Dynamics of Small Open Economies

Econ 4330 International Macroeconomics Spring 2011

Lecture 4 Part A

Asbjørn Rødseth February, 8 2011

$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r} W_t$$

Lessons about CA from the infinite horizon models

- Temporarily high output causes surpluses
- Temporarily high investment or gov. consumption cause deficits
- Temporarily high or low taxes do not matter.
- The more persistent output shocks are, the lower is the initial CA effect
- Expectations of fast-output growth produce deficits
- A high degree of impatience among consumers produce deficits
- Countries with a high marginal productivity of capital tend to get an initial deficit if they open up to international capital markets
- Deficits or surpluses may continue indefinitely

Do the conclusions fit with experience?

Do the conclusions survive if we enrich the theory?

The effect of output growth on the CA

Consumption growth determined by Euler equation: $u'(C_t) = \beta(1+r)u'(C_{t+1})$

Output growth determined by exogenous productivity growth (and investment)

Local consumption growth is independent of local output growth

Euler equation with CRRA-utility: $C_{t+1} = [\beta(1+r)]^{\sigma} C_t = (1+v)C_t$

v = rate of growth of consumption

If $\beta < 1$ and $\sigma < 1$, $v < r$. Assume this.

Solution of the small open economy model

From last weeks lecture with $I_t = 0$ and $G_t = 0$

$$W_t = (1+r)B_t + \frac{1+r}{r}\tilde{Y}_t$$

$$C_t = \frac{r-v}{1+r}W_t = (r-v)B_t + \frac{r-v}{r}\tilde{Y}_t$$

$$CA_t = rB_t + Y_t - C_t = Y_t - \frac{r-v}{r}\tilde{Y}_t + vB_t$$

Consequences of different rates of trend output growth

$Y_{t+1} = (1+g)Y_t$

g = outupt growth rate, $r > g$ assumed

$$\tilde{Y}_t = \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(1+g)^{s-t}Y_t = \left(\frac{r}{1+r}\right)\left(\frac{1}{1-\frac{1+g}{1+r}}\right)Y_t = \frac{r}{r-g}Y_t$$

Insert this in the current account equation:

$$CA_t = Y_t - \frac{r-v}{r}\left(\frac{r}{r-g}\right)Y_t + vB_t = \frac{v-g}{r-g}Y_t + vB_t$$

$(v-g)/(r-g)$ is the savings rate out of current income from production

From interest income vB_t is saved, while $(r-v)B_t$ is spent

Stricter rule than for the Petroleum Fund!

$v > 0 \Rightarrow$, $B_t > 0$ contributes to $CA_t > 0$, and increased B_{t+1}.

$$CA_t = \frac{v-g}{r-g} Y_t + vB_t$$

The country with low output growth

$g < v$ Output growth lower than consumption growth

- share of output saved $(v-g)/(r-g)$, is positive.
- savings rate can be huge even if *v-g* is small
- high savings needed to raise consumption faster than income.
- starting level of consumption is low
- net foreign assets will be increasing

The country with high output growth

$g > v$: Output growth higher than consumption growth

- share of output saved $(v-g)/(r-g)$, is negative
- starting level of consumption is high
- net foreign debt is increasing

What happens to net foreign assets in the long run?

$$B_{t+1} = B_t + CA_t = (1+v)B_t + \frac{v-g}{r-g} Y_t \qquad (1)$$

Asset ratio: $b_t = B_t / Y_t$ (negative of the debt ratio)

Divide by Y_t on both sides of (1) :

$$\frac{B_{t+1}(1+g)}{Y_t(1+g)} = (1+v)\frac{B_t}{Y_t} + \frac{v-g}{r-g}$$

$$b_{t+1}(1+g) = (1+v)b_t + \frac{v-g}{r-g}$$

$$b_{t+1} = \frac{1+v}{1+g} b_t + \frac{v-g}{(1+g)(r-g)} \qquad (2)$$

First order difference equation, solution see Berck and Sydsæter

Solution of (2) is

$$b_s = \left(\frac{1+v}{1+g}\right)^{s-t}\left(b_t + \frac{1}{r-g}\right) - \frac{1}{r-g}$$

or

$$b_s = \left(\frac{1+v}{1+g}\right)^{s-t}\left(b_t - \overline{b}\right) + \overline{b}$$

where $\overline{b} = -\frac{1}{r-g} < 0$ is the *stationary* level of b_t, the level that makes $b_{t+1} = b_t$

- for $g < v$ solution is *explosive*, movement is away from $\overline{b}$
- for $g > v$ solution is *stable,* movemenet is towards $\overline{b}$

If $b_t = \overline{b}$, then $W_t = 0$. All resources are needed to service the debt.

Proof: $W_t = (1+r)B_t + \frac{1+r}{r-g}Y_t = (1+r)\overline{b}Y_t + \frac{1+r}{r-g}Y_t = 0$

$$b_s = \left(\frac{1+v}{1+g}\right)^{s-t}\left(b_t + \frac{1}{r-g}\right) - \frac{1}{r-g}$$

High output growth country $g > v$

When $s \to \infty \Rightarrow b_s \to -\frac{1}{r-g}$

- Debt ratio goes to a (high) constant
- The share of consumption in output tends to zero.
- The share of GDP used to pay interest on the foreign debt tends to one.

Low output growth country $g < v$

- If $b_t > -1/(r-g)$ initially b_s will become positive and grow without limit
- The country will rely increasingly on interest income to finance consumption
- If $b_t < -1/(r-g)$ initially the country is bankrupt

The long run solution is not meaningful

Sooner or later the small country ceases to be small

Default risks

Is the present value budget constraint too permissive?

Constraints on the debt to GDP-ratio mayl force fast-growing countries to borrow less

Are preferences homothetic? If not, consumption growth rates change over time.

Constant growth rates forever? No!

The case with equal growth rates

Two countries, same β, σ and g

Since world consumption and output growth rates have to be equal, the world interest rate is determined by

$$1 + \nu = [\beta(1 + r)]^{\sigma} = 1 + g$$

Equal growth rates in (1) yields:

$$CA_t = gB_t$$

Net foreign assets grows with rate g

$$\frac{B_{t+1}}{B_t} = \frac{B_t + CA_t}{B_t} = \frac{B_t + gB_t}{B_t} = 1 + g$$

Consumption is

$$C_t = (r - g)B_t + Y_t$$

and grows with rate g

Overlapping generations and life-cycle saving

International Macro: Lecture 4

Asbjørn Rødseth

University of Oslo

February 8, 2011

Motivation

- Individuals have finite lives
- The economy persists
- Individual decision making, not dynastic

Different from infinite horizon model

- Positive relation between savings and growth
- Wealth to GDP ratios are bounded
- Timing of taxes matter for current accounts

Main assumptions

- Small open economy
- Output exogenous (endowment economy)
- Given world interest rate
- Consumers live for two periods
- Generations overlap
- No bequests and no gifts from children
- One representative consumer for each generation

Behavior of individual consumer

Utility function, generation borne at t:

$$U = u(c_t^Y) + \beta u(c_{t+1}^O) \tag{1}$$

Budget constraint:

$$c_t^Y + (1+r)^{-1} c_{t+1}^O = y_t^Y + (1+r)^{-1} y_{t+1}^O = w_t \tag{2}$$

w_t total life-time wealth of individual of generation t
First order condition:

$$u'(c_t^Y) = \beta(1+r)u'(c_{t+1}^O) \tag{3}$$

Example: log utility

$$u(c) = \ln c \tag{4}$$

Special case of CRRA-utility with $\sigma = 1$
First order condition:

$$1/c_t^Y = \beta(1+r)/c_{t+1}^O$$

or

$$c_{t+1}^O = \beta(1+r)c_t^Y$$

Insert in budget equation, solve and get:

$$c_t^Y = \frac{w_t}{1+\beta}, \qquad c_{t+1}^O = \frac{\beta(1+r)w_t}{1+\beta} \tag{5}$$

Saving when young

Individual saving (use (5) and (2)):

$$s_t^Y = y_t^Y - c_t^Y = \frac{1}{(1+\beta)(1+r)}\left[\beta(1+r)y_t^Y - y_{t+1}^O\right] \qquad (6)$$

$e =$ the growth rate of income from young to old,
$y_{t+1}^O = (1+e)y_t^Y$.

$$s_t^Y = \frac{1}{(1+\beta)(1+r)}\left[\beta(1+r) - (1+e)\right] y_t^Y$$

Savings rate of the young is then:

$$\mu = s_t^Y / y_t^Y = \frac{[\beta(1+r) - 1] - e}{(1+\beta)(1+r)} \qquad (7)$$

Saving when young

$$\mu = \frac{[\beta(1+r)-1]-e)}{(1+\beta)(1+r)}$$

Two reasons for saving:

- the return is high enough to overcome impatience $\beta(1+r) > 1$.
- income is lower when old $e < 1$

Retirement creates need for saving.

Saving when old

Saving when old is the negative of saving when young:

$$s^O_{t+1} = -s^Y_t$$

The sum of saving over the individual life-cycle is zero
+ Standard assumption $y^O << y^Y$

$$\Rightarrow$$

The young are saving, the old are dissaving.

Aggregate saving

The young save, the old dissave
+ Sum of savings over individual life-cycle is zero

$$\Rightarrow$$

Aggregate savings positive only if young are richer or more numerous than old.

Aggregation

Total savings

$$S_t = N_t s_t^Y + N_{t-1} s_t^O \tag{8}$$

N_t Size of young generation at t
Total financial assets of households at end of period t:

$$B_{t+1}^P = N_t s_t^Y \tag{9}$$

Total household income:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O \tag{10}$$

Growth and savings

- n growth rate of population $N_{t+1} = (1+n)N_t$
- g growth rate of income between generations $y^Y_{t+1} = (1+g)y^Y_t$
- e growth rate of income over life-cycle, $y^O_{t+1} = (1+e)y^Y_t$

$$
\begin{aligned}
S_t &= N_t \mu y^Y_t - N_{t-1}\mu y^Y_{(t-1)} \\
&= N_t \mu y^Y_t - N_t(1+n)^{-1}\mu y^Y_t(1+g)^{-1} \\
&= \mu N_t y_t[(1+n)(1+g)-1]/[(1+n)(1+g)] \qquad (11)
\end{aligned}
$$

No growth, no net saving

Growth and savings cont.

Aggregate output:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O = N_t y_t \frac{(1+n)(1+g)+(1+e)}{(1+n)(1+g)} \qquad (12)$$

Aggregate savings rate:

$$\begin{aligned} \frac{S_t}{Y_t} &= \mu \frac{(1+n)(1+g)-1}{(1+n)(1+g)+(1+e)} \\ &= \left(\frac{[\beta(1+r)-1]-e}{(1+\beta)(1+r)} \right) \left(\frac{n+g+ng}{2+n+g+ng+e} \right) \end{aligned} \qquad (13)$$

(Compare p. 150 in OR, where $n = 0$ and $\beta(1+r) = 1$).

Growth and savings cont.

Focus on case where $e < 0$ and $\beta(1 + r) = 1$

$$\mu = \frac{-e}{(1+\beta)(1+r)} = \frac{-\beta e}{1+\beta}$$

$$\frac{S_t}{Y_t} = -e\left(\frac{n+g+ng}{2+n+g+ng+e}\right)\frac{\beta}{1+\beta}$$

Savings rate is

- decreasing in e
- Increasing in n
- Increasing in g

Life-cycle model and timing of taxes

- Infinite horizon consumers: Compensates for tax reduction by saving more because they know they have to pay-back through higher taxes later
- Life-cycle consumer: Spends the part of the tax reduction that is going to be paid by future generations

Life-cycle model - evaluation

- Can explain that fast growing countries save more
- Net foreign assets stays within limits
- Retirement saving
- Precautionary saving
- Borrowing constraints
- Other life-cycle related motives
- Bequests without dynastic optimization

Investment and growth

Production function (constant returns):

$$Y = F(K, AN) = ANF(K/AN, 1) = ANf(k) \tag{14}$$

$k = K/AN$

First order condition:

$$f'(k) = r \tag{15}$$

k constant when r constant

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{K_t}{A_t N_t} = k \tag{16}$$

Investment and growth

$$K_{t+1} = K_t \frac{A_{t+1} N_{t+1}}{A_t N_t} = K_t(1+g)(1+n) \tag{17}$$

$$I_t = K_{t+1} - K_t = [(1+g)(1+n) - 1]K_t = (g+n+gn)K_t \tag{18}$$

- Investment rate high in fast-growing economies
- Feldstein-Horioka puzzle

Saving in corporations

- Norway 2008: Saving in the corporate sector six times saving in the household sector
- China 2008: 44 per cent of savings from household sector, 35 from corporate, 20 from government
- Households own corporations
- Governments own corporations
- Importance of income distribution for savings