Lecture 1: The intertemporal approach to the current account

Open economy macroeconomics, Fall 2006
Ida Wolden Bache

August 22, 2006
Intertemporal trade and the current account

- What determines when countries lend and when countries borrow in international capital markets?

- The current account (\textit{driftsbalansen})
  
  - Let $B_{t+1}$ be the value of an economy’s net foreign assets at the end of period $t$.
  
  - Definition of current account: net increase in foreign asset holdings

\begin{equation}
CA_t \equiv B_{t+1} - B_t
\end{equation}

or

\begin{equation}
CA_t = NX_t + rB_t
\end{equation}

where $NX_t$ denotes net exports
Table 1
Global current account balances

<table>
<thead>
<tr>
<th>Country or Aggregate</th>
<th>Current Account Balance, 2004</th>
<th>US Dollars (billions)</th>
<th>% of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Largest Deficits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>–668.1</td>
<td>–5.7</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>–55.3</td>
<td>–5.3</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>–42.1</td>
<td>–2.0</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>–39.8</td>
<td>–6.4</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>–15.5</td>
<td>–5.1</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>–15.0</td>
<td>–0.9</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>–12.7</td>
<td>–7.5</td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>–8.8</td>
<td>–8.8</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>–8.4</td>
<td>–0.4</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>–8.0</td>
<td>–3.9</td>
<td></td>
</tr>
<tr>
<td>(b) Largest Surpluses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>172.1</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>103.8</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>68.7</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>59.9</td>
<td>10.3</td>
<td></td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>51.6</td>
<td>20.5</td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>43.0</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>33.8</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>28.5</td>
<td>8.2</td>
<td></td>
</tr>
<tr>
<td>Singapore</td>
<td>27.9</td>
<td>26.1</td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>27.6</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>(c) Country Aggregates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>–668.1</td>
<td>–5.7</td>
<td></td>
</tr>
<tr>
<td>Other advanced economies</td>
<td>354.1</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Emerging economies</td>
<td>227.1</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Middle East</td>
<td>102.8</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>Developing Asia</td>
<td>93.0</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>World (discrepancy)</td>
<td>–86.3</td>
<td>—</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. The US current account in historical perspective
Percentage of GDP/GNP\(^1\)

---

1. GNP before 1929.

• The simplest possible model:
  
  – small open economy

  – two periods, labeled 1 and 2

  – one good at each date

  – endowment economy: output in each period is given: $Y_1$ and $Y_2$

  – all individuals are identical, population size normalised to one.

  – perfect foresight (no uncertainty)
• The representative consumer’s problem:

- Lifetime utility

\[
U = u(C_1) + \beta u(C_2), \quad 0 < \beta < 1
\]

where \( \beta \) is the subjective discount factor and \( u'(C) > 0, u''(C) < 0 \) and
\( \lim_{C \to 0} u'(C) = \infty \)

- Period budget constraints (\( B_1 = B_3 = 0 \))

\[
C_1 = Y_1 - B_2 \quad (4)
\]

\[
C_2 = Y_2 + (1 + r)B_2 \quad (5)
\]

where \( r \) is the (exogenous) world real interest rate

- Current account

\[
CA_2 = -B_2 = -(Y_1 - C_1) = -CA_1 \quad (6)
\]
- Intertemporal budget constraint

\[
C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}
\]

Present value of consumption \quad \text{Present value of output} \quad (7)

- Consumer’s optimisation problem: Maximise (3) subject to (7)

- From (7)

\[
C_2 = (1 + r)(Y_1 - C_1) + Y_2
\]

(8)

- Substitute into (3)

\[
U = u(C_1) + \beta u((1 + r)(Y_1 - C_1) + Y_2)
\]

(9)

- First-order condition with respect to \( C_1 \)

\[
u'(C_1) - \beta u'(C_2)(1 + r) = 0
\]

\[
u'(C_1) = \beta(1 + r)u'(C_2)
\]

(10)
This is the consumption Euler equation: at an optimum the consumer cannot increase utility by shifting consumption between periods.

* A one unit increase in consumption in period 1 increases utility by \( u'(C_1) \)

* Alternatively the consumer can save in period 1 and get \((1 + r)\) extra units of consumption in period 2 which increases utility by \( \beta(1+r)u'(C_2) \)

Consumers have incentive to *smooth* consumption over time

Rearrangement yields

\[
\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1 + r}
\]

Marginal rate of substitution of present for future consumption

Price of future consumption in terms of current consumption

Optimal consumption plan found by combining (10) and (7)
What determines whether a country runs a current account deficit or a current account surplus?

- Autarky real interest rate $r_A$: interest rate that would prevail in economy which could not borrow or lend internationally

- In autarky: $C_1 = Y_1$ and $C_2 = Y_2$

\[
\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1 + r_A} \tag{11}
\]

- An increase in $Y_1$ or a fall in $Y_2$ causes the autarky interest rate to increase

- If $Y_1 = Y_2 \implies \beta = \frac{1}{1+r_A}$

- Gains from intertemporal trade as long as $r \neq r_A$
Figure 1.1
Consumption over time and the current account
• Special case: $\beta = \frac{1}{1+r}$
  
  - Euler equation implies $C_1 = C_2 = C$ (perfect consumption smoothing)
  
  - Budget constraint yields

\[
C + \frac{C}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \quad \text{(12)}
\]

\[
\frac{2 + r}{1 + r} C = Y_1 + \frac{Y_2}{1 + r} \quad \text{(13)}
\]

\[
C = \frac{Y_1 (1 + r) + Y_2}{2 + r} \quad \text{(14)}
\]

- Autarky interest rate

\[
\frac{u'(Y_2)}{u'(Y_1)} = \frac{1 + r}{1 + r_A} \quad \text{(15)}
\]
− Assume economy initially expects $Y_1 = Y_2 \implies r = r_A$ and $CA_1 = 0$.

− Permanent changes in output ($dY_1 = dY_2 = dY$): no effect on $r_A$ or $CA_1$

− Temporary increase in output in period 1 ($dY_1 > 0, dY_2 = 0$): $r_A$ falls ($r_A < r$), $CA_1 > 0$

− Temporary increase in output in period 2 ($dY_1 = 0, dY_2 > 0$): $r_A$ increases ($r_A > r$), $CA_1 < 0$. 
• Adding government consumption

  – Period utility

    \[ u(C) + \nu(G) \]  \hspace{1cm} (16)

  – Assume balanced budget each period (no government deficits or surpluses)

  – Representative agent’s budget constraint

    \[ C_1 + \frac{C_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r} \]  \hspace{1cm} (17)

  – Euler equation same as above
What is the effect on the current account?

Assume $\beta = \frac{1}{1+r}$, $Y_1 = Y_2 = Y$, $G_1 > 0$ and $G_2 = 0$.

Closed form solution for consumption

\[
C + \frac{1}{1+r}C = Y - G_1 + \frac{Y}{1+r} \tag{18}
\]
\[
\frac{2+r}{1+r}C = \frac{2+r}{1+r}Y - G_1 \tag{19}
\]
\[
C = Y - \frac{1+r}{2+r}G_1 \tag{20}
\]

Current account

\[
CA_1 = Y - C - G_1 \tag{21}
\]
\[
= \frac{1+r}{2+r}G_1 - G_1 \tag{22}
\]
\[
= -\frac{G_1}{2+r} \tag{23}
\]
• Adding investment

  – Production function

    \[ Y = F(K), \quad F'(K) > 0, \quad F''(K) < 0, \quad F(0) = 0, \quad \lim_{K \to 0} F'(K) = \infty \]

    \[ \text{equation } (24) \]

  – Investment (ignoring depreciation)

    \[ I_t = K_{t+1} - K_t \]

    \[ \text{equation } (25) \]

  – Current account

    \[ CA_t = B_{t+1} - B_t = \underbrace{Y_t + rB_t - C_t - G_t - I_t}_{\text{National saving } S_t} = S_t - I_t \]

    \[ \text{equation } (26) \]

  – Current account balance = national saving minus investment
Returning to the two-period model

\[ B_1 = B_3 = 0, \ K_3 = 0 \implies I_2 = -K_2 \]

Period budget constraints

\[
\begin{align*}
Y_1 &= C_1 + G_1 + I_1 + B_2 \\
Y_2 + (1 + r)B_2 &= C_2 + G_2 + I_2
\end{align*}
\]  
(27)  
(28)

Intertemporal budget constraint

\[
\underbrace{C_1 + I_1 + \frac{C_2 + I_2}{1 + r}}_{\text{Present value of consumption and investment}} = \underbrace{Y_1 - G_1 + \frac{Y_2 - G_2}{1 + r}}_{\text{Present value of output}}
\]  
(29)

Solve for \( C_2 \) and substitute in for \( Y = F(K) \), \( I_2 = -K_2 \) and \( K_2 = I_1 + K_1 \)

\[
C_2 = (1 + r)(Y_1 - G_1 - C_1 - I_1) + Y_2 - G_2 - I_2
= (1 + r)(F(K_1) - G_1 - C_1 - I_1) + F(K_2) - G_2 + K_2
= (1 + r)(F(K_1) - G_1 - C_1 - I_1) + F(K_1 + I_1) - G_2 + K_1 + I_1
\]  
(30)
– Optimisation problem

\[
\max_{C_1, I_1} u(C_1) + \beta u \left( \frac{(1 + r)(F(K_1) - G_1 - C_1 - I_1)}{+F(K_1 + I_1) - G_2 + K_1 + I_1} \right)
\]  

(31)

– First-order condition with respect to \( C_1 \)

\[
u'(C_1) = \beta(1 + r)u'(C_2)
\]

(32)

– First-order condition with respect to \( I_1 \) (note that \( K_1 \) is given at date 1)

\[
-(1 + r) + F'(K_2) + 1 = 0
\]

\[
F'(K_2) = r
\]

(33)

* Note! Desired capital stock is independent of preferences

* Note! Government consumption does not crowd out investment
- **Equilibrium**

  - Assume for now $G_1 = G_2 = 0$

  - Intertemporal production possibilities frontier (PPF): the technological possibilities for transforming period 1 consumption into period 2 consumption (in autarky)

    $$C_2 = Y_2 + K_2$$
    $$= Y_2 + I_1 + K_1$$
    $$= F(I_1 + K_1) + K_1 + I_1$$
    $$= F(Y_1 - C_1 + K_1) + K_1 + Y_1 - C_1$$
    $$= F(F(K_1) - C_1 + K_1) + K_1 + F(K_1) - C_1$$

  - Maximum consumption in period 1 (horizontal intercept)

    $$C_1^{\text{max}} = F(K_1) + K_1$$  \hspace{1cm} (35)

  - Maximum consumption in period 2 (vertical intercept)

    $$C_2^{\text{max}} = F(F(K_1) + K_1) + K_1 + F(K_1)$$  \hspace{1cm} (36)
– Shape of PPF

\[
\frac{dC_2}{dC_1} = -F'(K_2) - 1 < 0, \quad \frac{d^2C_2}{dC_1^2} = F''(K_2) < 0 \quad (37)
\]

– Autarky equilibrium (point A)

\[
\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1 + F'(K_2)} = \frac{1}{1 + r_A} \quad (38)
\]

– Production in open economy (point B)

\[
F'(K_2) = 1 + r \quad (39)
\]

– Consumption in open economy (point C)

\[
\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1 + r} \quad (40)
\]
Figure 1.3
Investment and the current account