The foreign exchange market

Econ 4330 Lecture 6

Asbjørn Rødseth

University of Oslo

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Outline

The mean-variance model

The representative home investor maximizes

$$U = \mathrm{E}(\pi) - \frac{1}{2} R var(\pi) \tag{1}$$

subject to

$$\pi = (1 - f)i + f(i_* + e) - p \tag{2}$$

- R = relative risk aversion
- π = real rate of return
- $f = EF/PW$ = share of foreign currency in portfolio
- i, i_* = domestic and foreign interests rate
- e, p = expected rates of depreciation and inflation

Calculation of expected return and risk

$$\pi = (1 - f)i + f(i_* + e) - p$$

$$\mathrm{E}(\pi) = (1 - f)i + f(i_* + \mu_e) - \mu_p \tag{3}$$

$$var(\pi) = f^2\sigma_{ee} + \sigma_{pp} - 2f\sigma_{ep} \tag{4}$$

- Stochastic variables e and p
- Expectations μ_e and μ_p
- Variances σ_{ee}, σ_{pp}
- Covariance σ_{ep}

First-order condition

$$\frac{dU}{df} = \frac{d\mathrm{E}(\pi)}{df} - \frac{1}{2}R\frac{dvar(\pi)}{df} = 0 \tag{5}$$

Solution

$$f = \frac{\sigma_{ep}}{\sigma_{ee}} - \frac{r}{R\sigma_{ee}} = f_M + f_S \tag{6}$$

$r = i - i_* - \mu_e$ is the risk premium on kroner

1. *The minimum-variance portfolio* $f_M = \sigma_{ep}/\sigma_{ee}$
2. *The speculative portfolio* $f_S = -r/R\sigma_{ee}$

The minimum-variance portfolio

$$f_M = \frac{\sigma_{ep}}{\sigma_{ee}} \qquad b_M = \frac{-\sigma_{ep_*}}{\sigma_{ee}}$$

Examples:

1. *Relative purchasing power parity* $e = p - p_*$.
   - Assume inflation rates uncorrelated ($\sigma_{pp_*} = 0$)
   - $f_M = \sigma_{pp}/(\sigma_{p_*p_*} + \sigma_{pp}) = 1 - b_M$
   - No home bias
2. *Inflation and exchange rates uncorrelated* ($\sigma_{ep} = 0$, $\sigma_{ep_*} = 0$)
   - $f_M = 0$ and $1 - b_M = 1$
   - Strong home bias

Deviations from PPP create home bias, $1 - f_M > b_M$
Portfolio shares normally between 0 and 1 when $\sigma_{ep} > 0$

The speculative portfolio

$$f_S = -\frac{r}{R\sigma_{ee}} \qquad b_S = \frac{r}{R\sigma_{ee}}$$

- Symmetric, no home bias
- Goes towards currency with highest expected return
- Absolute level depends negatively on risk and risk aversion

Overall portfolio will have home bias: Domestic residents invest a larger share of their wealth in domestic currency than do foreigners

$$1 - f > b$$

Foreign exchange market equilibrium

$$F_p + F_* + F_g = 0 \tag{7}$$

$$F_p = fPW_p/E = \left[\frac{\sigma_{ep}}{\sigma_{ee}} - \frac{r}{R\sigma_{ee}}\right] PW_p/E \tag{8}$$

$$F_* = (1-b)P_*W_* = \left[1 + \frac{\sigma_{ep_*}}{\sigma_{ee}} - \frac{r}{R\sigma_{ee}}\right] P_*W_* \tag{9}$$

$$W_p = (B_{p0} + EF_{p0})/P, \qquad W_* = (B_{*0}/E + F_{*0})/P_* \tag{10}$$

Can be solved for E, F_g or r.

The portfolio composition effect

- Depreciation of kroner $E \uparrow$
- Reduced share of kroner in portfolio $B/EF \downarrow$
- Sell dollars, buy kroner to keep f constant $F \downarrow$, $B \uparrow$
- Supply of dollars directed towards Norges Bank increases

The portfolio composition effect

The supply of foreign currency to the central bank:

$$F^S = -f\frac{PW_p}{E} - (1-b)P_*W_* = -f\left[\frac{B_{p0}}{E} + F_{p0}\right] - (1-b)\left[\frac{B_{*0}}{E} + F_{*0}\right] \tag{11}$$

Slope:

$$\frac{dF^S}{dE} = f\frac{B_{p0}}{E^2} + (1-b)\frac{B_{*0}}{E^2} \tag{12}$$

- Positive slope if $0 < f < 1$, $0 < b < 1$, $B_{p0} > 0$, $B_{*0} > 0$
- Excessive speculation may reverse the slope
- Taken together governments are usually net borrowers, private sectors net lenders

The degree of capital mobility

$$F^S = -f\frac{PW_p}{E} - (1-b)P_*W_* = -f\left[\frac{B_{p0}}{E} + F_{p0}\right] - (1-b)\left[\frac{B_{*0}}{E} + F_{*0}\right]$$

The degree of capital mobility is

$$\kappa = \frac{dF^S}{dr} = \frac{1}{R\sigma_{ee}}[W_p + \frac{EP_*}{P}W_*]$$

- The basis is world private wealth
- Capital mobility is related to the speculative portfolio
- Historical values of exchange rate volatility (σ_{ee}) combined with relative risk aversion below 2 yield high capital mobility
- Traditional tests reject perfect capital mobility
- Or is it rational expectations that are rejected?

The equilibrium risk premium

$$r = R\sigma_{ee}(\bar{b} - \bar{b}_M) \tag{13}$$

where

$$\bar{b} = 1 - \frac{E(F_p + F_*)}{PW_p + EP_*W_*}$$

$$\bar{b}_M = 1 - \frac{f_M PW_p + (1 - b_M)EP_*W_*}{PW_p + EP_*W_*}$$

The equilibrium risk premium is a product of:

1. The exchange rate risk (σ_{ee})
2. The risk aversion of investors (R)
3. Risk exposure - the difference between the market portfolio and the minimum variance portfolio ($\bar{b} - \bar{b}_M$)

Market portfolio - mirror image of government portfolio

Observations on the risk premium

- Will be negative if the market contains less kroner than the MV portfolio
- $\sigma_{ee} = 0$ or $R = 0$ implies perfect capital mobility and $r = 0$ for any level of exposure.
- Interest rates are observed directly, expectations and risk premia difficult to measure.
- In surveys investors declare widely different expectations
- Interest rates often contain an (il)liquidity premium.

Impact of the current account

$$F^S = -f\frac{PW_p}{E} - (1-b)P_*W_* \tag{14}$$

For E, P and P_* constant the change over time is

$$\Delta F^S = -f\frac{P}{E}\Delta W_p - (1-b)P_*\Delta W_* \tag{15}$$

- Change in private wealth = current account + government deficit, $\Delta W_p = CA + GD$.
- Change in foreign wealth = - current account surplus, $\Delta W_* = -PCA/EP_*$

Hence,

$$\Delta F^S = \frac{P}{E}[-f(CA+GD) - (1-b)CA] = \frac{P}{E}[(1-f-b)CA - fGD] \tag{16}$$

Impact of the current account

$$\Delta F^S = \frac{P}{E}[(1 - f - b)CA - fGD] \tag{17}$$

- Home bias $1 - f > b$ ensures that a current account surplus increases supply of foreign currency
- Government deficit reduces supply of foreign currency when $f > 0$

Petroleum funds

$$\Delta F^S + \Delta F^{PF} = \frac{P}{E}[(1-f-b)CA + fGS]$$

- CA surplus due to high oil revenues creates appreciation pressure
- Government surplus GS reinforces appreciation
- Accumulate government surplus in Petroleum Fund and invest in foreign currency, $\Delta F^{PF} = PGS/E$

$$\begin{aligned}\Delta F^S &= \frac{P}{E}[(1-f-b)CA + fGS - GS] \\ &= \frac{P}{E}[(1-f-b)(CA-GS) - bGS]\end{aligned}$$

- Only CA surplus in excess of GS creates appreciation
- Slight depreciation pressure if foreigners sell kroner to finance their deficit