ECON4330 – International Macroeconomics

Postponed Exam – Solution Key

Total: 100 points

Instructor note. This is a companion solution key. Equivalent correct reasoning should receive credit even when notation or ordering differs. Full credit should require both the correct conclusion and the economic mechanism.

1. Current accounts, investment and the world interest rate [50 points]

(a) **Budget constraints and intertemporal budget constraint.** **[8 points]**

From the date-1 budget constraint,

$$B_2 = Y_1 - C_1 - K_2.$$

Substitute this into the date-2 budget constraint:

$$C_2 = Y_2 + AK_2^{1/2} + K_2 + (1+r)(Y_1 - C_1 - K_2).$$

Rearranging gives

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2 + AK_2^{1/2} + K_2}{1+r} - K_2.$$

Equivalently,

$$\boxed{C_1 + K_2 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2 + AK_2^{1/2} + K_2}{1+r}.}$$

Investment lowers date-1 resources one-for-one, but raises date-2 resources through both production, $AK_2^{1/2}$, and the remaining capital stock, K_2. The optimal investment choice compares this present cost with the present value of the future benefit.

Suggested grading: 4 points for substituting out B_2, 2 points for the correct present-value constraint, 2 points for the economic interpretation.

(b) **Euler equation, investment FOC and separation.** **[12 points]**

Write date-2 consumption as

$$C_2 = (1+r)(Y_1 - C_1 - K_2) + Y_2 + AK_2^{1/2} + K_2.$$

For consumption,

$$\frac{\partial C_2}{\partial C_1} = -(1+r).$$

The FOC for C_1 is therefore

$$\frac{1}{C_1} + \beta \frac{1}{C_2}\left[-(1+r)\right] = 0,$$

so

$$\boxed{\frac{1}{C_1} = \beta(1+r)\frac{1}{C_2}.}$$

Equivalently,

$$\boxed{C_2 = \beta(1+r)C_1.}$$

For investment,

$$\frac{\partial C_2}{\partial K_2} = -(1+r) + \frac{A}{2\sqrt{K_2}} + 1 = \frac{A}{2\sqrt{K_2}} - r.$$

The FOC for K_2 is

$$\beta \frac{1}{C_2}\left(\frac{A}{2\sqrt{K_2}} - r\right) = 0,$$

hence

$$\boxed{\frac{A}{2\sqrt{K_2}} = r.}$$

The left-hand side is the marginal product of capital. Since the capital stock itself can be consumed in period 2, the principal is returned; the relevant opportunity cost is the interest foregone, r, not the gross return $1+r$.

Solving,

$$\boxed{K_2^* = \left(\frac{A}{2r}\right)^2.}$$

This is the separation result: K_2 depends only on technology A and the world interest rate r. Preferences and income levels determine where the economy consumes along the intertemporal budget line after production has been chosen. Since $K_2^* = (A/2r)^2$, a higher r lowers investment.

Suggested grading: 4 points for the Euler equation, 4 points for the investment FOC, 2 points for explaining why the cost is r, 2 points for separation and the comparative static.

(c) **Numerical solution.** **[12 points]**

With $A = 2$ and $r = 0.20$,

$$\frac{2}{2\sqrt{K_2}} = 0.20 \quad \Rightarrow \quad \sqrt{K_2} = 5 \quad \Rightarrow \quad \boxed{K_2 = 25.}$$

Date-2 resources before debt repayment are

$$Y_2 + A\sqrt{K_2} + K_2 = 115 + 2 \cdot 5 + 25 = 150.$$

Define lifetime wealth, after the optimal production decision, as

$$W = Y_1 - K_2 + \frac{Y_2 + A\sqrt{K_2} + K_2}{1+r}.$$

Therefore

$$W = 100 - 25 + \frac{150}{1.20} = 75 + 125 = 200.$$

Since $\beta = 1$, the Euler equation gives $C_2 = 1.20 C_1$. Substituting into $C_1 + C_2/(1+r) = W$ gives

$$C_1 + C_1 = 200,$$

so

$$\boxed{C_1 = 100}, \qquad \boxed{C_2 = 120}.$$

Foreign assets are

$$B_2 = Y_1 - C_1 - K_2 = 100 - 100 - 25 = \boxed{-25}.$$

Hence

$$\boxed{\text{CA}_1 = -25.}$$

The current-account deficit finances investment, not extra date-1 consumption, because $C_1 = Y_1$. The country borrows 25 at date 1 to build capital, then repays $(1 + r)25 = 30$ at date 2 out of future resources.

Suggested grading: 3 points for K_2, 2 points for lifetime wealth, 3 points for C_1 and C_2, 2 points for B_2 and CA_1, 2 points for interpretation.

(d) **Current versus future income shocks.** **[8 points]**

In both experiments, the present value of lifetime resources rises by 12 and K_2 stays at 25. Since $\beta = 1$, half of any increase in lifetime wealth is spent in present-value terms on date-1 consumption. Thus

$$\Delta C_1 = \frac{12}{2} = \boxed{6}.$$

Case 1: Y_1 rises by 12. Then $Y_1 = 112$ and $C_1 = 106$. The current account becomes

$$\mathrm{CA}_1' = 112 - 106 - 25 = \boxed{-19}.$$

Relative to the baseline $\mathrm{CA}_1 = -25$, the current account rises by 6. The household consumes half the temporary current-income gain and saves the other half.

Case 2: Y_2 rises by 14.4. The present value is $14.4/1.20 = 12$, so again C_1 rises by 6 to 106. But current output has not changed. Therefore

$$\mathrm{CA}_1' = 100 - 106 - 25 = \boxed{-31}.$$

Relative to the baseline, the current account falls by 6. The country borrows more today because it wants to consume part of the higher future income before that income arrives.

Suggested grading: 2 points for recognizing that both shocks raise lifetime wealth by 12, 2 points for $\Delta C_1 = 6$, 2 points for the current-income case, 2 points for the future-income case and interpretation.

(e) **Productivity increase.** **[10 points]**

With $A = 2.4$ and $r = 0.20$,

$$\frac{2.4}{2\sqrt{K_2}} = 0.20 \quad \Rightarrow \quad \sqrt{K_2} = 6 \quad \Rightarrow \quad \boxed{K_2 = 36.}$$

Date-2 resources before debt repayment are

$$Y_2 + A\sqrt{K_2} + K_2 = 115 + 2.4 \cdot 6 + 36 = 165.4.$$

Lifetime wealth is

$$W = 100 - 36 + \frac{165.4}{1.20} = 64 + 137.833\ldots = 201.833\ldots.$$

With $\beta = 1$, $C_1 = W/2$ and $C_2 = 1.20C_1$, so

$$\boxed{C_1 \approx 100.92}, \qquad \boxed{C_2 \approx 121.10}.$$

The current account is

$$\mathrm{CA}_1 = 100 - 100.9167 - 36 \approx \boxed{-36.92}.$$

The productivity increase raises lifetime wealth, so households want to consume more today. It also raises the optimal capital stock from 25 to 36, so investment rises by 11. Both higher current consumption and higher investment raise date-1 absorption. In this example, the investment effect is especially important, so the current account falls from -25 to about -36.92. A positive future-productivity shock can therefore generate a current-account deficit today because the country borrows against higher future output and borrows to finance the larger investment program.

Suggested grading: 3 points for the new K_2, 2 points for lifetime wealth, 2 points for C_1 and C_2, 1 point for CA_1, 2 points for interpretation.

2. Short questions [20 points]

(a) **Persistence and the current account.** **[7 points]**

The coefficient multiplying the current output deviation is

$$\frac{1-\rho}{1+r-\rho}.$$

For a positive output deviation $\widetilde{z}_t > 0$, this coefficient is decreasing in ρ:

$$\frac{\partial}{\partial \rho}\left(\frac{1-\rho}{1+r-\rho}\right) = -\frac{r}{(1+r-\rho)^2} < 0.$$

A transitory positive shock raises current resources relative to permanent resources, so the country saves much of it and the current account rises. A highly persistent shock raises permanent income almost one-for-one with current income, so consumption rises with income and the current-account response is small.

An anticipated future oil boom raises permanent income before current output rises. Households want to consume part of the future income today. Since current output has not yet increased, the model predicts a current-account deficit before production starts, followed by higher output and eventual surpluses later.

Suggested grading: 3 points for the role of ρ, 2 points for the transitory vs. permanent interpretation, 2 points for the anticipated-oil-boom application.

(b) **Debt overhang.** **[7 points]**

Debt overhang arises when a country has inherited debt that will be repaid partly out of future output. New investment raises future output, but part of the marginal return is captured by existing creditors through higher repayment or larger recovery in default states. The debtor pays the full marginal cost of investment but keeps only part of the marginal benefit.

In reduced form, the investment FOC can be written

$$F'(K)\left[1-\tau(D,K)\right] = 1,$$

where $\tau(D,K)$ is the implicit creditor tax. Equivalently,

$$F'(K) = \frac{1}{1-\tau(D,K)} > 1.$$

Since $F'' < 0$, this implies $K < K^{FB}$: investment is below the first-best level. Higher debt usually widens the default region and raises the implicit tax, making the under-investment worse.

Debt forgiveness can be Pareto-improving on the wrong side of the debt Laffer curve. Reducing face value lowers the implicit tax, raises investment, and increases future output. The country benefits from higher net output. Creditors can also benefit if the higher investment and repayment probability raise the market value of the remaining debt.

Suggested grading: 3 points for the implicit tax mechanism, 2 points for under-investment, 2 points for the debt-relief/Laffer-curve argument.

(c) **Moral hazard and the Lucas paradox.** **[6 points]**

In the first-best, investment satisfies

$$\pi'(\bar{I})Z = 1 + r.$$

The borrower internalizes the full marginal product of investment, $\pi'(I)Z$, and invests until it equals the gross interest rate.

Under hidden action, the borrower chooses investment privately. The private FOC is

$$\pi'(I)\{Z - [P(Z) - P(0)]\} = 1 + r.$$

The term $P(Z) - P(0)$ is the extra repayment due in the good state. It lowers the borrower's private gain from increasing the success probability. Hence the private marginal benefit is below the social marginal benefit:

$$\pi'(I)\{Z - [P(Z) - P(0)]\} < \pi'(I)Z.$$

To satisfy the same right-hand side, concavity of π implies $I^* < \bar{I}$.

This helps explain the Lucas paradox because poor countries often need more external finance. Larger external finance needs make incentive problems more severe, limiting lending and lowering investment even when expected marginal products are high. Capital may therefore fail to flow from rich to poor countries at the rate predicted by the frictionless model.

Suggested grading: 2 points for comparing the two FOCs, 2 points for under-investment, 2 points for the Lucas-paradox link.

3. Exchange rates, intervention and monetary control [30 points]

(a) **Risk premium and foreign tightening.** **[6 points]**

The risk premium on domestic-currency assets is

$$r_k = i - i^* - e^e(E).$$

It is the excess expected return on domestic assets relative to foreign assets, after adjusting for expected depreciation. If $r_k > 0$, domestic assets must pay a premium; if $r_k = 0$, uncovered interest parity holds.

Holding E and i fixed, an increase in i^* lowers r_k:

$$\Delta r_k = -\Delta i^* < 0.$$

Domestic-currency assets become less attractive. Since $S_{r_k} > 0$, the public's net supply of foreign currency to the central bank falls. In words, investors want to shift toward foreign assets, creating depreciation pressure or reserve loss under a peg.

Suggested grading: 3 points for defining r_k, 3 points for tracing $i^* \uparrow \Rightarrow r_k \downarrow \Rightarrow S \downarrow$.

(b) **Peg without sterilization.** **[8 points]**

Under a peg, $E = \bar{E}$ is fixed and the central bank must adjust reserves so that

$$F_g = S(\bar{E}, r_k).$$

From part (a), $i^* \uparrow$ lowers r_k at the initial i and E, so S falls. Therefore

$$\Delta F_g < 0.$$

The central bank sells foreign reserves to satisfy the private demand for foreign currency and defend the peg.

Without sterilization,

$$\Delta(\overline{B} - B) = 0.$$

Hence the balance sheet gives

$$\Delta M = E\Delta F_g < 0.$$

The reserve loss contracts the money supply. With money-market equilibrium

$$\frac{M}{P} = m(i, Y), \qquad m_i < 0,$$

a lower real money supply requires a higher domestic interest rate i for given Y and P. Thus a foreign tightening is transmitted into domestic tightening. The rise in i partly offsets the fall in r_k, but the key qualitative result is

$$F_g \downarrow, \qquad M \downarrow, \qquad i \uparrow .$$

Suggested grading: 2 points for reserves falling, 3 points for the balance-sheet effect on money, 3 points for the money-market effect on i.

(c) **Peg with sterilization.** **[8 points]**

The FX-market pressure is the same as in part (b): at the initial interest rate, S falls and the central bank loses reserves,

$$\Delta F_g < 0.$$

Full sterilization means

$$\Delta M = 0.$$

From

$$\Delta M = E\Delta F_g + \Delta(\overline{B} - B),$$

it follows that

$$\Delta(\overline{B} - B) = -E\Delta F_g > 0.$$

Since $\Delta F_g < 0$, the central bank must increase its holdings of domestic bonds. It buys domestic bonds to offset the money contraction that would otherwise come from selling foreign reserves.

With M fixed, the money-market condition $M/P = m(i, Y)$ implies that i need not move on impact, holding Y and P fixed. Sterilization therefore insulates the domestic short rate in the short run.

The policy is limited because the reserve loss remains. Sterilization changes the monetary consequence of intervention, but it does not remove the underlying pressure on the peg. With finite reserves, repeated reserve losses cannot continue indefinitely; eventually the central bank must raise i, abandon sterilization, impose controls, or adjust the peg.

Suggested grading: 2 points for reserves falling, 3 points for the correct bond operation, 1 point for i on impact, 2 points for the reserve-limit explanation.

(d) **Floating exchange rate.** **[8 points]**

Under a float, reserves are fixed:

$$\Delta F_g = 0.$$

The FX market must clear through the exchange rate. The increase in i^* lowers r_k and shifts foreign-currency supply down at the initial exchange rate. To restore

$$F_g = S(E, r_k),$$

the exchange rate must depreciate:

$$\Delta E > 0.$$

There is no reserve sale. If the central bank does not change domestic bond holdings,

$$\Delta F_g = 0, \qquad \Delta(\overline{B} - B) = 0,$$

so

$$\Delta M = 0.$$

With money unchanged, the domestic short rate need not move on impact through the money market.

Comparison:

- Peg without sterilization: reserve losses contract money, so i rises. Monetary autonomy is low.
- Peg with sterilization: reserves fall but money is protected. Monetary autonomy is higher in the short run, but limited by finite reserves.
- Float: the exchange rate absorbs the external shock. Reserves and money need not move, so the central bank has the highest autonomy over the short rate.

Suggested grading: 3 points for depreciation under float, 2 points for no reserve/money movement, 3 points for comparing regimes.