The exam consists of three parts: A, B, and C. In the grading, part A is given 30 % weight, part B is given 20 % weight, and part C is given 50 % weight. The max page limit should be respected, but is not required (a shorter answer may be just as good).

Part A (20 %)

This part contains two short problems. You need to answer both to get full score.

a)

The policy rate of the central bank (the reserve rate) affects market rates via the banking sector through three channels: the bank lending channel, the bank capital channel, and the deposit channel. Explain each of these three channels and how they affect market rates.

b)

What is forward guidance and what is the "forward guidance puzzle"? Use the following model to explain the "forward guidance puzzle":

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa y_t$$
$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}).$$

Hint: solve the model forward.

Solution proposal:
a) The intuitions are as follows: Under the assumption that banks set lending rates as mark-ups over financing costs, the bank lending channel is the direct effect from the reserve rate (financing costs) to lending rates (higher costs, higher prices). The bank capital channel is an indirect effect where higher financing costs (higher reserve rate) make the constraints of the bank tighter. The bank responds by increasing lending rates more. The deposit channel is the direct effect that reserve rates have on deposit rates. Banks can finance themselves either by borrowing from the central bank at the reserve rate or by accepting deposits from consumers. A higher reserve rate makes it more attractive to use deposit as financing and they therefore increase the deposit rate to attract more funding.

b) Forward guidance is the promise of changing interest rates in the future. The forward guidance puzzle is the paradoxical effect in the standard New Keynesian model that changes

in future interest rates affect current inflation more than changes in the current interest rate. To see this, solve the model forward to get

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k y_{t+k}$$
$$y_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} (i_{t+k} - E_t\{\pi_{t+k+1}\})$$

A change in the interest rate in period $t+k$ affects output from t to $t+k$, and therefore has a more powerful effect on current inflation than a change in the current interest rate.

Part B (30 %)

In this problem, we are going to solve the firm problem with sticky prices. The firm problem is:

$$\max_{P_t^*, \{N_{t+k}, Y_{t+k}\}_{k=0}^{\infty}} E_t \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \left(P_t^*(i) Y_{t+k|t}(i) - W_{t+k} N_{t+k|t}(i) \right) \right\} \tag{1}$$

subject to

$$Y_{t+k}(i) = N_{t+k}(i)^{1-\alpha} \tag{2}$$
$$Y_{t+k|t}(i) = \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \tag{3}$$

where P_t^* is the optimal price in period t, $U_{c,t+k}$ is the marginal utility of consumption in period $t+k$, W is the aggregate nominal wage, N is hours, Y is aggregate output, P is the aggregate price, α is the curvature of the production function, and the notation $\cdot_{t+k|t}$ denotes the value of that variable in period $t+k$ for a firm that last reset its price in period t.

a)

The current period version of equation (3) is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \tag{4}$$

Interpret equation (4).

b)

Log-linearize equation (4).

c)

The Lagrangian for the firm problem is

$$\begin{aligned} L_t =& E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \Big[P_t^*(i) Y_{t+k|t}(i) - W_{t+k} N_{t+k|t}(i) \\ & - \zeta_{t+k|t}(i) \left(Y_{t+k|t}(i) - \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right) \\ & - \Psi_{t+k|t}(i) \left(Y_{t+k|t}(i) - N_{t+k|t}(i)^{1-\alpha} \right) \Big]. \end{aligned}$$

Find the first order conditions with respect to hours $N_t(i)$ and output $Y_t(i)$ and solve them for $\Psi_t(i)$ and $\zeta_t(i)$, respectively. What is $\Psi_t(i)$? What is $\zeta_t(i)$?

d)

One can combine the first order condition with respect to $P_t(i)$ with the two first order conditions we have found in (c) to get the price setting equation

$$\sum_{k=0}^{\infty} \underbrace{\theta^k}_{i} E_t \left\{ \underbrace{\beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} Y_{t+k|t}(i)}_{ii} \underbrace{\left[P_t^*(i) - \frac{\epsilon}{\epsilon - 1} \Psi_{t+k|t}(i) \right]}_{iii} \right\} = 0. \tag{5}$$

Interpret the price setting equation (5). The answer should explain the intuition for why each of the three terms (i, ii, and iii) are in the optimal price setting equation.

Solution proposal:
a) The equation describes how demand for good i depends on the price of good i relative to the aggregate price. If the price of good i is lower than the aggregate price, good i gets a larger share of aggregate demand. The opposite is true for a higher price. ϵ is the elasticity of substitution between goods and determines to what extent deviations in price from the aggregate price affects good i's demand. The higher ϵ is, the more sensitive is demand to prices.

b) $y_t(i) = -\epsilon(p_t(i) - p_t) + y_t$ (small letters denote log deviations).

c) First order conditions:

$$\Psi_t(i) = \frac{W_t}{(1-\alpha)N_t(i)^{-\alpha}} = \frac{W_t}{MPN_t(i)}$$
$$\zeta_t(i) = P_t^*(i) - \Psi_t(i)$$

Interpretations: $\Psi_t(i)$ is the shadow cost on the production function and equal to the nominal marginal cost (nominal wage divided by marginal product of labor). $\zeta_t(i)$ is the shadow cost of the demand equation and is equal to the nominal marginal profit (nominal price minus nominal marginal cost).

d) Firms set optimal prices as a mark-up over a weighted average of current and future expected marginal costs. Prices as a mark-up over marginal costs is described by (iii). The weights are determined by (i) how likely it is that the firm is still stuck with the same price (θ^k) and (ii) the relative importance of the future period (e.g. future has less weight (β^k) and periods with low consumption has a higher weight ($U_{c,t+k} \uparrow$)).

Part C (50 %)

We are going to use the New Keynesian model to see how the economy responds to a discount rate shock under an interest rate rule. We assume that the model is

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \tag{6}$$
$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\}\right) + z_t \tag{7}$$
$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t \tag{8}$$
$$z_t = \rho_z z_{t-1} + u_t^z \tag{9}$$
$$u_t^z \sim N(0, \sigma_z) \tag{10}$$
$$\kappa = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\left(\frac{1-\theta}{\theta}\right)(1-\beta\theta)\left(\frac{1-\alpha}{1-\alpha+\alpha\epsilon}\right) \tag{11}$$

a)

Insert (8) into (7) and guess that

$$\begin{aligned}\tilde{y}_t &= \psi_{yz} z_t \\ \pi_t &= \psi_{\pi z} z_t \\ E_t\{\tilde{y}_{t+1}\} &= \rho_z \psi_{yz} z_t \\ E_t\{\pi_{t+1}\} &= \rho_z \psi_{\pi z} z_t.\end{aligned}$$

Insert the guesses into the equation system and show that

$$\begin{aligned}\psi_{yz} &= (1-\beta\rho_z)\Lambda_z \\ \psi_{\pi z} &= \kappa\Lambda_z \\ \Lambda_z &= \frac{\sigma}{(1-\beta\rho_z)(\sigma(1-\rho_z)+\phi_y)+\kappa(\phi_\pi-\rho_z)}.\end{aligned}$$

b)

Figure 1 shows the impulse responses of the output gap, inflation, the nominal interest rate, and the discount rate shock to an initial shock to u_0^z of -0.5. We have used the following calibration: $\beta = 0.99$, $\sigma = 1$, $\phi = 5$, $\alpha = 0.25$, $\epsilon = 9$, $\theta = 0.75$, $\phi_\pi = 1.5$, $\phi_y = 0$, and $\rho_z = 0.5$. Explain why the negative discount rate shock affects the economy like it does in figure 1.

c)

We now change the calibration by reducing ϵ from 9 to 2. All other parameters are the same as in problem b. Figure 2 presents the impulse responses in the benchmark scenario and in the new calibration. Explain what ϵ is, how it changes the economic environment, and explain how agents alter their behavior. Next, explain why the aggregate responses of the economy change in the way presented in figure 2.

d)

We now change the calibration by reducing ϕ from 5 to 1. All other parameters are the same as in problem b. Figure 3 presents the impulse responses in the benchmark scenario and in the new calibration. Explain what ϕ is, how it changes the economic environment, and explain how agents alter their behavior. Next, explain why the aggregate responses of the economy change in the way presented in figure 3.

Figure 1: Impulse responses to a discount rate shock

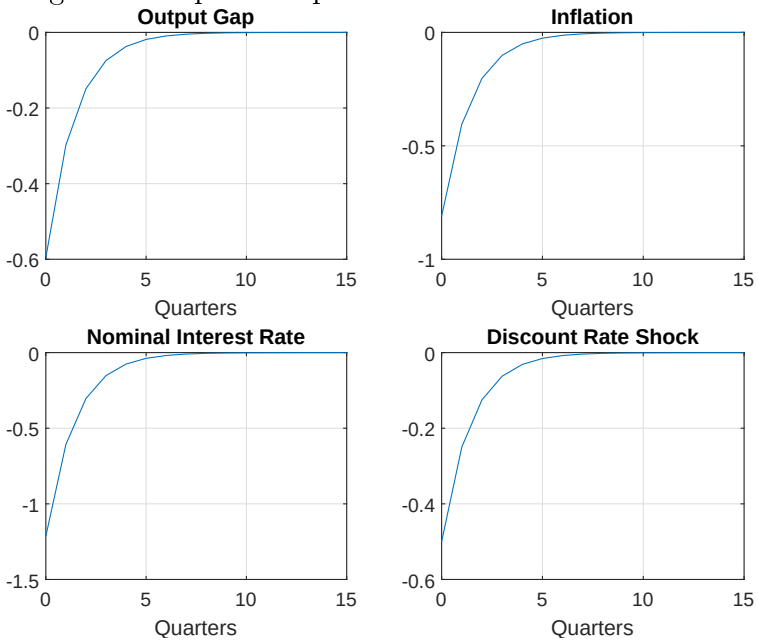
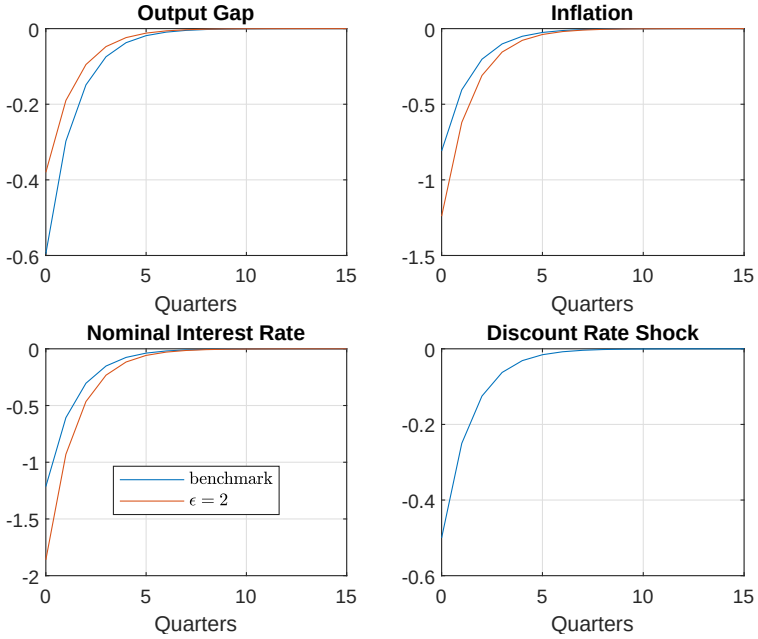


Figure 2: Impulse responses to a discount rate shock: variation in ϵ

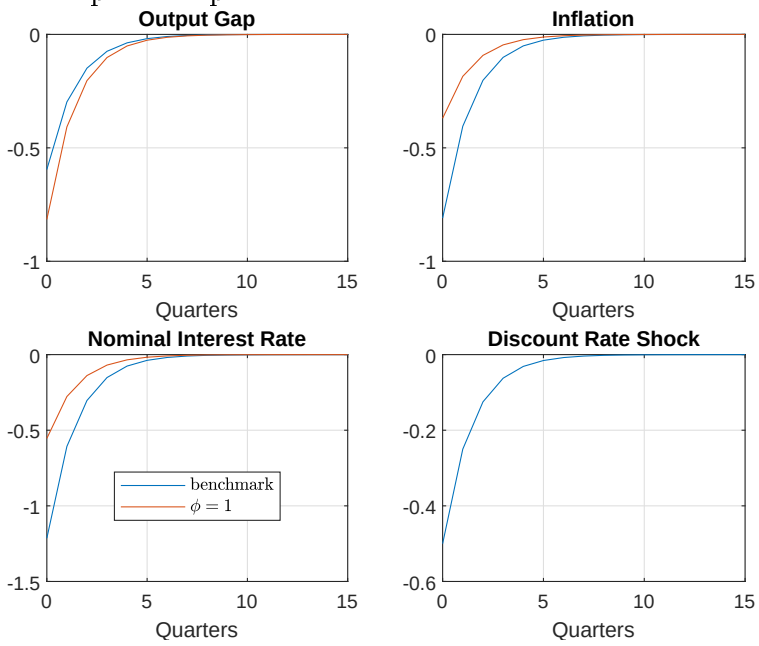


e)

We are now going to solve for the optimal simple rule: $\tilde{y}_t = \omega z_t$. Under the optimal simple rule, equation (8) can be rewritten as

$$\pi_t = \frac{\kappa}{1 - \beta\rho_z}\tilde{y}_t. \tag{12}$$

Figure 3: Impulse responses to a discount rate shock: variation in ϕ



Assume that the central bank minimizes the following loss function

$$L = \frac{1}{2}\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\lambda \tilde{y}_{t+k}^2 + \kappa \pi_{t+k}^2) \tag{13}$$

where $\lambda > 0$. Minimize equation (13) subject to equation (12). What is the optimal simple rule in this case? How can one implement this optimal simple rule with an interest rate rule of the form $i_t = \omega_i z_t$?

Solution proposal:
a) Solve by method of undetermined coefficients. Note that the equation system is slightly different from the term paper.

b) A negative discount rate shock is the same as a lower discounting, i.e. you want to shift consumption to future periods. Households therefore respond by reducing consumption. This reduction in aggregate demand also reduces inflation through the Phillips curve. The central bank follows a Taylor rule with a positive weight on inflation and therefore responds to the reduction in inflation by reducing the interest rate.

c) A reduction in ϵ implies that it is less costly to deviate from the aggregate price because all firms have more market power (or demand is less sensitive to prices). Firms are therefore

less concerned about having the wrong price in the future and become more willing to change prices in response to the negative discount rate shock. Hence, inflation is more sensitive to a change in output ($\kappa \uparrow$). The stronger inflation response (for a given output response) also implies that the central bank reduces the interest rate more (remember that the central bank only responds to inflation). The total response is less of an output response, more of an inflation response, and more of an interest rate response.

d) A lower ϕ implies that households are more sensitive to wage changes. The firms therefore do not need to reduce wages as much to reduce production. Hence, inflation is less sensitive to a change in output ($\kappa \downarrow$). Further, since the initial inflation response is lower, the central bank also reduces the interest rate less (again: the central bank only responds to inflation). The total response is more of an output response, less of an inflation response, and less of an interest rate response.

e) The solution to the optimal simple rule is $\pi_{t+k} = \tilde{y}_{t+k} = 0 \ \forall \ k \geq 0$, i.e. $\omega = 0$. To see this, solve the Phillips curve forward ($\pi_t = \frac{\kappa}{1-\beta\rho}\tilde{y}_t$) and either (i) solve by Lagrange and get the standard leaning against the wind condition under commitment rules and use the Phillips curve to replace for either variable in that condition to see that it must be 0 or (ii) replace for either π or $\tilde{y}$ in the loss function and see it immediately. This result is standard for demand shocks because the interest rate can move to perfectly offset the discount rate shock so that the shock does not affect inflation and output ("the divine coincidence"). The optimal rule for the interest rate is $i_t = \sigma z_t$.