \beta^t u(c_t)
\]

Period utility \( u(\cdot) \) is increasing and strictly concave: \( u' > 0, u'' < 0 \)

\( \beta \) is the discount factor, where \( \beta = 1/(1 + \rho) \) and \( \rho \) is the discount rate/time preference rate. A positive \( \rho \) reflects some impatience or time preference.
Intertemporal budget constraint

\[ a_{t+1} = (1 + r)(a_t + y_t - c_t) \]

where \( r \) is constant, \( a \) are assets, Finite horizon

\[ \sum_{t=1}^{T} c_t \leq a_0 + \sum_{t=1}^{T} y_t, \quad a_T = 0 \]

Infinite horizon

\[ \lim_{t \to \infty} \frac{a_t}{(1 + r)^t} = 0 \quad \text{(No-Ponzi-scheme)} \]

\[ \sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^t} \leq a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1 + r)^t} \]
Ponzi
Optimization

The optimization problem

$$\max_{\{c_t\}} \mathcal{L} = \sum_{t=0}^{T-1} \beta^t u(c_t) - \lambda \left[ a_0 + \sum_{t=0}^{T-1} \frac{y_t}{(1 + r)^t} - \sum_{t=0}^{T-1} \frac{c_t}{(1 + r)^t} \right]$$

First order conditions for $c_t$ and $c_{t+1}$:

$$u'(c_t) = \lambda$$

$$\beta (1 + r) u'(c_{t+1}) = \lambda$$
Euler equation

The intertemporal Euler equations are given by

\[
\beta (1 + r) u'(c_{t+1}) = u'(c_t), \quad t = 0, \ldots, T - 1
\]
\[
\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r
\]
\[
u'(c_{t+1}) = \frac{1 + \rho}{1 + r} u'(c_t)
\]

Consumption grows over time if \( r > \rho \). If \( r = \rho \) (and \( 1 + r = 1/\beta \)), then

\[
u'(c_t) = u'(c_{t+1})
\]
\[
c_t = c_{t+1}
\]
Permanent income

"It is better to have a permanent income than to be facinating" - Oscar Wilde

Optimal consumption in every period:

\[ c^* = \frac{1}{T} \left( a_0 + \sum_{t=0}^{T-1} \frac{y_t}{(1+r)^t} \right) = y^P \]

Saving will then be the difference between current income and permanent income (transitory income):

\[ s_t = y_t - c^* = y_t - y^P = y_t - \frac{1}{T} \sum_{t=0}^{T-1} \frac{y_t}{(1+r)^t} - \frac{1}{T} a_0 \]
Life-cycle model under uncertainty

In each period consumption is chosen so as to maximize

$$E_t \left[ \sum_{t=s}^{T-t} \beta^s u(c_{t+s}) \right]$$

given

$$a_{t+1} = (1 + r)(a_t + \tilde{y}_t - c_t)$$

where \( \tilde{y} \) is stochastic income (the source of uncertainty). This yields the stochastic Euler equation

$$E_t \left[ u'(c_{t+1}) \right] = \beta (1 + r) u'(c_t)$$
Hall’s random walk

Assuming quadratic utility (and constant $r = \rho = 0$)

$$E_t \left[ \sum_{t=0}^{T-1} c_t - \frac{a}{2} c_t^2 \right], \ a > 0$$

The stochastic Euler equation can the be reduced to

$$E_t (1 - ac_{t+1}) = 1 - ac_t$$
$$E_t c_{t+1} = c_t$$
$$c_{t+1} = c_t + \epsilon_{t+1}, \ E_t \epsilon_{t+1} = 0$$

Random walk (martingale) hypothesis: Only current consumption is required to predict future consumption.
Failure of the random walk hypothesis

"Excess sensitivity": current consumption is more closely tied to current income.

"Excess smoothness": consumption is not sufficiently sensitive to permanent innovations to income.

A large fraction of households consume all of their income in each period.

Possible explanations: precautionary saving and/or liquidity constraints, interest rates are not exogenous or constant.
Precautionary saving

Hall’s results based on quadratic utility gives certainty equivalence: consumption depends only expected future income and not uncertainty about that income.

For other functional forms of $u$ in which $u'''' > 0$

(the marginal utility is strictly convex), optimal $c_t$ also depends on the variability of the income stream. It follows that

$$E_t [u'(c_{t+1})] > u'[E_t (c_{t+1})]$$

Greater uncertainty about future income leads to reduced consumption today and more precautionary saving.
Liquidity constraints

Not all individuals are able to borrow as much as they would like at the same interest rate as they can save. Credit markets are imperfect: the borrowing rate exceeds the savings rate, there may be quantity constraints on borrowing, or collateral constraints (mortgages).

Impose a constraint in the model, for instance

$$a_t \geq 0, \ \forall t$$

When the constraint binds consumption will be equal to current income, and any increases in income will be fully consumed.
Buffer stock saving

In the US: Most households have little wealth. Consumption approximately tracks income, but small amounts of saving are held in the event of income falls or emergency spending. Most households exhibit buffer-stock saving behavior.

Assume impatient consumers ($\rho > r$)

Deaton (1991): general utility function and income process, but a liquidity constraint.

Carroll (1997): CRRA utility function and an income process with the possibility of zero income, $P [y_t = 0] > 0$.

Both models give buffer stock saving behavior. However, rule-of-thumb (non-optimizing behavior) may also explain observed behavior.
The interest rate and saving

Assume iso-elastic utility (and no uncertainty)

\[
\sum_{t=0}^{T-1} \beta^t \left( \frac{c_t^{1-\theta} - 1}{1 - \theta} \right)
\]

\[
a_{t+1} = (1 + r)(a_t + y_t - c_t)
\]

where \(\theta\) is the coefficient of relative risk aversion (\(1/\theta\) is then the elasticity of substitution between consumption at different dates). The corresponding Euler equation

\[
\beta^t c_t^{-\theta} = (1 + r) \beta^{t+1} c_{t+1}^{-\theta}
\]

\[
\frac{c_{t+1}}{c_t} = \left[ \beta (1 + r) \right]^{-\frac{1}{\theta}}
\]
The interest rate and saving, cont.

Higher interest rates have a positive effect on savings through the substitution effect (depending on the degree of intertemporal substitution).

The income effect will depend on net savings and previous accumulated assets.

The effect on aggregate savings will depend on the composition of households.

Tax policy affects substitution only (after-tax rate of return).

It matters whether the consumers consider the changes in the interest rate as transitory or permanent.