The exam consists of four parts: A, B, C, and D. In the grading, part A has 10% weight, part B has 30% weight, part C has 30% weight, and part D has 30% weight. Allocate your time accordingly.

Part A: Short questions on Macroprudential policy (10%)

Section 3 in the "Regulation of Monetary policy" states:

> *The operational target of monetary policy shall be annual consumer price inflation of close to 2 percent over time. Inflation targeting shall be forward-looking and flexible so that it can contribute to high and stable output and employment and to counteracting the build-up of financial imbalances.*

What is the rationale for including the phrase "counteracting the build-up of financial imbalances" in a monetary policy guideline?

Part B: The firm problem (30%)

The firm problem with sticky prices is:

$$\max_{P_t^*,\{N_{t+k|t},Y_{t+k|t}\}_{k=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t}\right) \right\} \tag{1}$$

subject to

$$Y_{t+k|t} = N_{t+k|t}^{1-\alpha} \tag{2}$$

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{3}$$

where P_t^* is the optimal price in period t, $U_{c,t+k}$ is the marginal utility of consumption in period $t+k$, W is the aggregate nominal wage, N is hours, Y is aggregate output, P is the aggregate price, α is the curvature of the production function, and the notation $\cdot_{t+k|t}$ denotes the value of that variable in period $t+k$ for a firm that last reset its price in period t.

a) Interpret equation (3). Where does it come from? What is ε?

b) Log-linearize equation (3).

c) The Lagrangian for the firm problem is

$$\begin{aligned} L_t =& \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \Big[P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \\ & - \zeta_{t+k|t} \left(Y_{t+k|t} - \left(\frac{P_t^*}{P_{t+k|t}} \right)^{-\varepsilon} Y_{t+k} \right) - \Psi_{t+k|t} \left(Y_{t+k|t} - N_{t+k|t}^{1-\alpha} \right) \Big]. \end{aligned}$$

Find the first order conditions with respect to hours $N_{t+k|t}$ and output $Y_{t+k|t}$ and solve for $\Psi_{t+k|t}$ and $\zeta_{t+k|t}$, respectively. What is $\Psi_{t+k|t}$ and what is $\zeta_{t+k|t}$?

d) One can combine the first order condition with respect to P_t^* with the two first order conditions we have found in (c) to get the price setting equation

$$\sum_{k=0}^{\infty} \underbrace{\theta^k}_{(1)} \mathbb{E}_t \left\{ \underbrace{\beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} Y_{t+k|t}}_{(2)} \underbrace{\left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} \Psi_{t+k|t} \right]}_{(3)} \right\} = 0. \tag{4}$$

Interpret the price-setting equation (4). The answer should explain the terms (1), (2), and (3) in the optimal price setting equation.

Part C: Optimal monetary policy (30%)

We will use the New Keynesian model to see how the economy responds to a cost-push shock under optimal monetary policy. The model is

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa x_t + u_t \tag{5}$$

$$x_t = \mathbf{E}_t\{x_{t+1}\} - \eta\left(i_t - \mathbf{E}_t\{\pi_{t+1}\}\right) \tag{6}$$

$$u_t = \rho u_{t-1} + \varepsilon_t^u \tag{7}$$

$$\varepsilon_t^u \sim N(0, \sigma_u) \tag{8}$$

where x is the output gap, π is inflation, i is the nominal interest rate, and u is a cost-push shock. $\kappa > 0$, $\eta > 0$, $\beta \in [0, 1)$, $\rho \in [0, 1)$, and $\sigma_u > 0$ are parameters. Assume further that the central bank minimizes this loss function:

$$L_t = \frac{1}{2}\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [\lambda x_{t+k}^2 + \pi_{t+k}^2] \tag{9}$$

with $\lambda \geq 0$.

a) Solve for optimal monetary policy under discretion and show that the first-order conditions can be combined to

$$x_{t+k} = -\frac{\kappa}{\lambda}\pi_{t+k} \quad \text{for} \quad k \geq 0. \tag{10}$$

b) Show that π_t and x_t can be written as functions of the cost-push shocks:

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} u_t,$$

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda(1 - \beta\rho)} u_t.$$

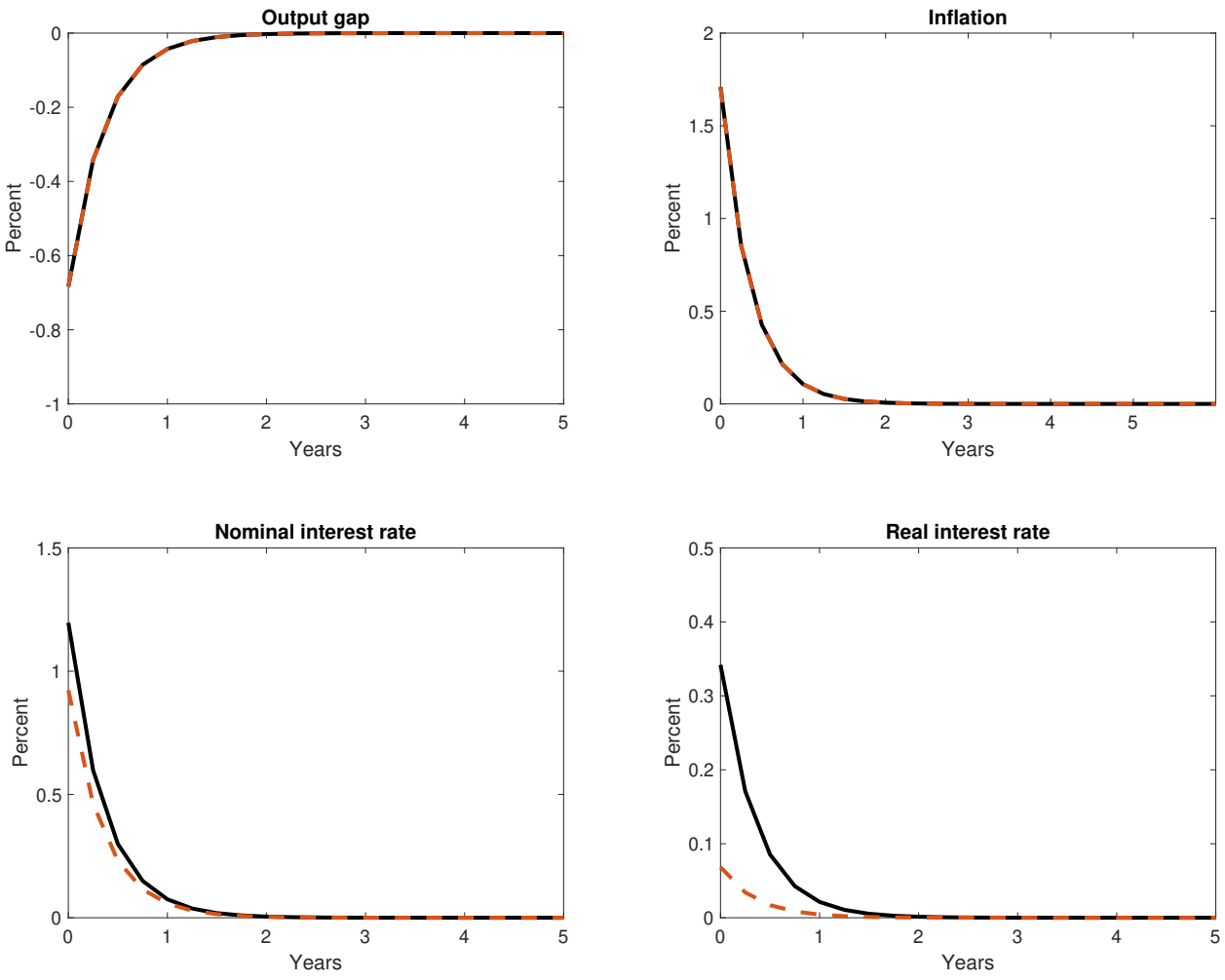


Figure 1: Impulse responses to a cost-push shock under two calibrations.

c) Figure 1 displays the response of the output gap, inflation, the nominal interest rate, and the real interest rate to a positive cost-push shock under discretion for two calibrations: one where $\eta = 1$ and one where $\eta = 5$. All other parameters are the same (κ, ρ, β, σ_u, λ). Why are the responses of output and inflation the same irrespective of the value of η? Which interest rate response (solid or dashed) corresponds to which calibration? Explain your answer.

Part D: Monetary policy in an open economy (30%)

In a model for a small, open economy the following three equations can be derived:

$$\pi = \pi^e + \left(\gamma_1 + \frac{\gamma_2}{\alpha_1 + \alpha_2}\right) y + u^{\text{open}} \tag{11}$$

$$\pi - \pi^* = -\frac{\lambda}{\gamma_1 + \frac{\gamma_2}{\alpha_1 + \alpha_2}} y, \tag{12}$$

$$y = -(\alpha_1 + \alpha_2)(r - \bar{r}), \tag{13}$$

where

$$u^{\text{open}} = u + \frac{\gamma_2}{\alpha_1 + \alpha_2}[\alpha_1((r^* - \rho) + e^e + z) - v], \tag{14}$$

$$\bar{r} = \rho + \frac{1}{\alpha_1 + \alpha_2} v + \frac{\alpha_2}{\alpha_1 + \alpha_2}((r^* - \rho) + e^e + z), \tag{15}$$

π is inflation, y is the output gap, π^e is expected inflation, π^* is the inflation target, v is a demand shock, u is an inflation shock, z is a risk-premium shock, and ρ, r^*, e^e, γ_1, γ_2, α_1, and α_2 are parameters.

a) Discuss the characteristics of a Central Bank's loss function in this model, and the condition for optimal monetary policy (12).

b) What determines the exchange rate in this model?

c) How should a central bank respond to the following shocks:

i. A negative demand shock?

ii. A negative risk premium shock?

d) What would be the optimal policy response to a negative demand shock if the authorities could use both fiscal and monetary policy?

Solution Proposal

Part A: The firm problem (10%)

The rationale is discussed in the lecture "Monetary policy in an open economy", on slide 21-33. The key rationale is that the central bank should counteract the build-up of financial imbalances because these may be a potential source for subsequent instability and recessions. By financial imbalances, the central bank typically thinks of debt and asset price developments. The financial imbalances typically affect the interest rate policy when the other targets (price growth and output) require a very low interest rate, such as in the aftermath of the financial crisis in 2008.

Part B: The firm problem (30%)

a) *The equation describes how demand for a good in period t + k depends on the price of that good P_t^* relative to the aggregate price P_{t+k}. If the price of good i is lower than the aggregate price, good i gets a larger share of aggregate demand. The opposite is true for a higher price. ε is the elasticity of substitution between goods and determines the extent to which relative price deviations affect demand. The higher ε is, the more sensitive demand is to prices.*

b) $y_{t+k|t} = -\varepsilon(p_{t+k|t} - p_{t+k}) + y_{t+k}$ *(small letters denote log deviations).*

c) *First order conditions:*

$$\Psi_t(i) = \frac{W_t}{(1-\alpha)N_t(i)^{-\alpha}} = \frac{W_t}{MPN_t(i)}$$
$$\zeta_t(i) = P_t^*(i) - \Psi_t(i)$$

$\Psi_t(i)$ *is the shadow cost on the production function and is equal to the nominal marginal cost (nominal wage divided by marginal product of labor).*

$\zeta_t(i)$ *is the shadow cost on the demand equation and is equal to the nominal marginal profit (nominal price minus nominal marginal cost).*

d) *Firms set optimal prices as a mark-up over a weighted average of current and future expected marginal costs. Prices as a mark-up over marginal costs is described by (3). The weights are determined by (1) how likely it is that the firm is still stuck with the same price (θ^k) and (2) the relative importance of the future period (e.g. future has less weight (β^k) and periods with low aggregate consumption has a higher weight ($U_{c,t+k} \uparrow$)).*

Part C: Optimal monetary policy (30%)

a) *Minimize the loss function subject to the Phillips curve, ignoring all future components. The problem is then*

$$\min_{x_t,\pi_t} \frac{1}{2}(\lambda x_t^2 + \pi_t^2 + F_t) - \gamma(\pi_t - \kappa x_t - f_t)$$

with the first-order conditions

$$\begin{aligned} \lambda x_t + \gamma\kappa &= 0 \\ \pi_t - \gamma &= 0. \end{aligned}$$

Combine these two to get (10).

b) *Use equation* (10) *to replace for* x_t *in the Phillips curve*

$$\pi_t = \frac{\lambda}{\lambda + \kappa^2}\left(\beta \mathbb{E}_t \pi_{t+1} + u_t\right).$$

Guess that $\pi_t = \psi u_t, \pi_{t+1} = \rho\psi u_t$ *and replace such that*

$$\psi u_t = \frac{\lambda}{\lambda + \kappa^2}\left(\beta\rho\psi u_t + u_t\right).$$

Divide through by u_t *and solve*

$$\psi = \frac{\lambda}{(1-\beta\rho)\lambda + \kappa^2}$$

which implies that

$$\pi_t = \frac{\lambda}{(1-\beta\rho)\lambda + \kappa^2} u_t, \quad x_t = -\frac{\kappa}{(1-\beta\rho)\lambda + \kappa^2} u_t.$$

c) *Note first that the solutions in problem (b) do not depend on* η*, hence, the output and inflation responses are the same irrespective of* η*. When the central bank follows optimal monetary policy under discretion, its only goal is to achieve the optimal trade-off between output and inflation. Changing* η *only affects how the central bank needs to change its interest rate to achieve this goal. Specifically, when* η *is higher, an interest rate increase (and thus a real interest rate increase) more strongly affects output and then inflation. Hence, when* η *is high (5), the only change is that the central bank needs to raise the interest rate less to achieve the same policy. The dashed line thus the interest rate response in the case with a high* η*.*

Part D: Monetary policy in an open economy (30%)

a) *Equation* (12) *is derived from minimizing a quadratic loss function given by* $L = \frac{1}{2}[(\pi - \pi^*)^2 + \lambda y^2]$. *This loss function means that the central bank's concern about output and inflation losses are symmetric (care equally much about positive and negative gaps), and that the relative weight of the targets is given by* λ. *The solution* (12) *states that the marginal loss of a recession* $-\lambda y$ *should equal to the marginal benefit of the recession in terms of lower prices* $(\gamma_1 + \frac{\gamma_2}{\alpha_1+\alpha_2})(\pi - \pi^*)$

b) *The exchange rate in this model is determined by uncovered interest parity, in which a shift in the domestic interest rate relative to the foreign interest rate affects the exchange rate. Specifically, a surprise increase in the interest rate today leads to an exchange rate appreciation today, followed by a gradual depreciation.*

c) *A negative demand shock* $v < 0$ *reduces output and increases prices. Output falls because demand contracts, while prices rise because the output contraction is followed by a real exchange rate depreciation, raising imported inflation. The central bank responds by lowering the interest rate somewhat. The total effect is higher prices, lower output, and a lower interest rate.*

A negative risk premium shock $z < 0$ *leads to an exchange rate appreciation, which reduces imported prices, and reduces net exports and thus output. The central bank responds to this reduction in output and inflation by reducing the interest rate, which somewhat dampens the appreciation of the exchange rate. The total effect is lower prices, higher output, and a lower interest rate.*

d) *If the authorities could use both fiscal and monetary policy, they could fully stabilize the negative demand shock by increasing government spending to stabilize output and raising the interest rate to stabilize the exchange rate.*