# The current account in an intertemporal equilibrium model 

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## Global imbalances


${ }^{4}$ CHN+EMA: China, Hong Kong SAR, Indonesia, Korea, Malaysia, Philippines, Singapore,
Taiwan Province of China, and Thailand; DEU+JPN: Germany and Japan; OCADC:
Bulgaria, Croatia, Czech Republic, Estonia, Greece, Hungary, Ireland, Latvia, Lithuania, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Turkey, and United Kingdom; OIL: Oil exporters; ROW: rest of the world; US: United States.

## Global imbalances



Source: IMF World Economic Outlook October 2007

| Country | Bill US <br> Dollars | Per cent of <br> GDP |  |
| :--- | ---: | ---: | :---: |
| China | 250 | 9,4 |  |
| Germany | 147 | 5,0 |  |
| Japan | 170 | 3,9 |  |
| United |  |  |  |
| Kingdom | -77 | $-3,2$ |  |
| United |  |  |  |
| States | -811 | $-6,2$ |  |

## Current accounts 2006



Source: IMF World Economic Outlook, Database

## Current accounts 2006

## Selected deficit countries

| Country | Per cent of GDP |
| :--- | :---: |
| Sao Tomé | -45.9 |
| Iceland | -27.3 |
| Latvia | -21.1 |
| Estonia | -15.5 |
| Greece | -9.6 |
| Portugal | -9.4 |
| Spain | -8.6 |

Figure 1. The US current account in historical perspective
Percentage of GDP/GNP ${ }^{1}$

Percentage of GDP/GNP1
Percentage of GDP/GNP ${ }^{1}$


1. GNP before 1929 .

Source: OECD, US Bureau of Economic Analysis; and for the pre-1946 period Bureau of the Census: Historical Statistics of the United States, Washington DC, 1975.

## Questions

- What determines current account deficits and surpluses?
- How are they affected by fiscal and monetary policy?
- Can deficits be sustained? For how long?
- Will they self-correct or do they warrant policy changes?
- How does the current account behave during business cycles?
- Is a current account deficit a threat to employment?
- Can a current account deficit force a country to devalue?


## Approaches

The intertemporal approach
Intertemporal general equilibrium models, explicit optimization over time, no nominal rigidities. Representative consumers and producers. Countries treated as if they were individuals. Obstfeld and Rogoff.

The traditional macro approach
Less focus on explicit optimization in micro, more focus on macro behavioral equations that seem to have empirical support. Nominal rigidities and unemployment problems. Rødseth.

## Current account - definition

Current account =

+ Trade account
Exports minus imports of goods and services
+ Primary income account
Payments for the use of labor and financial resources
+ Secondary income account
Redistribution (foreign aid, remittances etc)


## The accumulation equation

Net foreign assets at the beginning of the period

+ Current account surplus $\leftarrow$ Transactions
+ Revaluations
$=$ Net foreign assets at the end of the period

Current account surplus $=$ Net investment in foreign assets

## Relation to investment and saving

Saving = Net investment in real capital

+ Net investment in financial assets
= Investment in real capital at home
+ Current account surplus

Current account surplus $=$ Saving - Net investment in real capital

## Current account, saving and investment in real capital in per cent of GDP 2006

| Country | Current |  |  | Gov. |
| :--- | :---: | :---: | :---: | :---: |
| account | Saving | Investment | Surplus |  |
| Germany | 5,0 | 22,8 | 17,8 | $-1,6$ |
| Japan | 3,9 | 28,0 | 24,1 | $-4,1$ |
| United |  |  |  |  |
| Kingdom | $-3,2$ | 14,8 | 18,0 | $-2,7$ |
| United |  |  |  |  |
| States | $-6,2$ | 14,1 | 20,0 | $-2,6$ |
| World |  | 23,3 | 23,0 |  |

The figures are for gross saving and gross investment

## The simplest possible model

- The economy exists for two periods, labeled 1 and 2
- Small open economy. Everyone can borrow and lend at a given world market interest rate, $r$
- One good at each date, consumed in quantities $C_{1}$ and $C_{2}$
- Endowment economy: Output in each period is given: $Y_{1}$ and $Y_{2}$
- Representative consumer: All individuals are identical, population size normalized to one.
- Perfect foresight (no uncertainty)

The model of consumer saving from ECON1210 turned into a model of the current account of an entire country.


Period budget constraints ( $B_{2}=$ net lending to abroad)

$$
\begin{gather*}
C_{1}+B_{2}=Y_{1}  \tag{1}\\
C_{2}=Y_{2}+(1+r) B_{2} \tag{2}
\end{gather*}
$$

The current account is by definition

$$
\begin{gathered}
C A_{1}=S_{1}=Y_{1}-C_{1}=B_{2} \\
C A_{2}=S_{2}=Y_{2}+r B_{2}-C_{2}=-B_{2}=-C A_{1}
\end{gathered}
$$

The present-value budget constraint (from (1) and (2))

$$
\begin{equation*}
C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r} \tag{3}
\end{equation*}
$$

[Hint: Solve (2) for B, insert in (1).]

Consumer maximizes

$$
\begin{equation*}
U=u\left(C_{1}\right)+\beta u\left(C_{2}\right) \tag{4}
\end{equation*}
$$

Subject to present-value budget constraint

$$
\begin{equation*}
C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r} \tag{3}
\end{equation*}
$$

$\beta=1 /(1+\delta), 0<\beta<1$, subjective discount factor, $\delta>0$, discount rate
Assumptions

$$
u^{\prime}(C)>0, \quad u^{\prime \prime}(C)<0, \quad \lim _{C \rightarrow 0} u^{\prime}(C)=\infty
$$

First order condition

$$
\begin{array}{r}
\frac{\beta u^{\prime}\left(C_{2}\right)}{u^{\prime}\left(C_{1}\right)}=\frac{1}{1+r}  \tag{4}\\
\text { MRS }=\text { Price ratio }
\end{array}
$$

$1 /(1+r)=$ price of consumption in period 2 in terms of consumption in period 1

Two equivalent ways of writing the first order condition:

$$
\frac{\beta u^{\prime}\left(C_{2}\right)}{u^{\prime}\left(C_{1}\right)}=\frac{1}{1+r}(4) \quad \text { and } \quad u^{\prime}\left(C_{1}\right)=\beta(1+r) u^{\prime}\left(C_{2}\right)
$$

The consumption Euler equation: In optimum one cent yields the same (expected) return in terms of utility irrespective of whether it is spent on consumption now or invested and the proceeds spent on consumption next period.

$$
\text { Since } u^{\prime \prime}<0, \quad C_{2}>C_{1} \text { if, and only if, } \beta(1+\mathrm{r})>1
$$

$\beta(1+r)>1$ means that the interest rate exceeds the subjective discount rate.
$\beta(1+\mathrm{r})=1 \Rightarrow C_{1}=C_{2}=C$. Complete consumption smoothing.

Convex preferences mean that there is always some tendency to consumption smoothing.

Example 1: $\beta(1+\mathrm{r})=1, C_{1}=C_{2}=C$. Complete consumption smoothing.

Insertion in the budget constraint yields

$$
\begin{equation*}
C=\frac{(1+r) Y_{1}+Y_{2}}{2+r} \tag{5}
\end{equation*}
$$

The current account in this case is

$$
\begin{equation*}
C A_{1}=Y_{1}-C=Y_{1}-\frac{(1+r) Y_{1}+Y_{2}}{2+r}=\frac{Y_{1}-Y_{2}}{2+r} \tag{6}
\end{equation*}
$$

The main determinant of the current account is the difference between present and future income.
$C A_{1} / Y_{1}$ depends only on $\left(Y_{1}-Y_{2}\right) / Y_{1}$ not on the absolute level of income

Example 2: CRRA utility function

$$
\begin{equation*}
u(C)=\frac{1}{1-\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \tag{7}
\end{equation*}
$$

$\sigma$ is the intertemporal elasticity of substitution.

$$
u^{\prime}(C)=C^{-1 / \sigma}
$$

Hence, the first order condition can be written

$$
\begin{equation*}
\frac{\beta u^{\prime}\left(C_{2}\right)}{u^{\prime}\left(C_{1}\right)}=\beta\left(\frac{C_{2}}{C_{1}}\right)^{-\frac{1}{\sigma}}=\frac{1}{1+r} \tag{8}
\end{equation*}
$$

or

$$
\beta(1+r)=\left(\frac{C_{2}}{C_{1}}\right)^{1 / \sigma} \Leftrightarrow \frac{C_{2}}{C_{1}}=\lfloor\beta(1+r)\rfloor^{\sigma} \Leftrightarrow C_{2}=\lfloor\beta(1+r)\rfloor^{\sigma} C_{1}
$$

$C_{2}$ is proportional to $C_{1}$ with the factor of proportionality increasing in $r$.

## CRRA-example continued

From the first order condition and the budget equation

$$
\begin{equation*}
C_{1}=\frac{Y_{1}+(1+r)^{-1} Y_{2}}{1+(1+r)^{-1}[\beta(1+r)]^{\sigma}}=\frac{(1+r) Y_{1}+Y_{2}}{2+r+\left\{[\beta(1+r)]^{\sigma}-1\right\}} \tag{9}
\end{equation*}
$$

The current account is then

$$
\begin{equation*}
C A_{1}=Y_{1}-C_{1}=\frac{Y_{1}-Y_{2}+\left\{[\beta(1+r)]^{\sigma}-1\right\} Y_{1}}{2+r+\left\{[\beta(1+r)]^{\sigma}-1\right\}} \tag{10}
\end{equation*}
$$

Two motives for saving in the first period:

1. Consumption smoothing. Positive savings if $Y_{1}>Y_{2}$
2. Rate of return. Positive savings if $\beta(1+r)>1$, $(r>\delta)$

Strength of the last motive depends on the intertemporal substitution elasticity Savings rate (CA / Y) is independent of income level

CRRA-example continued

$$
\begin{equation*}
C_{1}=\frac{(1+r) Y_{1}+Y_{2}}{1+(1+r)+\left\{[\beta(1+r)]^{\sigma}-1\right\}} \tag{9'}
\end{equation*}
$$

Effects of $r$ on $C_{1}$

1) The substitution effect related to $[\beta(1+r)]^{\sigma}$. Always negative.
2) Two opposing income effects (one in the numerator, one in the denominator).

- Life-time income increases in proportion to $Y_{1}$
- The real value of life time income decreases in proportion to $C_{1}$ Net income effect is negative if $Y_{1}<C_{1}$, positive if $Y_{1}>C_{1}$

Hence, the effect of an increase in $r$ on the current account is unambiguously positive if $Y_{1}<C_{1}$, but may be negative if $Y_{1}$ is sufficiently greater than $C_{1}$ and the elasticity of substitution is low.

A high $r$ makes it less expensive to smooth consumption when $Y_{1}>Y_{2}$, more expensive when $Y_{2}>Y_{1}$.

## Summing up results on saving / current account

1. The absolute income level is not likely to be an important determinant of the CA.
2. Countries with high expected income growth should tend to have CA deficits, countries with low (negative) expected income growth to have surpluses.
3. Patient countries (with $\beta$ close to 1 ) should tend to have current account surpluses, impatient ones to have deficits.

Does the model help explaining:

- The US deficit and the Chinese surplus?
- The surpluses of oil-rich countries?

More questions:

- Are we in the first period?
- How is the world interest rate determined?
- Government deficits? Investment?


## Adding government consumption and taxes

Assumption: Government budget balanced in present value terms

$$
\begin{equation*}
T_{1}+\frac{1}{1+r} T_{2}=G_{1}+\frac{1}{1+r} G_{2} \tag{11}
\end{equation*}
$$

Budget constraint of the consumer

$$
C_{1}+\frac{C_{2}}{1+r}=\left(Y_{1}-T_{1}\right)+\frac{Y_{2}-T_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r}-\left[T_{1}+\frac{1}{1+r} T_{2}\right]
$$

Or after inserting from (6)

$$
\begin{equation*}
C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r}-\left[G_{1}+\frac{1}{1+r} G_{2}\right] \tag{12}
\end{equation*}
$$

Consumption Euler-equation unaffected provided that utility is separable in C and G :

$$
u(C)+v(G)
$$

In the example with $\beta(1+r)=1$

$$
\begin{equation*}
C_{1}=C_{2}=C=\frac{(1+r)\left(Y_{1}-G_{1}\right)+\left(Y_{2}-G_{2}\right)}{2+r} \tag{13}
\end{equation*}
$$

The current account is then

$$
\begin{equation*}
C A_{1}=Y_{1}-C-G_{1}=\frac{Y_{1}-Y_{2}-\left(G_{1}-G_{2}\right)}{2+r} \tag{14}
\end{equation*}
$$

Temporarily high government expenditures now can produce a current account deficit.

Taxes and the size of deficits do not matter (because of Ricardian equivalence)

## General equilibrium. Determination of $r$.

Two countries, (no explicit government sector)
Equilibrium condition $C A_{1}=-C A_{1}^{*}$.


World equilibrium in period 1.

## CRRA-example again

Assume same $\beta$ in both countries

$$
\begin{gathered}
C A_{1}=Y_{1}-C_{1}=\frac{Y_{1}-Y_{2}+\left\{[\beta(1+r)]^{\sigma}-1\right\} Y_{1}}{2+r+\left\{[\beta(1+r)]^{\sigma}-1\right\}} \\
C A_{1}^{*}=Y_{1}^{*}-C_{1}^{*}=\frac{Y_{1}^{*}-Y_{2}^{*}+\left\{[\beta(1+r)]^{\sigma}-1\right\} Y_{1}^{*}}{2+r+\left\{[\beta(1+r)]^{\sigma}-1\right\}}
\end{gathered}
$$

Equilibrium condition

$$
C A_{1}=-C A_{1}^{*}
$$

Equivalent to
$Y_{1}-Y_{2}+\left\{[\beta(1+r)]^{\sigma}-1\right\} Y_{1}=-Y_{1}^{*}+Y_{2}^{*}\left\{[\beta(1+r)]^{\sigma}-1\right\} Y_{1}^{*}$
Which can be solved to yield

$$
1+r=\frac{1}{\beta}\left(\frac{Y_{2}+Y_{2}^{*}}{Y_{1}+Y_{1}^{*}}\right)^{1 / \sigma}
$$

The world real interest rate depends positively on the world growth rate and negatively on patience.

A low elasticity of substitution means that the growth rate has a strong effect on the interest rate.

Why then such low real interest rates after 2001?

Since by definition $\beta=1 /(1+\delta)$, where $\delta$ is the subjectiv disount rate, we can also write

$$
1+r=(1+\delta)\left(\frac{Y_{2}+Y_{2}^{*}}{Y_{1}+Y_{1}^{*}}\right)^{1 / \sigma}
$$

