ECON4310 Answers to Exercise 2

Due 6/9 2010

1. (a) Since you are asked to discuss both the steady state and the balanced growth path, you can as well start with a graph like the one on slide 11 for Lecture 2. The determination of the steady state is illustrated by the point k_* in figure 1. A reduced savings rate means that for every level of k_t we get a reduced level (shift down) in k_{t+1}. The new steady state is k'_*. If the shift happens in period t, then $k_t = k_*$. The red curve shows how k_t gradually declines towards k'_*.
   $r = f'(k)$ and since $f'' < 0$, r increases as k goes down. Hence, r increases in a step-wise fashion period by period.

   (b) The steady state is characterized by

   $$sf(k_*) - \gamma k_* = 0$$

   Implicit differentiation of this equation yields

   $$\frac{dk_*}{ds} = -\frac{f(k_*)}{sf'(k_*) - \gamma} > 0$$

   The denominator can be signed in the following way: The curve depicted in figure 1 is

   $$k_{t+1} = [sf(k_t) + k_t]/(1 + \gamma)$$

   Its slope is $[sf'(k_t) + 1]/(1 + \gamma)$. At k_* this cuts the 45-degree line from above. This means that the slope there is less than one. Hence, $[sf'(k_*) + 1]/(1 + \gamma) < 1$ which is the same as $sf'(k_*) < \gamma$).

2. (a) The steady state, k_*, is determined by the condition that if $k_t = k_*$, then $k_{t+1} = k_*$. Inserting k_* for k_t and k_{t+1} in the accumulation equation yields the steady-state condition

   $$(\gamma + \delta)k_* = sf(k_*)$$

   A graph like the one in slide 9 in lecture 1 may be used to illustrate.

   (b) This is the same as asking whether the two curves in the graph always intersect for a strictly positive value of k. By assumption (the Inada conditions) $sf'(0) = \infty > \gamma + \delta$ and $sf'(\infty) = 0 < \gamma + \delta$ provided that $\gamma + \delta > 0$. Hence, in this case $sf(k) > (\gamma + \delta)$ when k is close to zero while $sf(k) < (\gamma + \delta)k$ for sufficiently high ks. However, if $\gamma + \delta < 0$, the two curves can only intersect at zero, and no steady state with $k > 0$ exists. (Is this case likely in practice?)

(c) Use the graph to show that if the economy starts away from the steady state, it moves closer to the steady state every period.

3. (a) From the production function

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}(A_{t+1}N_{t+1})^{\beta}L^{\delta}}{K_t^{\alpha}(A_tN_t)^{\beta}L^{\delta}}$$

Because $N_{t+1} = (1+n)N_t$ and $A_{t+1} = (1+g)A_t$, this expression can be reduced to

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha}[(1+g)(1+n)]^{\beta}$$

Try to insert the growth factor $1+\gamma$ for Y_{t+1}/Y_t and K_{t+1}/K_t in the equation above and see if it has a solution. Immediately we then get

$$1+\gamma = (1+\gamma)^{\alpha}[(1+g)(1+n)]^{\beta}$$

or multiplying with $(1+\gamma)^{-\alpha}$ on both sides

$$(1+\gamma)^{1-\alpha} = [(1+g)(1+n)]^{\beta}$$

Since the expressions on both sides obviously are positive, there exists a γ that solves the equation.

(b) Continuing from the previous equation we get the solution for $1+\gamma$ as

$$1+\gamma = [(1+g)(1+n)]^{\beta/(1-\alpha)} = [(1+g)(1+n)]^{(1-\alpha-\delta)/(1-\alpha)}$$

where in the last step we used that $\alpha+\beta+\delta = 1$.

(c) Positive growth in output per capita requires that $1+\gamma > 1+n$ or

$$[(1+g)(1+n)]^{\beta/(1-\alpha)} > 1+n$$

Multiply with $(1+n)^{-\beta/(1-\alpha)}$ on both sides:

$$(1+g)^{\beta/(1-\alpha)} > (1+n)^{1-\beta/(1-\alpha)} = (1+n)^{\delta/(1-\alpha)}$$

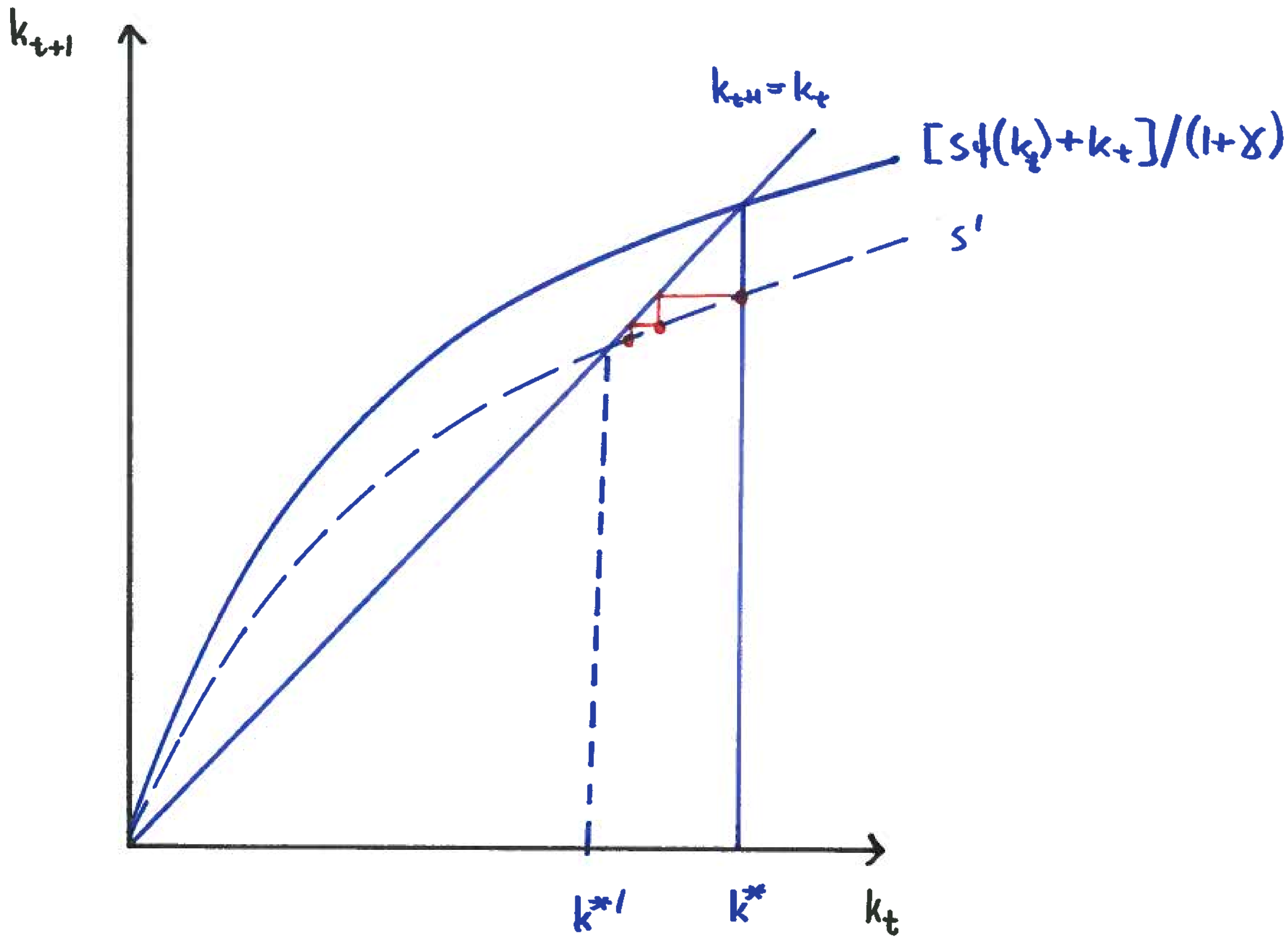
Invert

$$1+g > (1+n)^{\delta/\beta}$$

	δ	
n	0.20	0.05
0,005	0.17	0.04
0.020	0.66	0.17

Table 1: Required rate of technical progress. Per cent per year.

(d) Some keywords: i) Cobb-Douglas may exaggerate substitution possibilities; ii) Non-renewable resources; iii) Degrading of intensively used land


k_{t+1}
$k_{t+1}=k_t$
$[sf(k_t)+k_t]/(1+\gamma)$
s'
$k^{*\prime}$
k^*
k_t