Lecture 2: The intertemporal approach to the current account cont’d.

Open economy macroeconomics, Fall 2006
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What determines the world interest rate?

● A two-region world economy
  – Abandon assumption of small open economy
  – World is composed of two countries: Home and Foreign (*)
  – Ignore government spending

● Endowment economy
  – Global market clearing requires
    \[ Y_t + Y_t^* = C_t + C_t^* \]
    or
    \[ S_t + S_t^* = 0 \]
– Walras’s law: with two markets, only need to check that one of them clears to verify general equilibrium

– World market equilibrium in two-period model

\[ S_1(r) + S_1^*(r) = 0 \]
• How does saving depend on the interest rate?

  – Key concept: the intertemporal elasticity of substitution

    \[ \sigma(C') = -\frac{u'(C)}{C'u''(C')} \]

  – The sensitivity of intertemporal consumption allocations to interest rate changes depends on \( \sigma \)

  – Class of preferences with constant elasticity of intertemporal substitution (isoelastic preferences)

    \[ u(C) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \]

  – Special case: \( \sigma = 1 \rightarrow u(C) = \ln C \)
– Euler equation with isoelastic preferences

\[ C_1^{-\frac{1}{\sigma}} = \beta(1 + r)C_2^{-\frac{1}{\sigma}} \]

\[ C_2 = \beta^\sigma (1 + r)^\sigma C_1 \]

\[ \frac{C_2}{C_1} = \beta^\sigma (1 + r)^\sigma \]

– Define subjective rate of time-preference \( \delta \)

\[ \beta = \frac{1}{1 + \delta} \implies \delta = \frac{1 - \beta}{\beta} \]

– Euler equation becomes

\[ \frac{C_2}{C_1} = \left( \frac{1 + r}{1 + \delta} \right)^\sigma \]

– Consumption grows over time if the real interest rate exceeds the subjective rate of time-preference i.e., if \( r > \delta \)
• Derivation of the saving schedule (saving as a function of interest rates) with isoelastic preferences

– Substitute in for budget constraint $C_2 = (1 + r)(Y_1 - C_1) + Y_2$ in the Euler equation

\[ C_1 = \beta^{-\sigma}(1 + r)^{-\sigma}C_2 \]
\[ = \beta^{-\sigma}(1 + r)^{-\sigma}((1 + r)(Y_1 - C_1) + Y_2) \]

– Differentiate

\[ \frac{dC_1}{dr} = -\sigma(1 + r)^{-\sigma-1}\beta^{-\sigma}((1 + r)(Y_1 - C_1) + Y_2) \]
\[ +\beta^{-\sigma}(1 + r)^{-\sigma}(Y_1 - C_1) \]
\[ -\beta^{-\sigma}(1 + r)^{1-\sigma}\frac{dC_1}{dr} \]

– Collect terms and substitute in for $C_2$

\[ \frac{dC_1}{dr} = \frac{-\sigma(1 + r)^{-\sigma-1}\beta^{-\sigma}C_2 + \beta^{-\sigma}(1 + r)^{-\sigma}(Y_1 - C_1)}{1 + \beta^{-\sigma}(1 + r)^{1-\sigma}} \]
- Divide through by $\beta^{-\sigma}(1 + r)^{-\sigma}$

\[
\frac{dC_1}{dr} = \frac{-\sigma \frac{C_2}{1+r} + (Y_1 - C_1)}{\beta^\sigma (1 + r)^\sigma + 1 + r}
\]

- Substitute in for $C_2/C_2$ from Euler equation

\[
\frac{dC_1}{dr} = \frac{-\sigma \frac{C_2}{1+r} + (Y_1 - C_1)}{\frac{C_2}{C_1} + 1 + r}
\]
- A rise in $r$ has ambiguous effect on $C_1$ and hence on saving $S_1 = Y_1 - C_1$

* Substitution effect: an increase in the interest rate makes future consumption relatively cheaper $\rightarrow$ lower current consumption (increase saving)

* Terms-of-trade effect:

  · if $Y_1 - C_1 > 0$ interest rate increase makes the country richer (improves intertemporal terms of trade) $\rightarrow$ higher current consumption (lower saving)

  · if $Y_1 - C_1 < 0$ interest rate increase makes the country poorer (worsens intertemporal terms of trade) $\rightarrow$ lower current consumption (higher saving)
Closed form solution for consumption with isoelastic preferences

\[
\beta^\sigma (1 + r)^\sigma C_1 = (1 + r)(Y_1 - C_1) + Y_2 \\
(\beta^\sigma (1 + r)^\sigma + 1 + r) C_1 = (1 + r)Y_1 + Y_2 \\
C_1 = \frac{1}{\beta^\sigma (1 + r)^{\sigma-1} + 1} \left( Y_1 + \frac{Y_2}{1 + r} \right)
\]

* Three effects of an increase in the interest rate

1. Substitution effect (\(\div\)) 2. Income effect (\(+\)) 3. Wealth effect (\(\div\))

* Income effect dominates substitution effect if \(\sigma < 1\)

* With \(\sigma = 1\) (log case) income and substitution effects cancel \(\rightarrow dC_1/dr\) always negative

* Income effect + wealth effect = terms of trade effect
Figure 1.5
Global exchange equilibrium
• Comparative statics

  – Increase in $Y_1$ shifts saving schedule $SS'$ out $\rightarrow r$ falls

  – Increase in $Y_2$ shifts saving schedule $SS'$ in $\rightarrow r$ increases

  – Increase in $\beta$ shifts saving schedule $SS'$ out $\rightarrow r$ falls

• ‘Immiserizing growth’ (Bhagwati, The Review of Economic Studies Vol 25, 1958): Under certain circumstances, economic expansion may harm the growing country. The increase in output might lead to a sufficient deterioration in the terms of trade to reduce the real income of the growing country
World equilibrium in the two country model with investment

- Allow for different productivity levels
  \[ Y = AF(K), \quad Y^* = A^*F^*(K^*) \]

- Equilibrium requires
  \[ Y_1 + Y_1^* = C_1 + C_1^* + I_1 + I_1^* \]
  or
  \[ S_1 + S_1^* = I_1 + I_1^* \]
  or
  \[ CA_1 + CA_1^* = 0 \]
• Investment schedules

\[ A_2 F'(K_1 + I_1) = r \]
\[ A_2^* F^*(K_1^* + I_1^*) = r \]

Production functions concave: investment schedules slope downward

• Saving schedules with isoelastic preferences

  - Substitute in for budget constraint in the Euler equation

\[ C_1 = \beta^{-\sigma} (1 + r)^{-\sigma} C_2 \]
\[ = \beta^{-\sigma} (1 + r)^{-\sigma} \left[ \frac{A_2 F(K_1 + I_1) + K_1 + I_1}{+(1 + r)(A_1 F(K_1) - C_1 - I_1)} \right] \]
Differentiate

\[
\frac{dC_1}{dr} = -\sigma (1 + r)^{-\sigma-1} \beta^{-\sigma} C_2 + \beta^{-\sigma} (1 + r)^{-\sigma} \left( A_1 F(K_1) - C_1 - I_1 \right) - \beta^{-\sigma} (1 + r)^{1-\sigma} \frac{dC_1}{dr} + \beta^{-\sigma} (1 + r)^{-\sigma} \left( A_2 F'(K_2) + 1 - (1 + r) \right) \frac{dI_1}{dr} = 0 \]  

(enzyme theorem!)

Divide through by \(\beta^{-\sigma} (1 + r)^{-\sigma}\) and substitute in for \(C_2/C_2\) from Euler equation

\[
\frac{dC_1}{dr} = -\sigma \frac{C_2}{1+r} + \left( Y_1 - C_1 - I_1 \right) \frac{C_2}{C_1} + 1 + r
\]

Note! For a given value of the current account balance, the slope of the saving schedule is the same as in the endowment economy
Figure 1.7
Global intertemporal equilibrium with investment
• Comparative statics

  – Fall in $\beta$ shifts saving schedule $SS'$ inwards $\rightarrow r$ increases

  – A rise in future home productivity $A_2$

    * Shift in investment schedule $II'$

      \[
      A_2F'(K_1 + I_1) = r \\
      dA_2F'(K_2) + A_2F''(K_2)dI_1 = 0
      \]

      \[
      \frac{dI_1}{dA_2}\bigg|_{r \text{ constant}} = -\frac{F'(K_2)}{A_2F''(K_2)} > 0
      \]

    * Shift in saving schedule $SS'$

      \[
      C_1 = \frac{1}{\beta^\sigma(1+r)^{\sigma-1} + 1} \left( A_1F(K_1) - I_1 + \frac{A_2F(K_1 + I_1) - I_2}{1 + r} \right)
      \]

      \[
      \frac{dC_1}{dA_2}\bigg|_{r \text{ constant}} = \frac{F(K_2)}{\beta^\sigma(1+r)^\sigma + 1 + r} > 0
      \]
Figure 1.8
A rise in future Home productivity
Figure 1. United States' 10-Year Bond Rate
(Percent a year)
• Case study: The U.S. real interest rate and current account imbalances


  – A significant increase in the global supply of saving helps explain the increase U.S. current account deficit and the low level of world real interest rates

  – What has caused the increase in global saving?
    
    * Strong saving motive of rich countries with ageing populations

    * Low prospective returns to domestic investment in many mature industrialised countries (due to e.g., slowly growing workforces and high capital-to-labour ratios)
* Movement from large deficit to large surplus in the current account positions of developing countries

  · Response to the financial crises in the 1990s

  · High oil prices

  – High inflow of capital to the U.S. because of technology boom and high productivity in 1990s

  – Low U.S. saving because of high stock prices, increase in housing wealth and later, low real interest rates.