ECON4310 Answers to Exercise 1

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1. First-order condition:
$$\frac{\mu c}{\ell} = w$$
Marginal rate of substitution equals relative price.

   (a) Ten per cent

   (b) Zero per cent

   Leisure:
$$\ell = \frac{\mu}{1+\mu}(w + y_0)/w$$
   Consumption:
$$c = \frac{1}{1+\mu}(w + y_0)$$
   Labor supply:
$$1 - \ell = \frac{1}{1+\mu}\left[1 - \mu\frac{y_0}{w}\right]$$
   Constant fractions of total income are spent on leisure and on consumption. The wage elasticity of labor supply is positive for $y_0 > 0$ and approaches zero when y_0 goes to zero.

2. Marginal productivity conditions:
$$r = (1-\alpha)z(n/k)^{\alpha}$$
$$w = \alpha z(n/k)^{-(1-\alpha)}$$
   Share of labor:
$$wn/y = [\alpha z(n/k)^{-(1-\alpha)}]n/[zk^{1-\alpha}n^{\alpha}] = \alpha$$

3. From the answer to question 1 the marginal rate of substitution is $\mu c/\ell$. Since k has to be equal to $\bar{k}$, the marginal rate of transformation is the same as the marginal productity of labor (see question 2), which is the same as the right hand side of (4) (remember $n = 1 - \ell$).

   Solutions:
$$\ell = \mu/(\alpha + \mu)$$
$$c = zk^{1-\alpha}[\alpha/(\alpha+\mu)]^{\alpha}$$

4. The initial impact of a ten per cent increase in productivity is a ten per cent increase in total income. This should raise the demand for leisure by ten per cent. But ten per cent higher productivity also makes leisure ten per cent more expensive, which makes the consumer reduce the share of total income that is spent on leisure by ten per cent. With log utility the income and the substituion effects of the productivity increase cancels.

5. There are no distortionary taxes. Consumers and producers face the same real wage. Hence, MRS = MRT. The steps that lead to (5) are the same as those that lead to the solution for ℓ in question 3. The only difference is that μ is replaced by $\mu(1-\gamma)$. A higher γ leads to less leisure and a higher labor supply. Higher government consumption lowers real income, but has no direct impact on the real wage. The consumer responds by reducing consumption of both goods proportionally. This means less leisure and more work.