Solution suggestions to Seminar 6, Econ4310

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1 Ex 1. Risky assets

1. budget constraints:

$$C_0 + A_a + A_b = Y_0$$

$$C_1 = A_a(1 + r_a) + A_b(1 + r_b) + Y_1$$

2. FOC:

$$\begin{aligned} u'(C_0) &= \frac{1}{1+\rho} E(u'(C_1)(1 + r_a)) \\ u'(C_0) &= \frac{1}{1+\rho} E(u'(C_1)(1 + r_b)) \end{aligned}$$

The interpretations. The left-hand side gives the marginal utility (loss) to giving up a small amount of consumption in period 1, and using it to buy some of the asset a or b. The right-hand side gives the discounted expected marginal utility (gain) at period 2 from having an increased amount of the asset: part of the utility gain comes from the expected return of the additional amount of the additional asset brings.

3. From FOC we have

$$E(u'(C_1)(1 + r_a)) = E(u'(C_1)(1 + r_b))$$

which implies that

$$E(u'(C_1))E(1 + r_a) + cov(u'(C_1), (1 + r_a)) = E(u'(C_1))E(1 + r_b) + cov(u'(C_1), (1 + r_b)) \quad (1)$$

when $E(1 + r_a) = E(1 + r_b)$, the equation simplified to

$$cov(u'(C_1), (1 + r_a)) = cov(u'(C_1), (1 + r_b)) \quad (2)$$

Using the budget constraint for second period, we have

$$cov(u'(A_a(1 + r_a) + A_b(1 + r_b) + Y_1), (1 + r_a)) = cov(u'(A_a(1 + r_a) + A_b(1 + r_b) + Y_1), (1 + r_b))$$

Assume in addition that $var(r_a) = var(r_b)$, if return of asset a covary positively with Y_1 while return of b covary negatively with Y_1, we will have more of asset b. Asset a is not very useful in so far as it does not allow for consumption smoothing. In other words, it does not offer a high return when times are bad. Conversely, Asset b provide insurance against bad times and are useful for consumption smoothing. .

4. denote the asset of food processing company as a, then this means that $cov(Y_1, r_a) > 0$. The farmer should prefer other assets which is negatively correlated with his income, when expected returns are same, to reduce overall risk of consumption next period. However, exactly quantiity should be decided by the FOCs.

5. when assest a is risk free, we have

$$u'(C_0) = \frac{(1+r_a)}{1+\rho} E(u'(C_1))$$

and (1) becomes

$$(r_a - E(r_b)) = cov(u'(C_1), (1+r_b))/E(u'(C_1))$$

Which is so called consumption CAPM. The covariance between an asset's return and consumption is known as its consumption beta.

6. Equilibrium conditions requires that markets clear (goods market, market for assets). Since every agent is identical, so individual will hold 0 unit of assets. The equilibrium returns will be the ones which give optimal solution $A_a = A_b = 0$.

$$(E(r_b) - r_a) = -cov(u'(Y_1), (1+r_b))/E(u'(Y_1))$$

2 Ex 2 Efficiency wages

1. Factors can be behind efficiency wages:

- Nutrition
- Imperfect monitoring
- Unobserved differences in ability that are positively correlated with reservation wages
- Fairness-considerations – high wage induce loyalty and effort

2. The profit function is (suppose that w is the real wage)

$$AF(e(w)L) - w * L \tag{3}$$

FOC gives us

$$\begin{aligned} AF'(e(w)L)e'(w) &= 1 & (4) \\ AF'(e(w)L)e(w) &= w & (5) \end{aligned}$$

which implies that (solow condition)

$$\frac{w}{e(w)} e'(w) = 1$$

the elasticity of effort w.r.t wage is one at the optimal solution. (Note this is also the solution of the minimum problem of cost per effective labor input: $\min_w (w/e(w))$)

3. supply side: $\bar{L}(w)$ with $\bar{L}'(w) > 0$.

Demand side consists of N firms, each will provide wage wage w^* such that

$$\frac{w}{e(w)} e'(w) = 1$$

and willing to employ L^* workers such that

$$AF'(e(w^*)L)e'(w^*) = 1$$

.

so if $\bar{L}(w^*) = NL^*$, full employment, if $\bar{L}(w^*) > NL^*$, involuntary unemployement, $\bar{L}(w^*) < NL^*$ not enough workers, competition between firms will bid up wages above w^* until there is equilibrium.

4. An increase in the technology A, leads to an increase in the demand of workers L^* but not change the offered wage rate. Labor supply will be unchanged.

5. the goverment increase tax on wage income, so the labor productivity function takes the form

$$e = e(w(1-\tau))$$

the FOC conditions are now

$$\begin{aligned} pAF'(e(w(1-\tau))L)e'(w(1-\tau))(1-\tau) &= 1 \qquad (6) \\ pAF'(e(w(1-\tau))L)e(w(1-\tau)) &= w \qquad (7) \end{aligned}$$

so the solow conditin is now

$$\frac{w^{\#}(1-\tau)}{e(w^{\#}(1-\tau))}e'(w^{\#}(1-\tau)) = 1$$

so the new optimal offered wage $w^{\#}$ will increase (if the original optimal offered wage is w^*, then $w^{\#} = w^*/(1-\tau)$). the optimal demaned labor will go down. On the other hand, labor supply may also shift.