

Dynamics of Small Open Economies

Lecture 2, ECON 4330

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The models we have looked at so far are characterized by:

- A representative agent
- Two periods
- Perfect foresight
- Partial eq. if r is taken as constant
- General eq. if $CA_1(r) + CA_1^*(r) = 0$

Last lecture: Open economy??

So far, no reference to exchange rates, terms of trade, etc. Why not?

- Models we have time to look at: *Real* models where there are no nominal variables. Hence no nominal exchange rates either
- When each country produces a single good (identical across borders) with no trade frictions, the price is equal in all countries \rightarrow the real exchange rate equals one
- In chapter 4 we return to the issue of terms of trade and real exchange rates

Last lecture: Simple two-period endowment model

Home maximizes its utility $U = u(C_1) + \beta u(C_2)$ subject to the budget constraint $C_2 = (1 + r)(Y_1 - C_1) + Y_2$. First-order condition:

$$u'(C_1) = \beta(1 + r)u'((1 + r)(Y_1 - C_1) + Y_2) \quad (1)$$

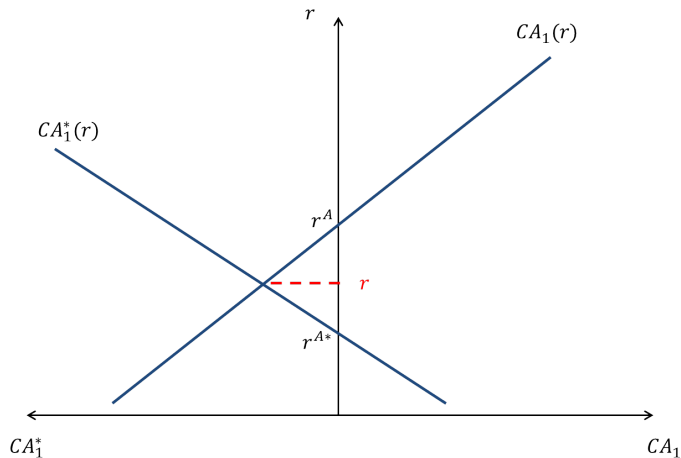
determines optimal period 1 consumption. (1) implicitly defines $C_1(r)$, such that the 'current account function' is:

$$CA_1(r) = Y_1 - C_1(r) \quad (2)$$

Same calculations can be made for Foreign. Gives $CA_1^*(r)$. Equilibrium real interest rate is found by imposing

$$CA_1(r) = -CA_1^*(r) \quad (3)$$

Last lecture: Simple two-period endowment model II



Last lecture: Adding investment

If we add capital [$Y_t = F(K_t)$], the new set of optimality conditions includes

$$A_2 F'(K_2) = r$$

i.e., the country will invest such that the rate of return at home equals the world interest rate.

Last lecture: Adding government spending

Not covered in the lecture was how to add government spending. Will look more at that in seminar 1, but for now accept that the only effect of government spending (G_t in period t) is simply that it reduces income by the same amount.

Last lecture: WHY?

Why were we looking at these models? Starting block for building microfounded models to understand the main determinants behind the current account and (if general equilibrium) world interest rate. According to our models, the main reasons for running a current account *deficit* are:

- (Expectations of) positive future income growth
- (Expectations of) higher productivity in the future (or a fall in this period's productivity)
- A temporary increase in government spending ($G_1 > G_2$)
- A sudden destruction of the initial capital stock (war?)
- A relatively low world interest rate (lower than a country's autarky rate – $r^A > r$)

Further, the world interest rate should reflect some measure of the world discount rate, growth rate, as well as people's elasticity of substitution.

Last lecture: WHY? Real life example

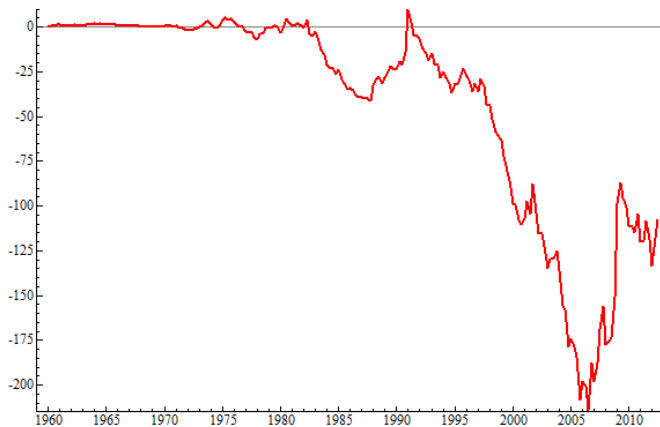


Figure: Quarterly US current account (billions)

Last lecture: WHY? Real life example II

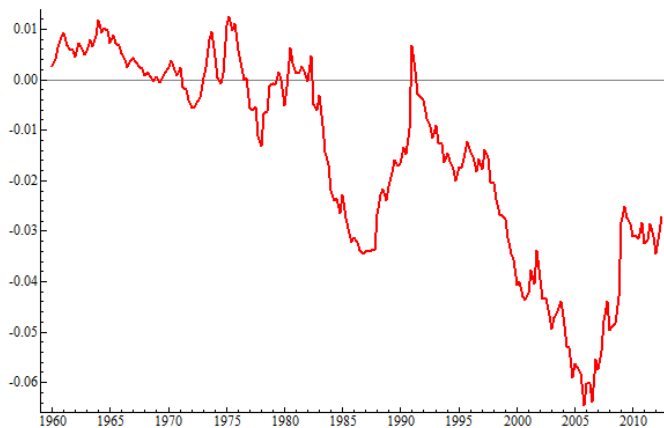


Figure: US current account relative to GDP

Last lecture: WHY? Real life example III

Can the GDP growth rate be helpful to explain CA/GDP ?

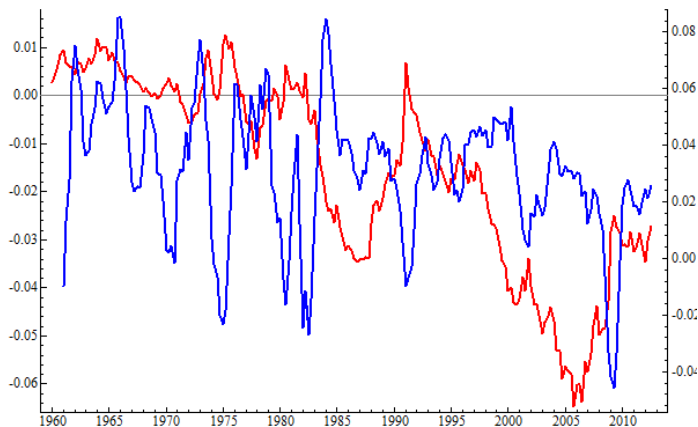


Figure: US current account relative to GDP (red, left axis) and annual real GDP growth rate (blue, right axis)

Last lecture: WHY? Real life example III

What about the US real interest rate? (As a proxy for the world interest rate)

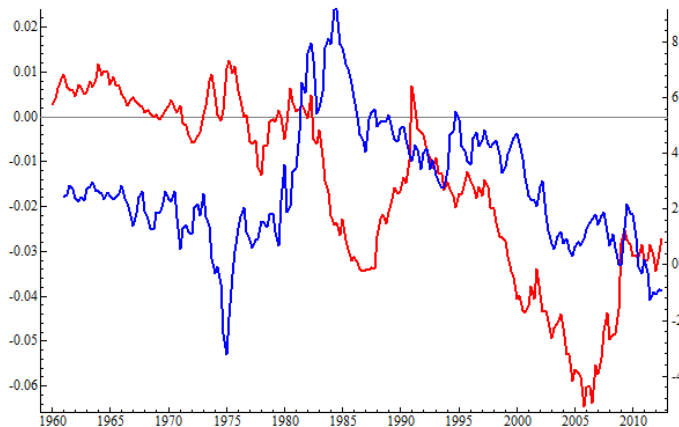


Figure: US current account relative to GDP (red, left axis) and annual real interest rate (blue, right axis)

Today's lecture

Seems like we're not fully there yet, but first models show some relevance. Today we will consider two changes/extensions that generalize the two-period results:

- Infinite horizon
- Uncertainty

Outline

- 1 Infinite horizon model
 - CES example
 - Solution for the current account
- 2 When is a country bankrupt?
- 3 Stochasticity
- 4 Implications for optimal investment
- 5 Fundamental CA when output is exogenous
- 6 Empirical relevance

Model

First consider the model from last lecture, but generalized to infinite horizon. Still perfect foresight.

Model II

Utility function:

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \dots = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \quad (4)$$

Production function, capital accumulation, current account definition and NFA-accumulation is defined precisely as before:

$$Y_s = A_s F(K_s) \quad (5)$$

$$K_{s+1} = K_s + I_s \quad (6)$$

$$CA_s = Y_s - C_s - I_s - G_s + r_s B_s \quad (7)$$

$$B_{s+1} = B_s + CA_s \quad (8)$$

for any period $s \geq t$. (Notice that I have added government spending)

Model III

What about the budget constraint? For every period s we have (as in the two-period model):

$$C_s + G_s + I_s + B_{s+1} = Y_s + (1 + r_s)B_s \quad (9)$$

The *intertemporal* budget constraint for a constant world interest rate is

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (C_s + I_s + G_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s \quad (10)$$

Model IV

The derivation of the intertemporal budget constraint (will be done in a seminar problem) rests on assuming

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T B_{t+T+1} \geq 0 \quad (11)$$

This is the *no Ponzi-game condition*. Further, utility maximization will require

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T B_{t+T+1} = 0$$

since you want to consume as much as possible.

Model V

To solve the model, we maximize utility—given by (4)—subject to (5)-(9) and (11). More compactly:

$$\max_{\{B_{s+1}, K_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} u([1+r]B_s - B_{s+1} + A_s F(K_s) - [K_{s+1} - K_s] - G_s)$$

given K_t , B_t and subject to the no-Ponzi condition (11).

Model VI

Let's optimize ignoring the no Ponzi condition. First-order condition for B_{s+1} :

$$\beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1})(1+r) = 0$$

where $C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s$. Can be re-arranged to give the (now familiar) consumption Euler equation:

$$u'(C_s) = \beta(1+r)u'(C_{s+1}) \quad (12)$$

This must hold for $s = t, t+1, t+2, \dots$

Model VII

First-order condition for K_{s+1} :

$$\beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1}) [1 + A_{s+1} F'(K_{s+1})] = 0$$

where $C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s$. Can be re-arranged to give

$$u'(C_s) = \beta[1 + A_{s+1} F'(K_{s+1})] u'(C_{s+1}) \quad (13)$$

This too must hold for $s = t, t+1, t+2, \dots$

Implications

- The first-order conditions (12) and (13), together with the initial conditions and no-Ponzi condition (11), describes the optimum path.
- In the two-period model we found a neat expression for optimal capital. Is it still there? If we combine (12) and (13), it is easy to see that they imply

$$A_{s+1}F'(K_{s+1}) = r$$

Hence consumption and investment decisions are still separated. The country will invest in capital as long as it pays off more than the world interest rate.

Implications II

So in principle, much is carried over from the two-period model. The Euler equation is standard, and investment requires $A_{s+1}F'(K_{s+1}) = r$, just as before. It is therefore essential to be able to interpret these equations.

Example

For an analytical example, let us once again assume CES-utility:

$$u(C) = \frac{1}{1 - \frac{1}{\sigma}} C^{1-1/\sigma}$$

Since

$$u'(C) = C^{-1/\sigma}$$

the Euler equation in this case is

$$C_{s+1} = [\beta(1+r)]^\sigma C_s = (1+g)C_s$$

where $g = [\beta(1+r)]^\sigma - 1$.

Example II

- What does this imply for the consumption path? Since the Euler equation holds for $s = t, t + 1, t + 2, \dots$, we get that

$$C_s = (1 + g)^{s-t} C_t$$

- So the optimal consumption in any period s is given by $(1 + g)^{s-t} C_t$. Consumption grows at a rate g .
- Consumption grows (forever) if $\beta(1 + r) = \frac{1+r}{1+\delta} > 1$, i.e. if the world interest rate is high enough ($r > \delta$). Put differently, for a given interest rate, the patient countries will have consumption growth. Impatient will have consumption declining forever.
- Realistic? One problem is that we treat r as exogenous.

Example III

How does this relate to the present-value budget constraint? Re-call equation (10):

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - I_s - G_s)$$

Insert for optimal consumption and let the RHS define W_t ('lifetime wealth').

$$\sum_{s=t}^{\infty} \left(\frac{1+g}{1+r} \right)^{s-t} C_t = W_t$$

As long as $g < r$ (which holds for $\beta < 1$ and $\sigma \leq 1$), consumption is not growing faster than the interest rate, and the sum converges. By the rules for geometric sums,

$$\sum_{s=t}^{\infty} \left(\frac{1+g}{1+r} \right)^{s-t} = \frac{1}{1 - \frac{1+g}{1+r}} = \frac{1+r}{r-g}$$

Example IV

It follows that

$$C_t = \frac{r - g}{1 + r} W_t$$

- $g = 0$: Consume the permanent income from your total wealth
- $g > 0$: Consume less than your permanent income. Gives you a rising consumption path
- $g < 0$: Consume more than the permanent income. Gives you a declining consumption path

The CA

What about the current account? To analyze the CA-dynamics, introduce the concept *permanent level* of X_t :

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \tilde{X}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} X_s$$

\tilde{X}_t is thus the hypothetical constant value of X which has the same present value as a given sequence $\{X_s\}_{s=t}^{\infty}$ at the interest rate r . Since \tilde{X}_t is constant for all s , we have:

$$\tilde{X}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} X_s$$

The CA II

We will use this trick to get an easy-to-interpret equation for the CA. First re-write the definition of W_t :

$$\begin{aligned} W_t &= (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - I_s - G_s) \\ &= (1+r)B_t + \frac{1+r}{r} \left(\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t \right) \\ &= \frac{1+r}{r} \left(rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t \right) \end{aligned}$$

The CA III

Then assume CES-utility again, in which case

$$\begin{aligned} C_t &= \frac{r-g}{1+r} W_t \\ &= rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t - \frac{g}{1+r} W_t \end{aligned}$$

The CA IV

Since the current account is defined as

$$CA_t = rB_t + Y_t - C_t - I_t - G_t$$

it follows that under CES-utility:

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{g}{1+r} W_t \quad (14)$$

O&R label (14) the fundamental equation for the current account. Gives an easy way to interpret the driving forces behind current account deficits and surpluses.

The CA V

So what drives the CA-development?

- The fruits from temp. high output will be spread across periods
- The pain from temp. high rates of investment will be reduced by borrowing abroad
- So to if government spending is temp. high
- But there may be an extra effect when $g \neq 0$, which stems from the desire to be on an upward or downward sloping consumption path

This shows us that the infinite horizon model in many ways *generalize* the results from our two-period model. Note two limitations:

- Exogenous interest rate
- Perfect foresight
- No-Ponzi is assumed to be satisfied. 'Unsustainable CA deficits' do not exist.

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What is a sustainable level of foreign debt?

Write the intertemporal budget constraint (10) as:

$$-(1+r)B_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - C_s - I_s - G_s)$$

From here we see that a country's foreign debt ($-B$) reflects the discounted sum of all future trade surpluses.

What is a sustainable level of foreign debt? II

Introduce the *break-even trade balance* tb^* as the constant trade balance to output level that makes the intertemporal budget constraint hold, and assume that $Y_s = (1 + g)^{s-t} Y_t$. The intertemporal budget constraint is then

$$-(1 + r)B_t = \sum_{s=t}^{\infty} \left(\frac{1}{1 + r} \right)^{s-t} tb^* (1 + g)^{s-t} Y_t$$

which can be re-arranged as

$$tb^* = (r - g) \frac{-B_t}{Y_t}$$

(assuming that $r > g$).

What is a sustainable level of foreign debt? III

- For a given foreign debt to GDP ratio, $-B/Y$, and for given estimates of the future world interest rate and country-specific growth rate, we can calculate the required trade balance (as share of GDP) the country must adjust to if it is to satisfy its intertemporal budget constraint.
- Assume $-B/Y = 1$, so $tb^* = r - g$. Not unreasonable to assume $r = 6\%$. If the expected growth rate is 2 percent, this means that the country must run a constant trade surplus of 4% of GDP *forever*.
- Will this make sure it repays all its debt?

What is a sustainable level of foreign debt? IV

Examples: Greece and Spain. Both have a net international investment position to GDP of (slightly below) 1. Assume optimistically $g = 2\%$ for both. Need $tb^* = 4\%$. Current situation:

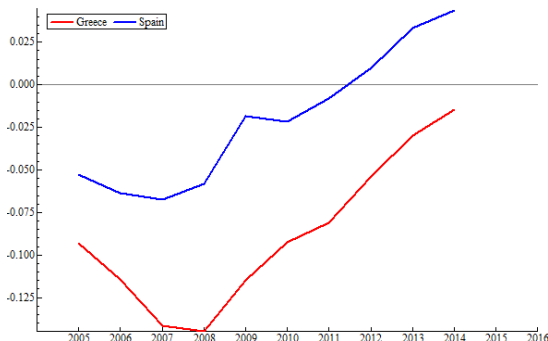


Figure: Trade balance to GDP for Greece and Spain

Looks like it is improving. But some of it is because of a fall in GDP..

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Stochastic model

Now it is time to introduce uncertainty in the model.

- Future levels of output, government spending, etc. are stochastic
- But we will still have the possibility of investing in a riskless bond paying an interest rate r
- Rational expectations/Model-consistent expectations

Stochastic model II

The utility function will be assumed to be given by

$$U_t = \mathbf{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \quad (15)$$

(15) is a simple generalization of the utility function used so far. Except for this change, all other conditions are unchanged. We maximize (15) subject to (5)-(8), and in addition (9) and (11), or alternatively (10). More compactly:

$$\max_{\{B_{s+1}, K_{s+1}\}_{s=t}^{\infty}} \mathbf{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s) \right\}$$

given K_t , B_t and subject to the no-Ponzi condition (11).

Stochastic model III

We will look at two interesting points.

- How does uncertainty affect the optimal investment decision?
- What is the new ‘fundamental’ current account equation for an exogenous income process?

Stochastic model IV

First-order condition for B_{s+1} :

$$\mathbf{E}_t \{ \beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1})(1+r) \} = 0$$

where $C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s$. Can be re-arranged to give:

$$\mathbf{E}_t \{ u'(C_s) \} = \beta(1+r) \mathbf{E}_t \{ u'(C_{s+1}) \} \quad (16)$$

So the consumption Euler equation must hold *in expectation* for $s = t, t+1, t+2, \dots$. Special case for $s = t$:

$$u'(C_t) = \beta(1+r) \mathbf{E}_t \{ u'(C_{t+1}) \}$$

Stochastic model V

To look at the optimal investment decision, we find the first-order condition for K_{s+1} :

$$\mathbf{E}_t \{ \beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1}) [1 + A_{s+1} F'(K_{s+1})] \} = 0$$

where $C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s$. Can be re-arranged to give

$$\mathbf{E}_t \{ u'(C_s) \} = \beta \mathbf{E}_t \{ [1 + A_{s+1} F'(K_{s+1})] u'(C_{s+1}) \} \quad (17)$$

This must hold for $s = t, t+1, t+2, \dots$. Special case for $s = t$:

$$u'(C_t) = \beta \mathbf{E}_t \{ [1 + A_{t+1} F'(K_{t+1})] u'(C_{t+1}) \}$$

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Implications for optimal investment

- With perfect foresight, we could combine the FOCs to get $A_s F'(K_s) = r$. Still possible?
- Consider the special case of $s = t$. The two conditions can be combined to give

$$\mathbf{E}_t \left\{ \frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)} A_{t+1} F'(K_{t+1}) \right\} = r$$

but it **cannot** be simplified further since, in general, C_{t+1} and A_{t+1} are correlated.

Implications for optimal investment II

To learn more, we can re-write the LHS as

$$\mathbf{E}_t \left\{ \frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)} \right\} \mathbf{E}_t \{ A_{t+1}F'(K_{t+1}) \} + \text{cov}_t \left(\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1}) \right) =$$

$$\mathbf{E}_t \{ A_{t+1}F'(K_{t+1}) \} + \text{cov}_t \left(\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1}) \right)$$

Implications for optimal investment III

So the new condition for optimal investment is

$$\mathbf{E}_t \{ A_{t+1} F'(K_{t+1}) \} = r - \text{cov}_t \left(\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}, A_{t+1} F'(K_{t+1}) \right)$$

The covariance is likely to be negative. If A_{t+1} turns out to be high, this will make consumption, C_{t+1} , high as well, reducing the marginal utility $u'(C_{t+1})$. You require a 'risk premia', and invest less than under perfect foresight. This is similar to consumption-CAPM which you will learn about in other courses.

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Fundamental CA

Assume that:

- $\beta(1+r) = 1$ (implying no trend growth in consumption in the absence of technological growth).
- Output is given by an exogenous process $Y_t = \rho Y_{t-1} + \varepsilon_t$
- Quadratic utility:

$$u(C) = C - \frac{a_0}{2} C^2$$

Fundamental CA II

With quadratic utility, marginal utility is simply $1 - a_0 C$. The Euler equation becomes

$$\mathbf{E}_t \{1 - a_0 C_s\} = \mathbf{E}_t \{1 - a_0 C_{s+1}\}$$

which implies

$$\mathbf{E}_t C_{s+1} = \mathbf{E}_t C_s$$

For $s = 1$ we get Hall's random walk result:

$$\mathbf{E}_t C_{t+1} = C_t \tag{18}$$

Consumption is a random walk

Hall's random walk once more

Hall's model predicts that

$$\mathbf{E}_t C_s = C_t$$

which is, as we've seen, a property it shares with random walks. Another implication, which can be motivated both by RE and by the RW itself, is that when expectations are rational,

$$\mathbf{E}_t C_s = C_s + \eta_s$$

where η_s is the *expectational error*. More importantly, $\mathbf{E}_t(\eta_s \eta_v) = 0$ for $s \neq v$. This gives us a testable prediction since $\mathbf{E}_t C_{t+1} = C_{t+1} + \eta_{t+1}$, and hence:

$$\Delta C_{t+1} = \eta_{t+1}$$

⇒ If consumption follows a random walk, then consumption growth should be completely unpredictable, and uncorrelated over time.

Fundamental CA III

What level of consumption? We know that the present-value budget constraint must hold:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s)$$

Since it holds for sure, it must also hold in expectation:

$$\mathbf{E}_t \left\{ \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s \right\} = (1+r)B_t + \mathbf{E}_t \left\{ \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s) \right\}$$

Fundamental CA IV

Since $\mathbf{E}_t C_{s+1} = \mathbf{E}_t C_s$, it follows that $\mathbf{E}_t C_{t+i} = C_t$ for any $i > 0$. Hence:

$$\begin{aligned} \mathbf{E}_t \left\{ \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s \right\} &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \mathbf{E}_t C_s \\ &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_t \\ &= \frac{1+r}{r} C_t \end{aligned}$$

We find consumption for period t to be:

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \mathbf{E}_t \{ (Y_s - G_s) \} \right]$$

Fundamental CA V

This case is an example of *certainty equivalence*. Even though the future is uncertain, the agent acts as if expected values were certain to be realized. Put differently: Risk is irrelevant. Why do we get that here?

- Quadratic utility
- Non-stochastic interest rate r

Fundamental CA VI

Then set $G_t = 0$. To see how the current account reacts to shocks, assume that income is an AR-process

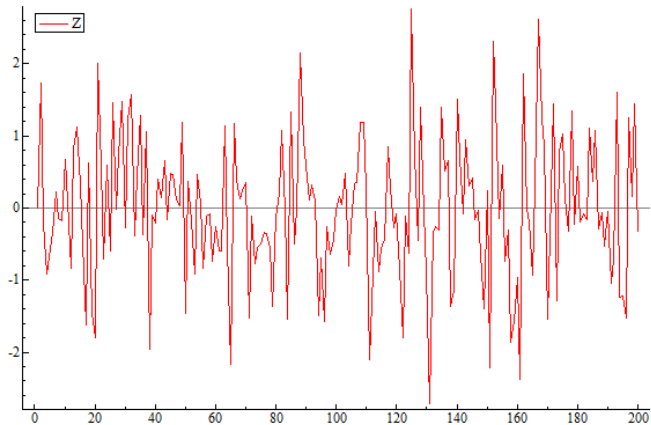
$$Y_t - \bar{Y} = \rho(Y_{t-1} - \bar{Y}) + \epsilon_t$$

ϵ_t is usually assumed to be iid $N(0, \sigma^2)$, while $0 \leq \rho \leq 1$. The AR-parameter ρ measures how persistent the process is.

- If $\rho = 0$, then income varies randomly around its mean
- If $0 < \rho < 1$, then income may deviate persistently away from its mean, but it will always return to it.

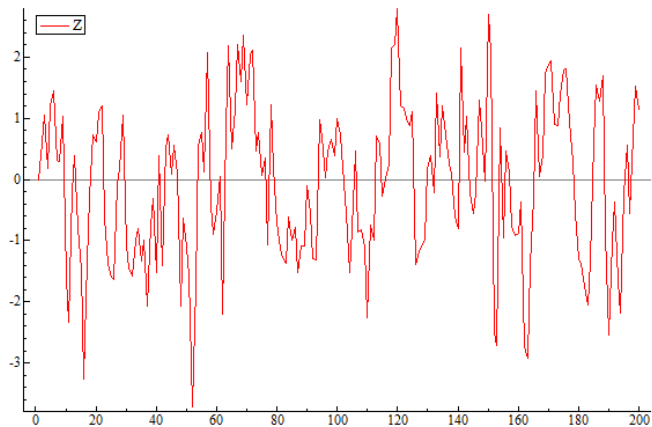
Fundamental CA VII

When $\rho = 0$:



Fundamental CA VIII

When $\rho = 0.5$:



Fundamental CA IX

In the special case of $\rho = 1$, we see that

$$Y_t = Y_{t-1} + \epsilon_t$$

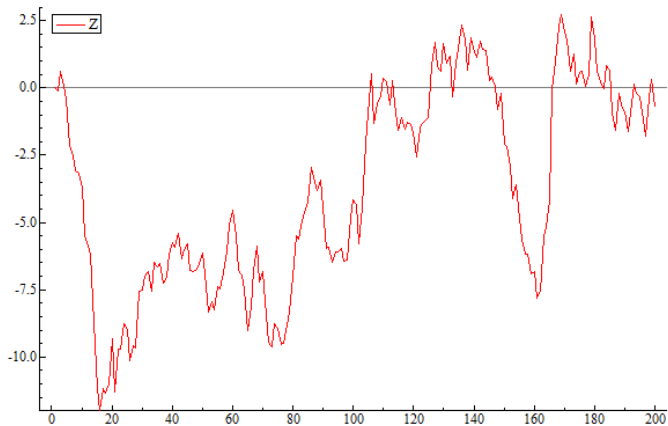
which after iterating backwards will give us

$$Y_t = Y_0 + \sum_{s=1}^t \epsilon_s$$

Hence any shock to Y_t is permanent, and the process is accumulating shocks over time.

Fundamental CA X

When $\rho = 1$:



Fundamental CA XI

Since $\mathbf{E}_t \varepsilon_{t+s} = 0$ for $s = 1, 2, \dots$, the expected value of an AR-process is given by

$$\mathbf{E}_t \{Y_s - \bar{Y}\} = \rho^{s-t}(Y_t - \bar{Y})$$

We can therefore start out with

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \mathbf{E}_t Y_s \right]$$

or:

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \mathbf{E}_t (Y_s - \bar{Y})$$

Fundamental CA XII

Inserting for what the expected values are, we get

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \rho^{s-t} (Y_t - \bar{Y})$$

and after using the formula for an infinite geometric sum:

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r-\rho} (Y_t - \bar{Y})$$

Fundamental CA XIII

Since the current account is

$$CA_t = Y_t - C_t + rB_t$$

we can insert for C_t to find

$$CA_t = \frac{1 - \rho}{1 + r - \rho} (Y_t - \bar{Y})$$

This is a slightly different, but very similar, version of the fundamental current account equation (14), but now derived in a stochastic environment. Only source of stochasticity is income. Deviations from long-run mean will cause current account surpluses or deficits. What happens if $\rho = 1$?

Impulse response functions

In standard stochastic macro models, an important component is the set of **structural shocks** that you include. In our previous example, income was the only source of uncertainty, and in principle we can regard it as a structural shock in this context. The structural shocks are what's driving the development over time. The structural shocks are typically AR(1)-processes, just like what we assumed for $Y_t - \bar{Y}$.

Impulse response functions II

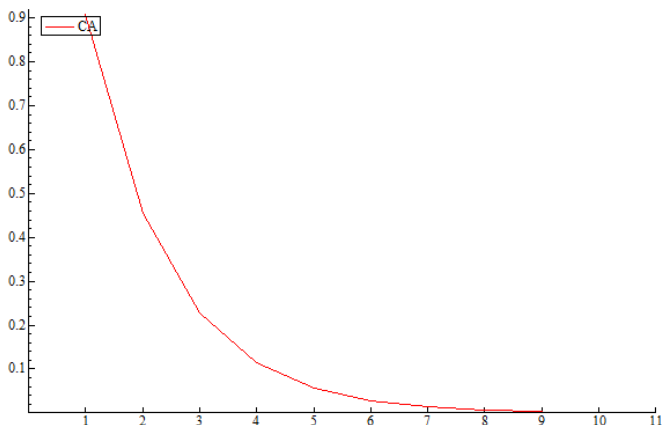
Impulse-response functions show how the endogenous variables of the model react to shocks. In our previous example, the structural (income) shock was:

$$Y_t - \bar{Y} = \rho(Y_{t-1} - \bar{Y}) + \epsilon_t$$

An impulse-response function will in this case show how C and CA react to a shock in income (ϵ_t), assuming that there are no other shocks afterwards ($\epsilon_{t+s} = 0$).

Impulse response functions III

For instance, if $\rho = 0.5$ and $r = 0.05$, the IRF for CA is:



(Since the model is so simple, it is really just a re-scaling of $Y - \bar{Y}$)

Outline

- 1 Infinite horizon model
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 - Solution for the current account
- 2 When is a country bankrupt?
- 3 Stochasticity
- 4 Implications for optimal investment
- 5 Fundamental CA when output is exogenous
- 6 Empirical relevance

Empirical relevance of stochastic CA model

Is the stochastic current account model empirically relevant? OR section 2.3.5 goes through one way to shed light on this issue. What they basically do is this: First define 'permanent levels' of output, government spending and investment. Then use the solution for consumption when we have quadratic utility to get

$$CA_t = (Y_t - E_t \tilde{Y}_t) - (I_t - E_t \tilde{I}_t) - (G_t - E_t \tilde{G}_t)$$

Then specify $Z = Y - G - I$ such that $CA_t = Z_t - E_t \tilde{Z}_t$. By re-arranging the current account equation, one can show that **the current account reflects the discounted sum of all expected future growth in Z** . Therefore, if the theory holds, CA_t should be a good predictor of future growth in Z .

Empirical relevance of stochastic CA model II

To test this (you don't need to understand all the details) they:

- Specify a forecast model for $\Delta Z_t = Z_t - Z_{t-1}$, which is assumed to be a VAR model for CA_t and ΔZ_t .
- Derive what the forecasts of the model imply for the discounted sum of all expected future growth in Z
- And finally use this to produce 'predictions' for the current account

Comparing predicted and actual values show a pretty good fit of the model for Belgium, Canada, Denmark, Sweden but not the UK.