Solution suggestions to Seminar 5, Econ4310

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1 Ex 1.Savings

1. Period by period budget constraints:

$$
\begin{aligned}
c_{y,t} &= 1 - a_{t+1} && (1)\\
c_{o,t+1} &= (\omega_{o,t+1}) + (1 + r_{t+1})a_{t+1} && (2)
\end{aligned}
$$

2. optimization problem faced by consumer:

$$
\begin{aligned}
\max Eu_t &= \max_{c_{y,t}} \ln(c_{y,t}) + E\ln(c_{o,t+1})\\
&= \max_{c_{y,t}} \ln(c_{y,t}) + \frac{1}{2}(\ln(c^H_{o,t+1}) + \ln(c^L_{o,t+1}))
\end{aligned}
$$

where $c^H_{o,t+1} = 1 + \Delta + (1 + r_{t+1})(1 - c_{y,t})$ and $c^L_{o,t+1} = 1 - \Delta + (1 + r_{t+1})(1 - c_{y,t})$
FOĊ:

$$
1/c_{y,t} = \frac{(1 + r_{t+1})}{2}\left(\frac{1}{1 + \Delta + (1 + r_{t+1})(1 - c_{y,t})} + \frac{1}{1 - \Delta + (1 + r_{t+1})(1 - c_{y,t})}\right)
$$

or in terms of a_{t+1}

$$
\frac{2}{1 - a_{t+1}} = \left(\frac{1}{\frac{1+\Delta}{(1+r_{t+1})} + a_{t+1}} + \frac{1}{\frac{1-\Delta}{(1+r_{t+1})} + a_{t+1}}\right) \quad (3)
$$

3. it is impossible for the old to borrow from the young (why?). And all young people are ex ante identical , so there will be no borrow/lending between them, which means that only the solution where savings be zero relavent. (what if there is money? or what if there is a social planner?)

4.Use FOC (3) , we have

$$
r_{t+1} = -\Delta^2.
$$

This is because the young will only lend to each other and there will be excessive supply of "savings" (no one wants to borrow), which leads to a decrease of interest rate.

5. In standard theory: higher uncertainty lead to higher savings. (a_{t+1} increase with Δ which can be seen as a measure of variance for $\omega_{o,t+1}$). precautionary savings increase for given interest rate.

In our case, as above suggestted, higher uncertain should lead to higher savings, but increased supply of "savings" brings down the interest rate as well. The two effects cancel against each other, so the equilibrium savings remain to be zero.

6. under qudratic funcion, the size of uncertainty does not matter (only expectation matters), so optimal savings will be zero and the equilibrium interest rate is always zero(you can try to verify this, by looking at the FOC with qudratic utility).

2 Ex 2. Tobin's q.

Tobin's q, $q(t)$, is the increase in the firm's market value as measured in time t dollars, resulting from the purchase of one more unit of capital at time t.

Marginal q matters, but it is easier to obtain the average q. Decreasing return to capital: marginal q is less than average q.

Let $C(.)$ be the adjusment costs of capital, then $1 + C'(I(t))$ gives all time t marginal costs of this time t investment. If $q(t) > 1 + C'(I(t))$, additional time t investment adds more to the firm's market value than the investment costs, and so the firm should go forward with more investment. If $q(t) < 1 + C'(I(t))$ the contemplated time t investment expenditure is too large and should be lessened. But if $q(t) = 1 + C'(I(t))$, investment expenditures are optimal, and a firm faces no incentive to revise its decision about I(t) either upward or downward.

3 Ex 3. Goverment deficit

From the exercise, we have

- $b = 0.6$
- $\gamma = 0.02$
- $r = i - f = 0.055 - 0.025 = 0.03$
- $\tau = 0.2$
- $g = 0.25$
- $i = 0.055$

1. To prevent the debt/GDP ratio to grow further, we need to have $b_{t+1} = 0.6$ as well, since

$$b_{t+1} = (b_t(1 + r) - \tau + g)/(1 + \gamma)$$

we have

$$0.6 = (b_t(1 + r) - \tau + g)/(1 + \gamma)$$

so

$$g^* = 0.6(0.02 - 0.03) + 0.2 = 0.194$$

which means that the goverment spending can be no more than 19.4 percent of GDP.

2.Real/nominal primary surplus is $20\% - 19.4\% = 0.6\%$ of real/nominal Gdp. the total nominal deficits will be nominal interest payment on debt plus the nominal primary deficit (where GDP_N denotes nominal GDP)

$$0.006 * GDP_N - 0.6 * GDP_N * i$$

and the nominal deficits relative to GDP equals to

$$0.006 - 0.6 * 0.055 = -0.027 = 2.7\%$$

3. Although the GDP growth is greater than the real interest rate, the primary deficit is too big to be compensated, so the debt/GDP ratio will continue to increase.

$$\begin{aligned} b_{t+1} &= (b_t(1 + r) - \tau + g)/(1 + \gamma) \\ &= (0.6(1 + 0.03) - 0.2 + 0.25)/(1 + 0.04) \\ &= 0.642 \end{aligned}$$

4. The constant interest rate assumption is not realastic. Running deficits means higher goverment spending and raises aggregate demand, which in turn may lead to increased interest rate.