**Welfare costs of business fluctuations**  

What are the costs of business fluctuations?

**Lucas(1987, 2003)**
- Representative agent framework, no unemployment
- consider costs of variability in consumption, without taking source of fluctuation into account
- show that costs of fluctuations are very small

**Here:**

Recessions:

Loss of output and consumption (low output and consumption and high leisure, households would want to work more)
Theory-based measure of the variations in aggregate economic efficiency:

The gap between
- the marginal product of labor and
- households consumption/leisure tradeoff

- Use representative agent – no unemployment, only variation in hours.

- Neglect costs of efficient fluctuations (consumption variability is costly even if it is caused by efficient fluctuations driven by technology shocks)

- Neglect costs of distortions of relative prices and wages (due to price and wage stickiness)
Inefficiency gap  \[ \text{gap}_t = \text{mrs}_t - \text{mpn}_t \]
**Firms**

Assume standard production function function (constant elasticity of output with respect to hours)

\[ Y = AN^\alpha \]

\[ MPN = \frac{dY}{dN} = A\alpha N^{\alpha-1} = \alpha \frac{Y}{N} \]

Lower case denotes logs:

\[ mpn_t = \log(\alpha) + y_t - n_t \]

\[ mpn_t = y_t - n_t \quad \text{(neglecting constant)} \]

Relate to markups in goods and labor markets

**Wage-taking firms and no labor adjustment costs**

**Price markup = price - nominal marginal cost**

\[ \mu_p^t = p_t - (w_t - mpn_t) \]

\[ = mpn_t - (w_t - p_t) \]

\[ = (y_t - n_t) - (w_t - p_t) \]

\[ = - s_t \]

\( s_t \) is log of wage share or log of real unit labor cost: \( S = WN/PY \)
Households

\[ U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi} \]

\[ MRS_t = -\frac{U_{Nt}}{U_{Ct}} = C_t^\sigma N_t^\varphi \]

MRS is the marginal rate of substitution between consumption and leisure, i.e. the marginal disutility of work, measured in terms of consumption.

Logs

\[ mrs_t = \sigma c_t + \varphi n_t - \bar{\xi}_t \]

(\bar{\xi}_t reflect changes in preferences, low frequency)

Boom:
N and C are high => \( U_{Nt} \) high and \( U_{Ct} \) low
=> \( MRS_t \) high

Wage markup
\[ \mu^w_t = (w_t - p_t) - mrs_t \]

(difference between wage and marginal disutility of work, expressed in terms of consumption)
Can estimate the inefficiency gap empirically by
\[
gap_t = \text{mrs}_t - \text{mpn}_t = \sigma c_t + \phi n_t - \xi_t - (y_t - n_t)
\]

Can also decompose the gap in two markups by adding and subtracting the real wage
\[
\begin{align*}
\text{gap}_t &= [\text{mrs}_t - (w_t - p_t)] - [\text{mrs}_t - (w_t - p_t)] \\
\text{gap}_t &= - \mu_w t - \mu_p t \\
&= - \{ \mu_w t + \mu_p t \}
\end{align*}
\]

In steady state
\[
\text{gap} = - (\mu_p + \mu_w) < 0
\]

Empirical decomposition in price markup and wage markup is only possible if the observed wage reflects the true cost of hiring an additional unit of labor (the wage is allocational)

This is not possible if there is wage smoothing so that the firm provides “insurance” to risk averse workers, i.e. that wage is smoothed over the cycle so as to reduce income fluctuations, while employment is nevertheless set at the efficient level.
Measurement
\[ \text{gap}_t = \text{mrs}_t - \text{mpn}_t = \sigma c_t + \phi n_t - \xi_t - (y_t - n_t) \]

Baseline case:
Inverse of Frisch wage elasticity of labor supply
\[ \phi = 1 \quad \text{(micro data: inverse 0.05-0.5 \quad \text{macro data: unity or higher})} \]

Coefficient of relative risk aversion (equal to inverse of intertemporal elasticity of substitution)
\[ \sigma = 1 \quad \text{(direct estimates: 3-10, \quad \text{balanced growth: unity})} \]

Bias against finding large costs of fluctuations

Allow for low-frequency changes in preferences \( \xi_t \), by fitting a third-order polynomial in time
\[ \text{Gap}_t = c_t + 2n_t - y_t \]
Changes in wage markup dominate variation in efficiency gap

Wage markup $c_t + n_t$ dominates

While price markup $y_t - n_t$ fluctuates less
The gap is interpreted as a measure of inefficient changes in markups.

However, we must explore an alternative interpretation, namely that labor supply preferences $\xi_t$ change, in which case the short run volatility in our gap measure does not indicate inefficiency.

Yet no support for alternative interpretation:

One can reject test of no-Granger causality of detrended GDP, nominal interest rate and yield spread on gap measure (i.e. gap depends on these variables, which it should not if the gap just indicate labor supply preferences).

Furthermore, monetary policy shock also affects gap.
Welfare costs

\[
GAP = \frac{MRS_t}{MPN_t} = \exp\{-\mu\} = 1 - \Phi < 1,
\]

where upper bars denote values along a constant gap path, and \( \mu \) is (minus) the steady-state value of our (log) gap variable. A second-order approximation of the period utility

\[
\Delta_t \equiv U(C_t, N_t) - U(\tilde{C}_t, \tilde{N}_t)
\]

\[
= \tilde{U}_{c,t}\tilde{C}_t\left(\tilde{c}_t + \frac{1 - \sigma}{2} \tilde{c}_t^2\right) + \tilde{U}_{n,t}\tilde{N}_t\left(\tilde{n}_t + \frac{1 + \varphi}{2} \tilde{n}_t^2\right),
\]

where the tildes denote log deviations from the underlying constant-gap path, that is, \( \bar{x}_t \equiv \log(X_t/X_t) \), and where \( \varphi \equiv - (\tilde{U}_{nn,t}\tilde{N}_t)/\tilde{U}_{n,t} \) and \( \sigma \equiv - \tilde{U}_{cc,t}\tilde{C}_t/\tilde{U}_{c,t} \).

Assume that

\[
\tilde{c}_t = \tilde{y}_t = \tilde{n}_t \quad \Rightarrow mpn_t = y_t - n_t = \text{constant}
\]
\[- \frac{\bar{U}_{n,t} \bar{N}_t}{\bar{U}_{c,t} \bar{C}_t} = 1 - \Phi.\]

Hence, we can rewrite the second-order approximation as

\[
\Delta_t = \bar{U}_{c,t} \bar{C}_t \left( \Phi \bar{y}_t - \frac{1}{2} [(\sigma + \varphi) - (1 - \Phi)(1 + \varphi)] \bar{y}_t^2 \right).
\]

(19)

**NB:** typo in (19): (1-Φ) should be Φ

\[
\text{gap}_t = (\sigma + \varphi) \bar{y}_t,
\]

where \(\text{gap}_t = \text{gap}_t - \text{gap}\). Using the previous expression to substitute for \(\bar{y}_t\) in equation (19), we obtain

\[
\frac{\Delta_t}{\bar{U}_{c,t} \bar{C}_t} = \frac{1}{\sigma + \varphi} (\Phi \text{gap}_t - \psi \text{gap}_t^2)
\]

\[
\equiv \omega(\text{gap}_t)
\]

(20)

where \(\psi = \frac{1}{2} \left[ 1 - \frac{(1 - \Phi)(1 + \varphi)}{\sigma + \varphi} \right] \).

Notice that \(\omega(\text{gap}_t)\) is the period efficiency loss or gain from the gap's deviations from its steady-state value, expressed as a percentage of the frictionless level of consumption \(\bar{C}_t\). The first term in the parentheses, the linear term,

**NB:** typo in expression for ψ: (1-Φ) should be Φ
of the average welfare cost over time analogous to those found in the literature, we take the unconditional expectation of equation (20) to obtain

\[ E\left( \frac{\Delta_t}{U_{c,t}C_t} \right) = -\frac{\psi}{\sigma + \varphi} \text{var}(\text{gap}_t), \tag{21} \]

where \( \text{var}(\text{gap}_t) \) is the variance of our gap measure. Notice that, as a result of the concavity of \( \omega \), the expected welfare effects of fluctuations in the gap variable are negative, that is, these fluctuations imply losses in expected welfare. This
Figure 6.—The Welfare Effects of Postwar U.S. Fluctuations

$\sigma=1$, $\varphi=1$, $\mu=0.50$

Welfare Fluctuations
of quarterly consumption

$\sigma=1$, $\varphi=5$, $\mu=0.50$

Welfare Fluctuations
of quarterly consumption

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### Table 4.—Welfare Costs of Fluctuations (1960–2004)

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\varphi = 1$</th>
<th>$\varphi = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010</td>
<td>0.043</td>
</tr>
<tr>
<td>5</td>
<td>0.027</td>
<td>0.059</td>
</tr>
<tr>
<td>10</td>
<td>0.049</td>
<td>0.080</td>
</tr>
</tbody>
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Note: Based on calibration $\mu = 0.5$. The data were filtered using a third-order polynomial in the time. Welfare computations cover the sample period 1960:1–2004:3.

### Table 5.—The Welfare Costs of Recession Episodes

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\varphi$</th>
<th>Percentage of One Year’s Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1970s</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-4.58</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>-6.18</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>-2.88</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-4.89</td>
</tr>
</tbody>
</table>

Note: See table 4.
Discussion

Focus in this paper:

Recessions involve lost output, and fluctuations involve inefficient tradeoff consumption – leisure

Other points:

- representative agent, no unemployment
- recession give increase utility of leisure
- neglect costs associated with effects on income distribution
- neglect other costs of unemployment
  - self confidence,
  - loss of human capital
  - social costs, etc
- costs associated with wage and price inflation when wages and prices are rigid
- costs of efficient fluctuations
- Assume that mean (gap) = 0
  - if downturns are more persistent, then the welfare loss would be much greater
  - neglect hysteresis (may be caused by loss of human capital)