

# ECON4310 Fall 2011 Seminar 4

Week 44

## 1 A real business cycle model

Consider an economy with no underlying growth. Agents are price takers. The aggregate production function is

$$Y_t = e^{z_t} K_t^\alpha (N h_t)^{1-\alpha} \quad t = 1, 2, \dots, \infty \quad (1)$$

where  $Y_t$  is output in period  $t$ ,  $K_t$  the capital stock,  $N$  the number of workers,  $h_t$  the number of hours each of them works,  $0 < \alpha < 1$  a constant parameter and  $z_t$  is a stochastic variable which evolves according to

$$z_t = \rho z_{t-1} + \sigma \epsilon_t \quad t = 1, 2, \dots, \infty \quad (2)$$

Here  $\rho$  and  $\sigma$  are constant parameters and the  $\epsilon$ s are independently distributed stochastic variables with expectation 0 and variance 1.

Consumers in the economy maximize:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \quad (3)$$

where

$$u(c, 1 - h) = \ln c + \mu \ln(1 - h) \quad (4)$$

They accumulate wealth according to the budget equation

$$a_{t+1} = (1 + r_t)a_t + w_t h_t - c_t \quad t = 1, 2, \dots, \infty \quad (5)$$

where  $a_t$  is net assets the consumer carries from period  $t - 1$  to period  $t$ ,  $r_t$  the rate of return on these assets,  $w_t$  the hourly wage rate and  $c_t$  consumption.

1. How do you interpret  $z_t$ ? Explain what the values of  $\rho$  and  $\sigma$  tell us about how  $z_t$  evolves over time. What kind of real world events may  $\epsilon_t$  represent?
2. Derive the first-order conditions that characterize producer behavior in this economy. Assume that the rate of depreciation,  $\delta$ , is between zero and one.

3. How is the ultimate budget constraint for the consumer usually formulated in the present setting and how is it justified?
4. Derive the first-order conditions for the consumer's choice of consumption and leisure. Interpret these.
5. Use the first order conditions to discuss the relationship between the wages and the relative labor supplies in two successive periods .
6. With the equations above and those you have derived so far we have almost finished the formulation of a general equilibrium model. What equation(s) are still missing? Write them down.
7. Suppose there is a positive shock  $\epsilon_t$ . Based on the model, describe what you expect will happen to the time paths of output, labor supply, consumption, investment, the capital stock, the wage rate and the interest rate. Try to give intuitive explanations of the mechanisms that operate. Formal derivations are not expected.

## 2 Labor market in RBC models

*From the 2009 exam where it had weight 1/3*

Discuss the treatment of the labor market in real business cycle theories and how these theories explain fluctuations in employment.