ECON4310 Fall 2010 Answers to Exercise 5

Week 40

1 Government in steady state

1. The budget constraint of the consumer is

$$(1+\theta)C_1 + (1+r)^{-1}(1+\theta)C_2 = (1-\tau)W \tag{1}$$

(This can be derived from separate budget constraints for the two periods). With utility function

$$U = \ln C_1 + (1+\rho)^{-1}\ln C_2 \tag{2}$$

we get first order condition:

$$\frac{C_2}{C_1} = \frac{1+r}{1+\rho} \tag{3}$$

Use this to insert for C_2 in (1) and solve for C_1:

$$C_1 = \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{1-\tau}{1+\theta}\right)W \tag{4}$$

(Note the symmetric effect of τ and θ on C_1).

2.

$$s = \frac{W - \tau W - (1+\theta)C_1}{W} = 1 - \tau - (1+\theta)\left(\frac{1+\rho}{2+\rho}\right)\left(\frac{1-\tau}{1+\theta}\right) = \frac{1-\tau}{2+\rho} \tag{5}$$

The labor income tax is paid only in the first period. The consumer, who wants to smooth consumption, distributes the effect of the tax on the two periods by saving less. The consumption tax is paid in both periods. The need to save for retirement is unaffected by the level of the tax.

3. Let τ' be the total amount of taxes collected per efficiency unit of labor and let $\tilde{g}$ be government consumption per efficiency unit of labor. The law of motion for government debt per efficiency unit of labor is then

$$b_{t+1}(1+n)(1+g) = (1+r_t)b_t + \tilde{g} - \tau' \tag{6}$$

(This can be derived from the corresponding equation in levels). $b_{t+1} = b_t = b$ yields the stationarity condition

$$\tau' = \tilde{g} + [(1 + r_*) - (1 + n)(1 + g)]b \tag{7}$$

τ' is related to τ and θ by

$$\tau' = \tau w_* + \theta c_* \tag{8}$$

where c_* is total consumption (old and young) per efficiency unit of labor. (Nothing more is expected in an answer. However, it is possible to go on by relating c_* to w_*).

4. $$k_* + b = \frac{s}{(1 + n)(1 + g)}(1 - \alpha)k_*^\alpha \tag{9}$$

   On the r.h.s k_*^α is output per efficiency unit of labor and $(1 - \alpha)$ is labor's share of output. The product $(1 - \alpha)k_*^\alpha$ then represents labor income in the economy. The expressions derives from the first order conditions for the producers. s is the savings rate that was derived above. The product $s(1 - \alpha)k_*^\alpha$ is then the total savings of the young. These are the amounts available to invest in real capital and government bonds to be carried over to next period. By dividing with $(1 + n)(1 + g)$ we convert the r.h.s. to be measured relative to next periods efficiency units of labor, making the two sides of the equation commensurable. The equality results from the equilibrium condition for the capital market.

5. That the economy is dynamically efficient implies that $(1 + r_*) > (1 + n)(1 + g)$, which again means that a higher debt level necessitates a tax increase. You may want insert for s in (9) and rewrite it as

   $$k_* = \frac{1}{(1 + n)(1 + g)} \frac{1 - \tau}{2 + \rho}(1 - \alpha)k_*^\alpha - b \tag{10}$$

   Figure 1 illustrates the equilibrium. (Naturally we focus on the stable steady state). An increase in b reduces the supply of savings that are available for investment in real capital. Hence, it reduces k_*. If it is financed by an increase in the tax on labor income, it reduces total savings and, hence, the supply of funds for investment in real capital declines further. The latter affect is missing if the tax is on consumption.

2 Zero substitution elasticity in an overlapping generations model

1. The budget constraint for a young consumer is

   $$c_{y,t} + \frac{1}{1 + r_{t+1}}c_{o,t+1} = w_t \tag{1}$$

By setting $c_{o,t+1} = c_{y,t}$ in (1) we find

$$c_{y,t} = \frac{1 + r_{t+1}}{2 + r_{t+1}} w_t \tag{2}$$

The savings rate is

$$s_t = \frac{w_t - c_{y,t}}{w_t} = \frac{1}{2 + r_{t+1}} \tag{3}$$

Note that a higher interest rate reduces the need to save for old age. For later use, note that $a_{t+1} = s_t w_t$.

2. Producer behavior is standard with first-order conditions for capital

$$f'(k_t) = \alpha k_t^{\alpha - 1} = r_t \tag{4}$$

and for labor

$$f(k_t) - k_t f'(k_t) = (1 - \alpha) k_t^{\alpha} = w_t \tag{5}$$

(Make sure you know how to derive these from first principles).

3. The capital stock in period $t+1$ is determined by the savings of the young in period t

$$K_{t+1} = L_t a_{t+1} = L_t \frac{w_t}{2 + r_{t+1}} = L_t \frac{1 - \alpha}{2 + r_{t+1}} k_t^{\alpha} \tag{6}$$

Divide by L_t and you get

$$k_{t+1}(1 + n) = \frac{1}{2 + r_{t+1}} (1 - \alpha) k_t^{\alpha} \tag{7}$$

4. See Figure 2. From (4) the demand function for capital is

$$k_{t+1} = r_{t+1}^{-1/(1-\alpha)} \tag{8}$$

Hence,

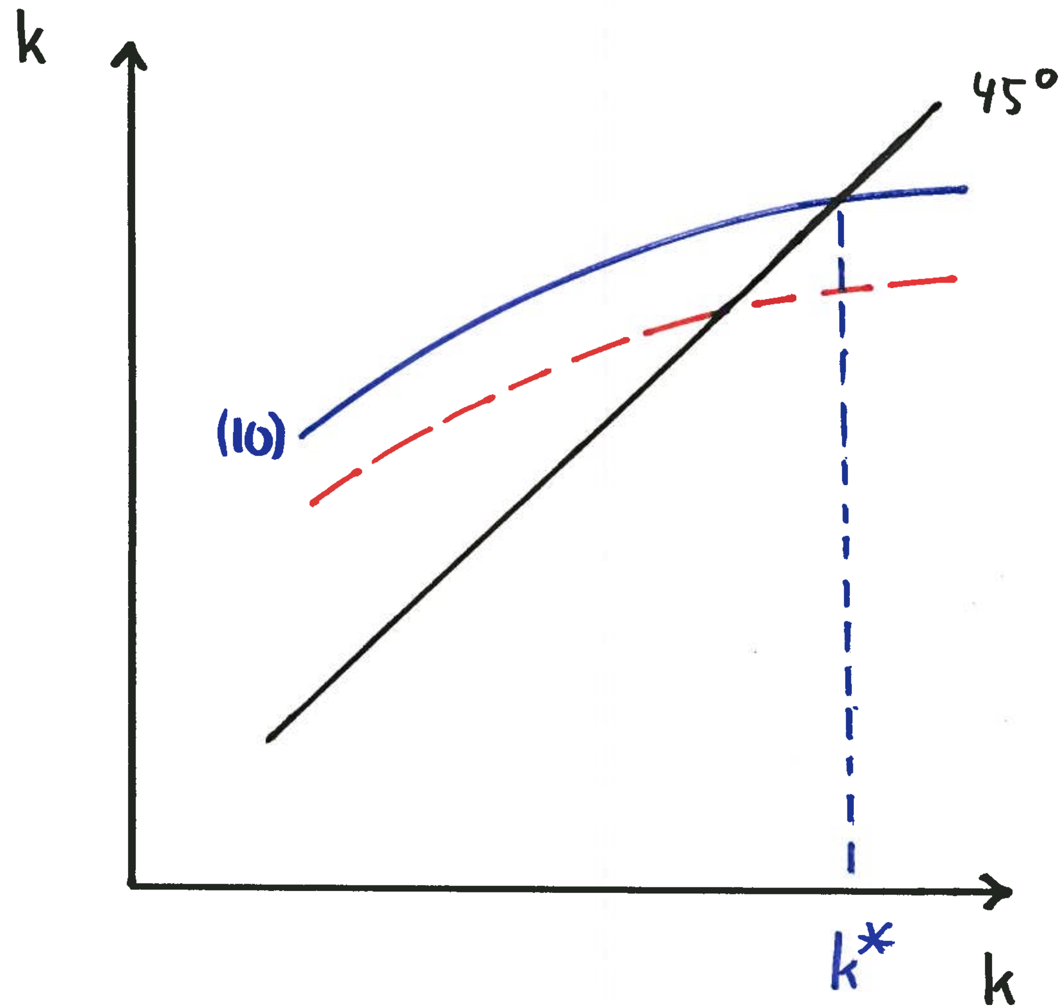
$$El_{r_{t+1}} k_{t+1} = -1/(1 - \alpha)$$

This is negative, but greater than one in absolute value. The elasticity of the supply curve is from (7):

$$El_{r_{t+1}} k_{t+1} = -r_{t+1}/(2 + r_{t+1})$$

This is also negative, but less than one in absolute value. Demand always reacts more to the interest rate than supply. Hence, the curves intersect only once. A higher interest rate reduces demand for capital more than supply.

5. See the shift in the graph. A higher k_t raises w_t and, hence, raises the supply of savings for every r_{t+1}.


k
45°
(10)
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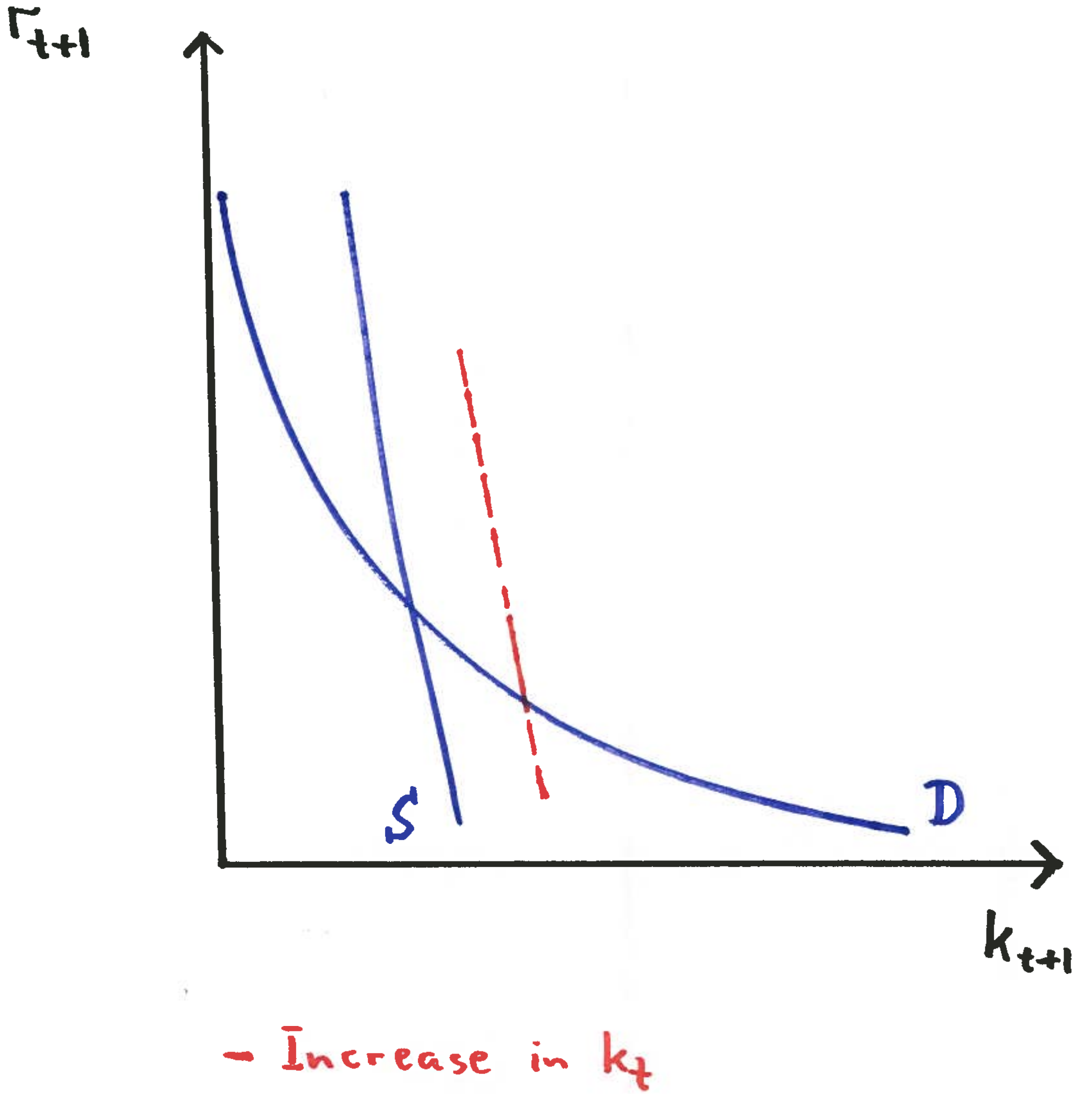

r t+1
k t+1
S
D
- Increase in k t