The current account in an intertemporal equilibrium model. Part 2

Econ 4330 Open Economy Macroeconomics Spring 2011

Second lecture

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Last time: Two-period endowment economy with representative consumer

Consumer maximizes

$$U = u(C_1) + \beta u(C_2) \quad (1)$$

Subject to present-value budget constraint

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \qquad (2)$$

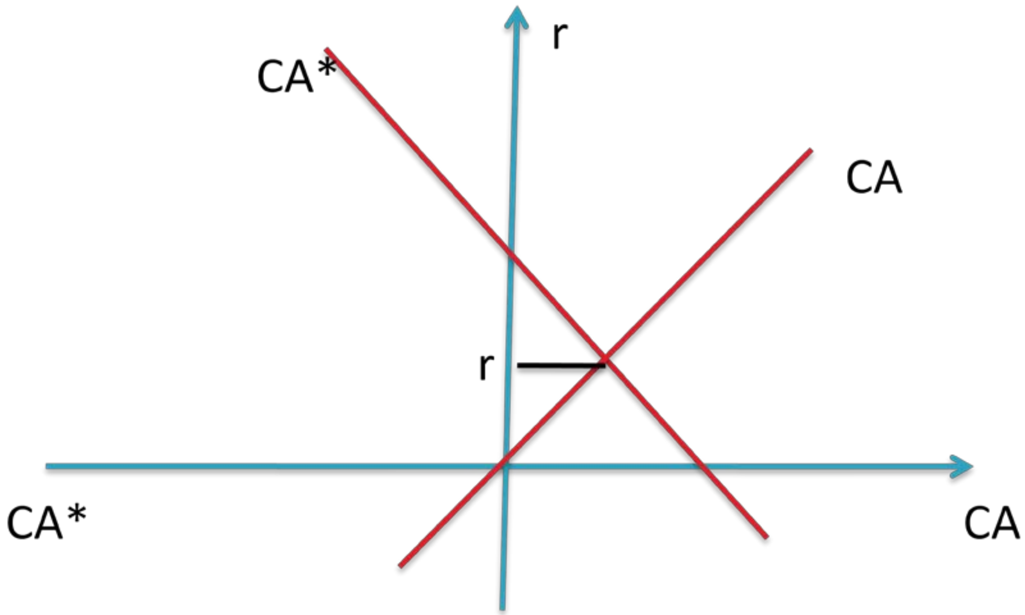
First order condition

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r} \quad (3)$$

World equilibrium. Determination of r.

Two countries, no (explicit) government sector, endowment economies.

Equilibrium condition $CA_1 = -CA_1^*$.



World equilibrium in period 1.

World consumption equals world output in each period. Hence,

$$\frac{C_2 + C_2^*}{C_1 + C_1^*} = \frac{Y_2 + Y_2^*}{Y_1 + Y_1^*} = \frac{Y_2^W}{Y_1^W}$$

Complete consumption smoothing is impossible.

First-order conditions

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+r} \qquad (4)$$

Potential gains from trade:

- More opportunities for consumption smoothing
- The more impatient may move consumption forward, the more patient may wait and increase total consumption

Equality of the two MRS ensures that the potential gains are fully exploited

High growth in world output → Higher interest rate needed for equilibrium

Last time: CES-example with same β and σ in both countries:

$$u(C) = \frac{1}{1-1/\sigma} C^{1-1/\sigma}$$

First order conditions:

$$\beta \left(\frac{C_2}{C_1}\right)^{-\frac{1}{\sigma}} = \beta \left(\frac{C_2^*}{C_1^*}\right)^{-\frac{1}{\sigma}} = \frac{1}{1+r}$$

Implies same consumption growth in both countries:

$$\frac{C_2}{C_1} = \frac{C_2^*}{C_1^*} = \frac{Y_2^W}{Y_1^W}$$

$$1 + r = \frac{1}{\beta}\left(\frac{Y_2^W}{Y_1^W}\right)^{1/\sigma} = (1+\delta)\left(\frac{Y_2^W}{Y_1^W}\right)^{1/\sigma} \qquad (5)$$

$$1 + r = (1 + \delta) \left(\frac{Y_2^W}{Y_1^W} \right)^{1/\sigma} \qquad (5)$$

- Positive output growth means $r > \delta$
- Higher output growth means higher world interest rate
- Effect is stronger the lower is the elasticity of substitution

$$CA_1 > 0 \Leftrightarrow \frac{Y_2}{Y_1} < \frac{Y_2^W}{Y_1^W}$$

- Same consumption growth everywhere
- Countries with slow income growth need to save in first period
- Contries with high income growth want to borrow in first period

Including investment

Production function

$$Y_t = A_t F(K_t), \qquad t = 1,2 \qquad (6)$$

A_t an exogenous productivity factor

K_t capital stock at beginning of period t

Assumptions: $F' > 0,\ F'' < 0,\ F(0) = 0$

Assuming no depreciation, capital accumulates according to

$$K_t = K_{t-1} + I_{t-1}, \qquad t = 2,3 \qquad (7)$$

K_1 is given by past history, $K_3 = 0$ since the economy ends there

By implication: $I_2 = -K_2$

Period budget constraints

$$C_1 + B_2 + K_2 = Y_1 + K_1 \qquad (8)$$

$$C_2 = Y_2 + K_2 + (1+r)B_2 \qquad (9)$$

Elimination of B_2 yields present value budget constraint

$$C_1 + \frac{C_2}{1+r} = K_1 + Y_1 + \frac{Y_2 - rK_2}{1+r} \qquad (10)$$

Separation of consumption and investment decisions:

1. Maximize total wealth (rhs of (10)). Since K_1 and $Y_1 = F(K_1)$ are given, this amounts to maximizing $Y_2 - rK_2$ with respect to K_2. Implicitly this also determines I_1.
2. Maximize utility with respect to C_1 and C_2 given total wealth. Same problem as in Lecture 1, same Euler equation.

Wealth maximization:

Max $Y_2 - rK_2 = A_2F(K_2) - rK_2$

First order condition:

$$A_2F'(K_2) = r \qquad (11)$$

Two ways of providing for the future:

- Lending to abroad, constant returns
- Investing in productive capital at home, diminishing return

Do the latter until returns are equalized.

Since $I_1 = K_2 - K_1$, I_1 depends negatively on *r* and K_1, positively on A_2

Effects of exogenous variables on the current account of a small open economy

$$CA_1 = S_1 - I_1 = A_1F(K_1) - C_1 - I_1 = A_1F(K_1) - C_1 - (K_2 - K_1) \qquad (12)$$

1) An increase in *r* now has three different types of effects on the CA:
   i) The usual positive substitution effect on savings.
   ii) Income effects on savings.

      Net borrowers ($B_1 < 0$) lose real income, consume less and save more.

      Net lenders ($B_1 > 0$) gain and save less.

   iii) A positive effect because an increase in *r* reduces investment demand I_1.

Total effect is ambiguous for net lenders; the investment effect iii) diminishes the ambiguity.

A digression on the income effects

Their sign can be found by looking at the consumer's budget constraint, conveniently rewritten as

$$(1+r)C_1 + C_2 = (1+r)(K_1 + Y_1) + Y_2 - rK_2 \text{ or}$$

$$(1+r)C_1 + C_2 = K_1 + (1+r)Y_1 + Y_2 - rI_1$$

Does an increase in *r* tighten or relax this constraint? (Does it increases the lhs more or less than the rhs given the initial values of C_1, C_2 , Y_1, Y_2 and I_1)?

Answer: An increase in r raises income more than expenditure if, and only if, $Y_1 - I_1 > C_1$ or $B_1 = Y_1 - C_1 - I_1 > 0$.

- An increase in r also changes K_2 and Y_2, but since we are starting from an optimum, this net effect of this on real income is zero (the envelope result).
- At the world level gains and losses from an increase in r are netted out.

Back to the other exogenous variables

2) An increase in A_1 works like an exogenous increase in Y_1 in the exchange economy. CA_1 is improved. No effect on investement.

3) An increase in A_2 now has two effects that lead to a deterioration of the current account:
   i) For a given K_2, Y_2 is increased. As in the endowment economy, this leads to increased $C_1$and a deterioration in CA_1.
   ii) From $A_2 F'(K_2) = r$ we see that the optimal K_2 increases. Hence, I_1 increases and CA_1 deteriorates.

   The increase in K_2 that comes out of ii) has on the margin no net effect on income in period 2 since K_2 is optimized initially (the envelope theorem).

4) An increase in K_1 has two opposing effects

   i) It increases total wealth. Part of this is spent on C_1. Hence, CA_1 deteriorates, but less than the increase in K_1.

   ii) Since K_2 is unaffected, the increase in K_1 reduces I_1 and improves CA_1 one for one.

In this case the second effect obviously dominates. Countries with a high initial capital stock will *ceteris paribus* tend to have a CA surplus.

World equilibrium with investment

New opportunities:

- The sum of world output over the two periods can be increased by consuming less and investing more in the first period.
- The distribution of world consumption between the two periods can be smoothed by adjusting investment in the first period.

Equilibrium is characterized by

- *efficiency in production*

$$A_2 F'(K_2) = A_2^* F'(K_2^*) = r \qquad (13)$$

Productive capital is distributed in a way that maximizes second period output

- *efficiency in distribution*

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+r} \qquad (14)$$

Consumers cannot increase utility by exchanging goods between them.

- *overall efficiency*

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+A_2F'(K_2)} = \frac{1}{1+A_2^*F'(K_2^*)} \qquad (15)$$

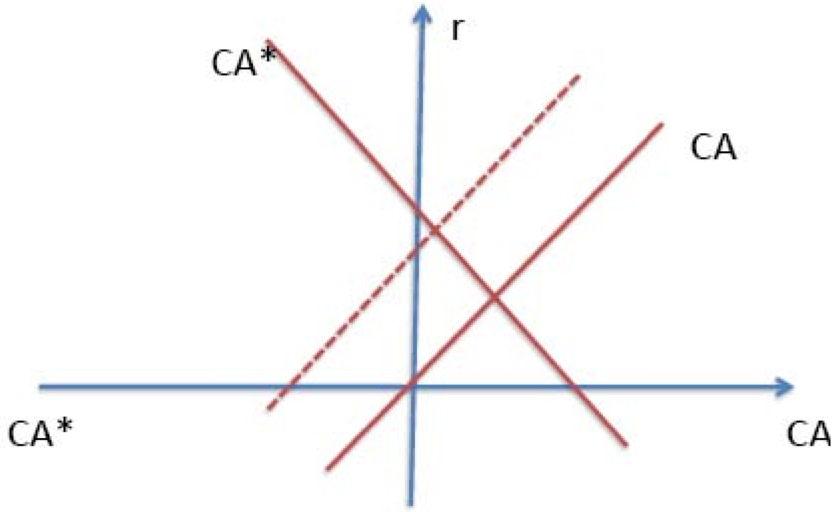
MRS = MRT No gain from moving output between the periods

Standard results on the efficiency of competitive equilibrium apply.

Standard results on the gains from trade apply.

New source of gains from trade: More efficient use of capital.

Two-country equilibrium, period 1



Increase in A_2 shifts $CA_1$to the left.

$A_2\uparrow \quad \rightarrow \quad r\uparrow, \quad$ CA↓, CA*↑

$$A_2F'(K_2) = r$$

Opposing effects on $I_1 = K_2 - K_1$ from A_2 and r.

- Increased return to investment
- More demand for C_1

Net effect on I_1 ambiguous, negative if strong desire for consumption smoothing.

K_2^* and I_1^* ↓ , since r is up and A_2^* is unchanged.

How are Home and Foreign welfare affected by an increase in A_2?

All effects on Foreign come through the increase in *r.*

→ Foreign gains if it is a net lender, loses if it is a net borrower.

Home has in addition a direct positive effect from A_2.

→ If a net lender, home gains on both accounts

→ If a net borrower, home gains on A_2 and loses on *r*. Net effect ambiguous.

Immiserizing growth, most likely if

- Strong preference for smoothing of consumption (low σs)
- Low or no response of investment to interest rate
- High debt

Real problem or theoretical curiosity?

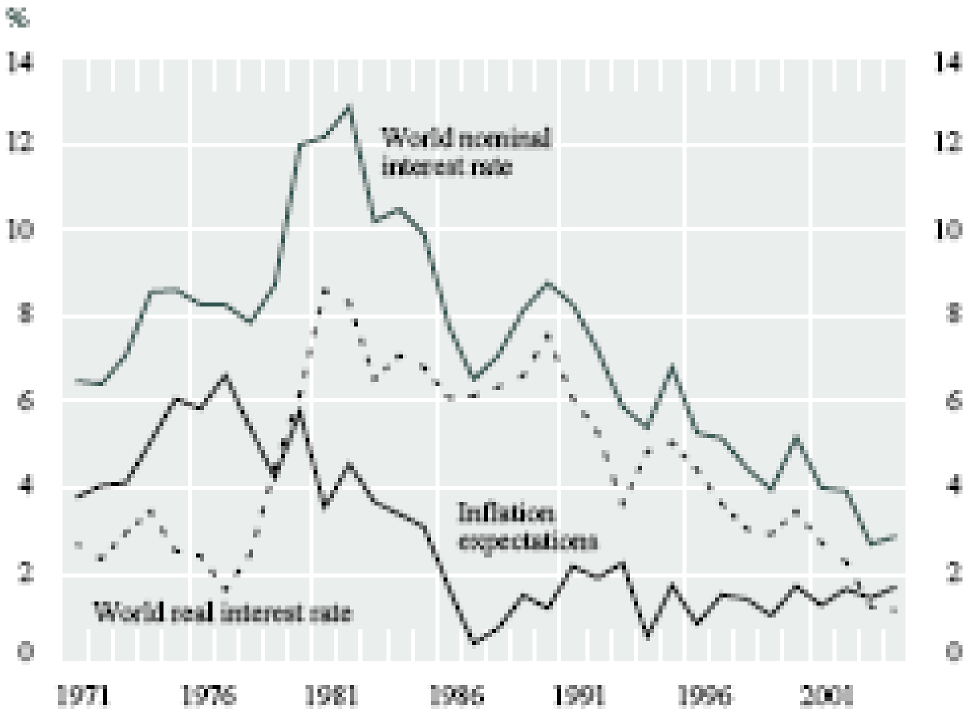
Current account, saving and investment in real capital in per cent of GDP 2006

Country	Current account	Saving	Investment
Germany	5,0	22,8	17,8
Japan	3,9	28,0	24,1
Developing Asia	6,1	43,9	37,9
United Kingdom	-3,2	14,8	18,0
United States	-6,2	14,1	20,0
World	0,3	23,3	23,0

The figures are for *gross* saving and *gross* investment

Chart 1

World Interest Rates and Inflation Expectations



Source: World Bank, BIS, IMF, Bank of Canada calculations

Bank of Canada Review • Winter 2006–2007

Infinite horizons – constraints

- Preparing for lecture 3: *Dynamics of small open economies (OR Ch.2)*

Finite horizon:

Loans have to be paid back with interest in the end

Infinite horizon:

Loans can be rolled over ad infinitum as long as (at least a sufficient part of) the interest is paid

Foreign assets evolves according to

$$B_{s+1} - B_s = Y_s + rB_s - (C_s + G_s + I_s), \quad s = t, t+1, t+2, ... \qquad (1)$$

- B_s is net foreign assets at end of period $s-1$

Budget constraints with infinite horizons

Two suggestions:

1) *The present value constraint:*

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+\mathrm{r}}\right)^{s-t}(\mathrm{C_s}+\mathrm{I_s}+\mathrm{G_s}) \leq (1+\mathrm{r})\mathrm{B_t}+\sum_{s=t}^{\infty}\left(\frac{1}{1+\mathrm{r}}\right)^{s-t}\mathrm{Y_s} \qquad (2)$$

The present value of expenditure should not exceed the value of the initial asset holding plus the present value of income.

2) *The no-Ponzi-game condition:*

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1} \geq 0 \qquad (3)$$

Debt shouldn't grow faster than the interest rate. Do not pay all interest with new loans!

More on the budget constraints

- The present value constraint works only if income in the long run grow less than the interest rate (otherwise the sums diverges)
- The no-Ponzi-game condition implies the present value constraint when the income growth rate is below the interest rate
- If the growth rate is above the intrest rate, there are objections also to the no-Ponzi-game condition

Last four slides contain more for those interested!

Here:

- Focus first on the case where the interest rate is above the growth rate
- Treat the two budget constraints as equivalent

The debt limit: Reorganize (3)

$$-(1+r)B_t \leq \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} X_s$$

where $X_t = Y_t - (C_t + I_t + G_t)$ is the *trade surplus*

- Future *trade* surpluses must be sufficiently large

If the trade surplus is kept constant equal to X, how large does it have to be?

$$-(1+r)B_t \leq \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} X = \frac{1}{1-(\frac{1}{1+r})}X = \frac{1+r}{r}X$$

Required trade surplus: $X \geq rB_t$

Required trade surplus, per cent of GDP

Debt to GDP ratio	Real interest rates	
	0.02	0.04
0.5	0.01	0.02
1.0	0.02	0.04
1.5	0.03	0.06
2.0	0.04	0.08

- How large trade surpluses are achievable?
- Default risk may give rise to a lower debt limit
- Debt limits are on individual borrowers, not on nations
- What if the economy is growing?

What if the economy is growing?

Balanced growth with rate γ per period: $Y_{t+1} = (1+\gamma)Y_t,\ X_{t+1} = (1+\gamma)X_t$

Ratio of trade surplus to GDP constant

When $r > \gamma$ we need

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s = \sum_{s=t}^{\infty} \left(\frac{1+\gamma}{1+r}\right)^{s-t} X_t = \frac{1+r}{r-\gamma} X_t \geq -(1+r)B_t$$

Required trade surplus: $X_t \geq -(r-\gamma)B_t$

Implication for debt if $X_t = -(r-\gamma)B_t$:

$$B_{t+1} = (1+r)B_t + X_t = (1+r)B_t - (r-\gamma)B_t = (1+\gamma)B_t$$

Foreign debt grows at the same rate as the rest of the economy.

$$X_t \geq -(r-\gamma)B_t$$

$r > \gamma$ For a quantitative example, replace r with $r - \gamma$ in table above

$r < \gamma$ Present value of trade surpluses does not exist ("are infinite")

- present value constraint breaks down
- no-Ponzi-game condition may still be applied, but yields disputable results
- if X_t and Y_t grow at the rate γ, and the (constant) ratio of the trade surplus to GDP is x, then it can be shown that the ratio of net foreign assets to GDP converges to

$$\bar{b} = \frac{x}{r-\gamma}$$

- negative x, may result in a huge debt ($\bar{b} < 0$), but debt doesn't explode
- current account deficits may seem unproblematic for a while
- r doesn't stay below γ forever. Debt crisis when r goes up?

Deriving the present value budget constraint

$$B_{s+1} - B_s = Y_s + rB_s - (C_s + G_s + I_s), \quad s = t, t+1, t+2, ... \qquad (1)$$

- B_s is net foreign assets at end of period $s-1$

1.Start with $s = t$. Use (1) to calculate B_{t+1}

2. Set $s = t + 1$ in (1), insert for B_{t+1} from step 1 and calculate B_{t+2}

3. Continue for $s = t + 2, t + 3, \dots, t + T$

$$B_{t+T+1} = (1+r)^{T+1} B_t + \sum_{s=t}^{t+T} (1+r)^{T-(s-t)} \, [Y_s - (C_s + G_s + I_s)], \qquad (2)$$

In order to get present values, divide by $(1+r)^T$:

$$(1+r)^{-T}B_{T+1} = (1+r)B_t + \sum_{s=t}^{t+T}(1+r)^{-(s-t)}\,[Y_s - (C_s + G_s + I_s)],$$

Reorganize:

$$\sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t}(C_s + I_s + G_s) + \left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = (1+r)B_t + \sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t} Y_s \qquad (3)$$

Take the limit as $T \to \infty$ and you get

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s + I_s + G_s) = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s \qquad (4)$$

provided that all the limits exist and that

$$LIM = \lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = 0 \qquad (5)$$

$$LIM = \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0 \qquad (5)$$

Suppose *LIM < 0.* This means

- for large T, $B_{t+T+1} < 0$ and growing in absolute value at a rate greater than or equal to *r*.
- the country finances all interest payments by acquiring new debt.
- creditors will not accept that this goes on forever.
- *LIM* has to be greater than or equal to zero.

-

Suppose *LIM > 0*. This means that

- - for large T, $B_{t+T+1} > 0$ and growing at a rate greater than or equal to *r.*
- the country is providing resources to others without getting anything in return.
- consumption can be increased at no cost.
- utility maximization demands *LIM* to be zero or negative.

Conclusion: LIM = 0.

Consistent with this:

- With an infinite horizon debt can be rolled over forever as long as some of the interest is paid from present income.
- To continue acquiring foreign assets forever can be consistent with utility maximization as long as some of the interest received is actually consumed.