# Solution Proposal

## Part A: Short questions on financial stability (10%)

**a)** Financial stability means that the financial system is resilient to shocks, so that it is able to perform its key functions (intermediation, executing payments, redistributing risk) also in bad times. Policymakers care about financial stability because financial crises are associated with deep, long-lasting recessions: credit dries up, asset prices fall, and the government may have to deploy substantial fiscal resources to rescue the financial system. The costs of financial instability are largely borne by agents other than those who take on the risks (an externality), thereby motivating policy intervention.

**b)** Highly indebted households are vulnerable to shocks to income, interest rates, and house prices. When a negative shock hits, highly indebted households must cut consumption sharply to service their debt (or rebuild their balance sheets), thereby amplifying the downturn. Falling asset prices (e.g., housing) reduce collateral values, which further tightens households' access to credit. In addition, if households default, banks incur losses, which may force them to tighten credit supply to households and firms, further amplifying the downturn. High household debt thus both increases the probability of a crisis and amplifies the downturn if a crisis occurs.

## Part B: The household problem (30%)

**a)** The Lagrangian for the household problem is

$$\mathcal{L}_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} + \lambda_{t+k} \left( B_{t+k-1} + W_{t+k} N_{t+k} + D_{t+k} - P_{t+k} C_{t+k} - \frac{B_{t+k}}{1+i_{t+k}} \right) \right].$$

The first-order conditions with respect to $C_t$, $N_t$, and $B_t$ are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial C_t} &= C_t^{-\sigma} - \lambda_t P_t = 0 \quad \Rightarrow \quad \lambda_t = \frac{C_t^{-\sigma}}{P_t}, \\
\frac{\partial \mathcal{L}_t}{\partial N_t} &= -N_t^{\phi} + \lambda_t W_t = 0 \quad \Rightarrow \quad N_t^{\phi} = \lambda_t W_t, \\
\frac{\partial \mathcal{L}_t}{\partial B_t} &= -\frac{\lambda_t}{1+i_t} + \beta \mathbb{E}_t\{\lambda_{t+1}\} = 0 \quad \Rightarrow \quad \lambda_t = \beta(1+i_t)\mathbb{E}_t\{\lambda_{t+1}\}.
\end{aligned}$$

Combining the first two conditions gives the labor supply equation (4):

$$N_t^{\phi} = C_t^{-\sigma} \frac{W_t}{P_t}.$$

Substituting $\lambda_t = C_t^{-\sigma}/P_t$ into the third condition gives the Euler equation (5):

$$C_t^{-\sigma} = \beta(1 + i_t)\mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right\}.$$

**b)** Equation (4) is the intratemporal optimality condition (labor supply). The household works until the marginal disutility of an extra hour of work ($N_t^{\phi}$) equals the marginal utility of the consumption that this extra hour of work can buy ($C_t^{-\sigma} W_t/P_t$). The trade-off is between leisure and consumption within the period. Equation (5) is the intertemporal optimality condition (the Euler equation). The household saves until the marginal utility cost of giving up one unit of consumption today ($C_t^{-\sigma}$) equals the discounted expected marginal utility gain from consuming the gross real return on savings tomorrow. The trade-off is between consumption and saving today.

**c)** Take logs of the Euler equation under certainty equivalence (no Jensen's inequality & expectations pass through logs):

$$-\sigma c_t = \log \beta + \log(1 + i_t) - \sigma \mathbb{E}_t\{c_{t+1}\} - \mathbb{E}_t\{p_{t+1} - p_t\}.$$

Use the approximation $\log(1 + i_t) \approx i_t$, the definitions $\pi_{t+1} = p_{t+1} - p_t$ and $\rho = -\log \beta$, and rearrange:

$$-\sigma c_t = -\rho + i_t - \sigma \mathbb{E}_t\{c_{t+1}\} - \mathbb{E}_t\{\pi_{t+1}\}.$$

Dividing by $-\sigma$ gives

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho\right),$$

which is equation (6).

**d)** Equation (6) says that consumption growth is increasing in the gap between the real interest rate ($i_t - \mathbb{E}_t\{\pi_{t+1}\}$) and the household's discount rate $\rho$: when the real interest rate is high, saving is attractive, so the household postpones consumption. The parameter $1/\sigma$ is the elasticity of intertemporal substitution: a higher $\sigma$ (stronger preference for a smooth consumption path) implies that consumption reacts less to changes in the real interest rate.

## Part C: A monetary policy shock (30%)

**a)** Insert the Taylor rule (9) into the dynamic IS equation (8) and use the guesses. From the Phillips curve (7):

$$\psi_{\pi v} v_t = \beta\rho\psi_{\pi v} v_t + \kappa\psi_{yv} v_t \quad \Rightarrow \quad \psi_{\pi v} = \frac{\kappa}{1-\beta\rho}\psi_{yv}.$$

From the IS equation:

$$\psi_{yv} v_t = \rho\psi_{yv} v_t - \left(\phi_\pi \psi_{\pi v} v_t + v_t - \rho\psi_{\pi v} v_t\right) \quad \Rightarrow \quad (1-\rho)\psi_{yv} = -(\phi_\pi - \rho)\psi_{\pi v} - 1.$$

Substituting the expression for $\psi_{\pi v}$:

$$(1-\rho)\psi_{yv} = -(\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}\psi_{yv} - 1 \quad \Rightarrow \quad \psi_{yv}\left[(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)\right] = -(1-\beta\rho).$$

Hence

$$\psi_{yv} = -(1-\beta\rho)\Lambda_v, \qquad \psi_{\pi v} = -\kappa\Lambda_v, \qquad \Lambda_v = \frac{1}{(1-\beta\rho)(1-\rho) + \kappa(\phi_\pi - \rho)}.$$

Note that the Taylor principle ensures $\Lambda_v > 0$.

**b)** A positive monetary policy shock raises the nominal interest rate over and above what the Taylor rule prescribes, which translates into a higher real interest rate. The higher real interest rate makes saving more attractive, and households postpone consumption, so the output gap falls. Lower demand reduces firms' marginal costs (lower hours, lower wages), and firms lower prices (or raise them less), so inflation falls through the Phillips curve. The effects are amplified by expectations: because the shock is persistent, households expect lower future output and future inflation.

**c)** The real interest rate follows directly from the IS equation:

$$r_t = \mathbb{E}_t\{y_{t+1}\} - y_t = (\rho - 1)\psi_{yv} v_t = (1-\rho)(1-\beta\rho)\Lambda_v\, v_t > 0.$$

The nominal interest rate follows from the Taylor rule:

$$i_t = \phi_\pi \pi_t + v_t = \left(1 - \phi_\pi \kappa \Lambda_v\right) v_t.$$

The real interest rate must rise, and thus output must fall. The nominal interest rate, however, can fall if $\phi_\pi \kappa \Lambda_v > 1$: the shock lowers inflation, and the endogenous

response of the Taylor rule to lower inflation ($\phi_\pi \pi_t < 0$) can more than offset the direct effect of the shock. This is more likely when the shock is persistent (large $\rho$), the Phillips curve is steep (large $\kappa$), or the central bank responds strongly to inflation (large $\phi_\pi$). Generally, the stance of monetary policy in this model should be judged by the real interest rate rather than the nominal rate.

**d)** A higher $\rho$ increases $\Lambda_v$, so both $|\psi_{yv}|$ and $|\psi_{\pi v}|$ increase: the output gap and inflation fall by more on impact. The intuition is that the model is forward-looking. When the shock is more persistent, households expect the contraction to last longer. Expected lower future income lowers current consumption through the Euler equation (consumption depends on the entire path of future real rates), and expected lower future inflation raises the current real interest rate for a given nominal rate. Similarly, firms' pricing is forward-looking, so expected lower future marginal costs reduce inflation today. Hence, a more persistent monetary policy shock has larger immediate effects.

## Part D: Monetary policy in an open economy (30%)

**a)** Equation (13) is the open-economy Phillips curve: inflation depends on expected inflation, the output gap, the real exchange rate, and an inflation shock. The exchange rate enters directly because a depreciation (higher $e$) raises the price of imported goods, which feeds into consumer price inflation.

Equation (14) is the open-economy IS curve: demand falls when the real interest rate rises and rises when the exchange rate depreciates (higher exports, substitution away from imports).

Equation (15) is the (real) uncovered interest rate parity condition: if the domestic interest rate rises relative to the foreign rate, the exchange rate appreciates (lower $e$); a higher risk premium $z$ depreciates the currency.

Monetary policy affects inflation through three channels: (i) the direct demand channel (a higher $r$ reduces demand and thus inflation via the output gap); (ii) the exchange-rate demand channel (a higher $r$ appreciates the currency, which reduces net exports and demand); and (iii) the direct exchange-rate channel (the appreciation lowers import prices, which lowers inflation directly).

**b)** The central bank minimizes the loss function (16) subject to the Phillips curve. Define

$\tilde{\gamma} = \gamma_1 + \frac{\gamma_2}{\alpha_1+\alpha_2}$. The first-order condition with respect to $y$ is

$$(\pi - \pi^*)\frac{\partial \pi}{\partial y} + \lambda y = (\pi - \pi^*)\tilde{\gamma} + \lambda y = 0 \quad \Rightarrow \quad \pi - \pi^* = -\frac{\lambda}{\tilde{\gamma}} y,$$

which is equation (17). Optimal policy requires the inflation gap and the output gap to have opposite signs (or both to be zero): the marginal gain from bringing inflation closer to target must equal the marginal loss from moving output away from potential output. The effective slope of the Phillips curve is steeper in the open economy ($\tilde{\gamma} > \gamma_1$) because closing the output gap via the interest rate also moves the exchange rate, which affects inflation directly.

**c)** i. A positive risk premium shock raises $z$ in the uncovered interest parity condition, weakening the krone and pushing $e$ up. Because the depreciation raises imported inflation directly while also boosting net exports, it enters the reduced-form model both as a positive cost-push term in the open-economy Phillips curve and as an increase in the neutral real rate. This creates a trade-off, so an optimally behaving central bank cannot stabilize both targets. Delivering that outcome requires raising the real interest rate above the new, higher neutral rate, which in turn partly appreciates the krone and dampens some of the original shock.

ii. A positive inflation shock creates a trade-off between inflation and output stabilization. The central bank should raise the interest rate so that the output gap becomes negative (helped by the accompanying appreciation, which lowers import prices), thereby dampening the rise in inflation. Following (17), inflation ends up above target and output below potential, with the split determined by $\lambda$ and the effective Phillips curve slope $\tilde{\gamma}$. Because the exchange-rate channel makes the effective Phillips curve steeper, the required output loss for a given reduction in inflation is smaller than in a closed economy.

**d)** In the closed economy, a demand shock moves inflation and output in the same direction, so the interest-rate response that closes the output gap also stabilizes inflation, so there is no trade-off. In the open economy, this is no longer possible because the demand shock enters $u^{\text{open}}$: to neutralize a positive demand shock, the central bank must raise the interest rate, but the higher interest rate also appreciates the currency, which lowers inflation directly through import prices. If the central bank closes the output gap completely ($y = 0$), inflation falls below target because of the appreciation. If it instead completely stabilizes inflation, the output gap remains

positive.