The Life Cycle, Tax Policy, and the Current Account

Lecture 3, ECON 4330

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Outline

Summary from last lecture

Going from a two-period to an infinite horizon model (with perfect foresight) does not change the implications of the model too much. In particular, the period-by-period first-order conditions are almost exactly the same as when there are two periods.

Summary from last lecture II

The CES-example for this model gave a simple solution for the consumption:

$$C_t = \frac{r-g}{1+r} W_t$$

where $g = [\beta(1+r)]^\sigma - 1$ determines whether consumption is on an upward or downward sloping path. Introducing 'permanent levels' we found (still under CES-utility) the fundamental current account equation:

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{g}{1+r} W_t$$

which is more general than just discussing CA_1 (as we did in the two-period model).

Summary from last lecture III

When working with a stochastic model, more things change. Consumption and investment decisions are no longer separated, and risk may reduce the rate of investment. We saw that by combining the first-order conditions for B_{t+1} and K_{t+1}, which gave

$$\mathbf{E_t}\left\{A_{t+1}F'(K_{t+1})\right\} = r - cov_t\left(\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right)$$

Summary from last lecture IV

In a special case of the stochastic model, namely one without investment and with quadratic utility, we saw how that implied Hall's random walk result:

$$\Delta C_{t+1} = \eta_{t+1}$$

Furthermore, assuming a given AR-process for income, we could derive a different version of the fundamental current account equation:

$$CA_t = \frac{1-\rho}{1+r-\rho}(Y_t - \bar{Y})$$

Today

Today we shift focus from a representative agent to **overlapping generations**. Model-wise many similarities, but also considerable differences. Recall, the representative agent framework we have developed suggests that countries with a **higher growth rate tend to run current account deficits** (and also save less). Does that fit with what be observe in the world today?

Today II

Structure of a simple two-generations model:

	t	$t+1$	$t+2$	$t+3$
Old at time t:	c_t^o			
Young at time t:	c_t^y	c_{t+1}^o		
Young at time $t+1$:		c_{t+1}^y	c_{t+2}^o	
Young at time $t+2$:			c_{t+2}^y	c_{t+3}^o

... and so on ...

Today III

This may develop into an extremely technical lecture. To avoid that, we'll try to focus at the following points:

- Why Ricardian equivalence will fail to hold (verbal argument)
- Basic structure of an OLG model
- How to add investments
- Relationship between savings and growth
- GE effects

Outline

Failure of Ricardian equivalence

Recall that in a representative agent model, the timing of taxes was irrelevant. That will not hold in an OLG model. Assume that there are no government expenditures, making the taxes pure redistribution. The PV budget constraint of the government is:

$$(1+r)B_t^G + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\tau_s^y + \tau_s^o) = 0$$

Will illustrate one of the main differences between rep. agent and OLG models because:

- With represenative agent models, the timing of taxes is irrelevant (Ricardian equivalence).
- With overlapping generations, timing of taxes will matter. Why? Because the government can redistribute income across generations

Failure of Ricardian equivalence II

One example is to assume that at $t = 0$, the government hands out $d/2$ to both young and old, i.e. $\tau_0^y = \tau_0^o = -d/2$. This is financed by setting $\tau_t^y = \tau_t^o = rd/2$ for $t = 1, 2, \ldots$. Budget is still balanced since

$$\begin{aligned}
\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s}(\tau_s^y + \tau_s^o) &= \tau_0^y + \tau_0^o + \frac{1}{1+r}\sum_{s=1}^{\infty}\left(\frac{1}{1+r}\right)^{s-1}(\tau_s^y + \tau_s^o) \\
&= -[d/2 + d/2] + \frac{1}{1+r}\sum_{s=1}^{\infty}\left(\frac{1}{1+r}\right)^{s-1}(rd/2 + rd/2) \\
&= -d + \frac{1}{1+r}\frac{1+r}{r}rd \\
&= 0
\end{aligned}$$

Failure of Ricardian equivalence III

- The old generation at $t = 0$ will consume all they get, so c_0^o rises by $d/2$
- The young generation at $t = 0$ will spend some of what they get, but will save some of it (consumption smoothing + higher taxes in period $t = 1$)
- Hence aggregate consumption in period 0 will rise

Failure of Ricardian equivalence IV

What about period 1?

- The old generation at $t = 1$ will consume more than before, since some of the "gift" was saved. Compensates for higher taxes
- The young generation at $t = 1$ will consume less because of the higher taxes
- Sign of the change in aggregate consumption in period 1 is ambiguous.

In period 2 and onwards, aggregate consumption falls since all generations are paying higher taxes.

Failure of Ricardian equivalence V

Conclusion: Ricardian equivalence fails to hold in an OLG model. This is relevant in general, and also for an open economy model. Means that the current account can be affected by the timing of taxes!
Exceptions? See Section 3.7.1 (which is on the reading list). Barro (1974) shows how Ric. equiv. prevails also in an OLG model once you allow for people to care about future generations (roughly speaking).

Outline

Basic OLG structure

- Each generation lives for two periods, so the optimization problem of an individual is very similar to the two-period model from lecture 1.
- Production is carried out by a representative firm, which we describe later.
- From working for the firm young earn z_t^y and old z_t^o.
- Taxes paid are τ_t^y and τ_t^o.
- Young invest their savings in the international credit market. Yields interest rate r
- Perfect foresight.
- N_t young and N_{t-1} old persons at time t. Assume $N_t = (1+n)N_{t+1}$
- $P_t = N_t + N_{t-1}$ is population at time t. P_t will also grow at rate n

Basic OLG structure II

Conditional on income, the young individual at time t will maximize utility, given by

$$U_t^y = u(c_t^y) + \beta u(c_{t+1}^o) \tag{1}$$

subject to the budget constraint

$$c_t^y + \frac{c_{t+1}^o}{1+r} = z_t^y - \tau_t^y + \frac{z_{t+1}^o - \tau_{t+1}^o}{1+r} \tag{2}$$

This is the exact same problem as what we solved in Lecture 1. Euler equation describes optimum.

Basic OLG structure III

For instance, log utility (special case of CES with $\sigma = 1$, confer lecture notes) gives consumption levels:

$$c_t^y = \frac{1}{1+\beta}\left[(z_t^y - \tau_t^y) + \frac{(z_{t+1}^o - \tau_{t+1}^o)}{1+r}\right]$$
$$c_{t+1}^o = \frac{\beta}{1+\beta}(1+r)\left[(z_t^y - \tau_t^y) + \frac{(z_{t+1}^o - \tau_{t+1}^o)}{1+r}\right]$$

I will stick to the assumption of log utility throughout the lecture. Let us define the growth rate of income over the life-cycle as e_z:

$$1 + e_z = \frac{z_{t+1}^o - \tau_{t+1}^o}{z_t^y - \tau_t^y}$$

Consumptions levels are then

$$c_t^y = \frac{1}{1+\beta}\left[1 + \frac{1+e_z}{1+r}\right](z_t^y - \tau_t^y)$$
$$c_{t+1}^o = \frac{\beta}{1+\beta}(1+r)\left[1 + \frac{1+e_z}{1+r}\right](z_t^y - \tau_t^y)$$

Basic OLG structure IV

Then define individual saving when young, s_t^y, as income when young minus consumption when young:

$$s_t^y = z_t^y - \tau_t^y - c_t^y$$

while saving when old is simply

$$s_{t+1}^o = -s_t^y$$

Inserting for c_t^y the expression for savings becomes

$$s_t^y = \frac{1}{1+\beta}\left(\beta - \frac{1+e_z}{1+r}\right)[z_t^y - \tau_t^y]$$

or

$$s_t^y = \mu[z_t^y - \tau_t^y]$$

where $\mu = \frac{1}{1+\beta}\left(\beta - \frac{1+e_z}{1+r}\right)$ is the savings rate of a young individual.

Basic OLG structure V

Motives for saving? Decompose μ into two terms:

$$\mu = \lambda - (1-\lambda)\frac{1+e_z}{1+r}$$

- $\lambda = \beta/(1+\beta)$ is the savings rate when the agent has no old-age income ($e_z = -1$)
- If the agent recieves some income when old ($e_z > -1$), this reduces the savings rate
- A higher interest rate reduces the effect of future income growth since that lowers the present value of future income
- Income and substitution? These effects cancel out! (Remember: log utility)

If e_z is sufficiently low, we are guaranteed to find $\mu > 0$. $\Rightarrow$ The young are saving, the old are dissaving (life cyle model for consumption and savings). In addition, since savings over the life cyle is zero, the only way to get **aggregate** savings to be positive is through population growth or if the young are richer.

Basic OLG structure VI

With the savings rate defined, we can write consumption levels in a simpler way:

$$
\begin{aligned}
c_t^y &= (1-\mu)(z_t^y - \tau_t^y) \\
c_{t+1}^o &= [\mu(1+r) + (1+e_z)]\,(z_t^y - \tau_t^y)
\end{aligned}
$$

Outline

Representative firm

Output is produced by a representative firm with Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

A_t is productivity. This way of adding productivity is called 'labor augmented productivity'. We continue to assume away depreciation, making the capital accumulation equation:

$$K_{t+1} = K_t + I_t$$

Representative firm II

From where does the firm get L and K?

- Assume that the young inelastically supply 1 unit of labor each, while old do not work. Hence $L_t = N_t$.
- Capital can for simplicity be thought of as being rented from the international credit market at a rate r

If the firm behaves as a price-taking profit maximizer, we know that the first-order conditions are:

$$\alpha \left(\frac{K_t}{A_t L_t} \right)^{\alpha - 1} = r$$

$$(1 - \alpha) A_t \left(\frac{K_t}{A_t L_t} \right)^{\alpha} = w_t$$

Representative firm III

In an equilibrium we have $L_t = N_t$, so the first-order condition with respect to capital determines the capital stock as a function of N_t, A_t and r. Given the capital stock, the first-order condition with respect to labor determined the real wage, w_t.

Outline

Steady state: Firm

To simplify the analysis, let us consider the case where A_t grows smoothly at a rate g $(A_t = (1+g)A_{t-1})$.

- From foc for capital, this implies a constant capital to efficient labor ratio k

$$k = \left(\frac{\alpha}{r}\right)^{1/(1-\alpha)}$$

  such that K_t grows at $(1+g)(1+n)$.

- From foc for labor, we get the real wage growing at a rate g, $w_t = A_t w^*$, where

$$w^* = (1-\alpha)\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}$$

Steady state: Taxes

The government runs a pay-as-you-go pension system. It sets a constant tax rate τ for the young. Total tax revenues are split evenly among the old.

$$\tau_t^y = \tau z_t^y$$
$$\tau_t^o = -N_t \tau_t^y / N_{t-1}$$

Since $z_t^y = A_t w^*$, we see that

$$\tau_t^o = -(1+n)\tau A_t w^*$$

Steady state: Agents

We have $z_t^y = A_t w^*$, $\tau_t^y = \tau z_t^y$, $z_t^o = 0$ and $\tau_t^o = -(1+n)\tau A_t w^*$.Consumption and saving levels are:

$$\begin{aligned} c_t^y &= (1-\mu)(1-\tau)A_t w^* \\ s_t^y &= \mu(1-\tau)A_t w^* \\ c_{t+1}^o &= [\mu(1+r) + (1+e_z)]\,(1-\tau)A_t w^* \\ s_t^o &= -s_{t-1}^y \end{aligned}$$

where income growth over the life cycle is

$$1 + e_z = \frac{z_{t+1}^0 - \tau_{t+1}^o}{z_t^y - \tau_t^y} = \frac{(1+n)(1+g)\tau A_t w^*}{(1-\tau)A_t w^*} = \frac{\tau}{1-\tau}(1+n)(1+g)$$

Steady state: Agents II

What is the savings rate now? Recall,

$$\mu = \lambda - (1-\lambda)\frac{1+e_z}{1+r}$$

Inserting for the $1+e_z$:

$$\mu = \lambda - (1-\lambda)\frac{\tau(1+n)(1+g)}{(1-\tau)(1+r)}$$

- If $\tau = 0$, the savings rate is unaffected by population and technological growth
- But $\tau > 0$ makes n and g affect the savings rate since it creates a link between individuals' income and growth factors.

Steady state: Aggregation

The model description is finalized by aggregating over generations. Start by looking at aggregate before-tax labor income (denoted Z_t). Since we are in steady state with $z_t^o = 0$ and $z_t^y = A_t w^*$, we have:

$$Z_t = N_t A_t w^*$$

(this is also equal to aggregate post-tax income). Next: aggregate savings. Since $s_t^o = -s_{t-1}^y$, and $s_t^y = \mu(1-\tau)A_t w^*$:

$$S_t = \left[1 - \frac{1}{(1+n)(1+g)}\right]\mu(1-\tau)Z_t$$

Confirms our claim from earlier: Only possible to have positive aggregate savings if technology or population is growing (provided that $\mu > 0$).

Steady state: Aggregation II

How will any given amount of savings be divided between capital and foreign assets? Both assets yield an interest rate r, so the agents are indifferent. Import capital if autarky rate exceeds r, export if it is less. A country's *net foreign asset position* at the end of period t depends on how much *young* agents save relative to how much capital the firm needs.

$$B_{t+1} = S_t^y - K_{t+1}$$

Since the old dissave ($S_t^o = -S_{t-1}^y$), the standard identity linking savings, investment and the current account holds (of course):

$$S_t = S_t^y + S_t^o = (K_{t+1} + B_{t+1}) - (K_t + B_t) = I_t + CA_t$$

Steady state: Aggregation III

Finally, what about the link between labor income and GDP? Cobb-Douglas production implies constant factor shares, so

$$Z_t = (1 - \alpha) Y_t$$

What happens to the rest of GDP? It is paid to capital owners (either at home or abroad). Since $B_t = S^y_{t-1} - K_t$, we see that

$$rK_t = r\left(S^y_{t-1} - B_t\right)$$

This means that

- A fixed share $(1 - \alpha)$ of income is always paid to wage earners and a share α to capitalists
- If $B_t > 0$, then the share paid to capitalists goes 'directly' into the pockets of the old
- But if the capital stock is larger than what the old were saving, some of the capital income is sent abroad.

Outline

Application 1: Aggregate savings rate and growth

We found aggregate savings to be

$$S_t = \left[1 - \frac{1}{(1+n)(1+g)}\right] \mu(1-\tau) Z_t$$

while GDP and labor income is linked through $Z_t = (1-\alpha)Y_t$. So:[1]

$$\frac{S_t}{Y_t} = (1-\alpha)\mu(1-\tau)\left(1 - \frac{1}{(1+n)(1+g)}\right)$$

Assume that $\tau = 0$, so $\mu = \lambda$. Prediction by the model:

- $\Rightarrow$ Savings rate should be positively correlated with $(1+n)(1+g)$
- $\Rightarrow$ This predicition runs in contrast to the result in standard representative agent models (where higher growth reduces saving!)

[1] This is a more general expression than what you find on page 150 in O&R, which is for the endowment economy. Set $n = 0$ and $\alpha = 0$ in our expression $+$ $e = -1$ in their formula, to see that they are the same.

Application 1: Aggregate savings rate and growth II

Whether there is a positive link or not has been tested extensively in the literature.

- Modigliani (1970) finds support of a positive cross-sectional correlation
- Confirmed by Guiso, Jappelli and Terlizzese (1992)
- But some studies find a weaker link

(references in Obstfeld and Rogoff). Prelim. conclusion: Some support for life-cycle model.

Application 1: Aggregate savings rate and growth III

Carroll, Overland and Weil (2000, AER) make an interesting point:

- It does indeed look like S/Y and ‘growth’ are positively correlated
- But what causes what?
- Interpretation in a life-cycle model: Higher growth leads to higher savings rate. Why? Since the younger save more than the older dissave.
- Many studies indicate that most countries experience high growth rates *long before* their savings rate increased
- Claim that a representative agent model with habit formation does a better job.

Application 1: Aggregate savings rate and growth IV

Another problem for the life-cycle model occurs when we allow for taxes (or other ways to link old-age income to 'current' growth).

- An increase in $(1+g)(1+n)$ still leads to more savings (because of the 'chohort'-effect)
- But it will also reduce the savings rate μ, since agents foresee an improved pension-benefit.
- Thus not clear whether life-cycle models can explain a *strong* link between savings rates and growth

Application 2: Long-run level of NFA

We've assumed that the economy is in a steady state, which means that the capital intensity K/AL is constant. The economy continues to grow because of:

- Population growth (at a rate n)
- Productivity growth (assumed to grow at a rate g).

It follows that both output (Y_t) and the capital stock (K_t) will grow at a rate $(1+n)(1+g)$. It is intuitive that the steady state value of B also grows at the rate $(1+n)(1+g)$. That is easy to see from

$$S_t^y = B_{t+1} + K_{t+1}$$

since S_t^y and K grow at $(1+n)(1+g)$ ($S_t^y = N_t \mu (1-\tau) A_t w^*$).

Application 2: Long-run level of NFA

Insert for savings and divide by $A_t N_t$ to get:

$$\mu(1-\tau)w^* = (1+n)(1+g)(b_{t+1}+k_{t+1})$$

where $b_t = B_t/A_t N_t$. Hence the net foreign asset per efficient worker is:[2]

$$\bar{b} = \frac{\mu(1-\tau)w}{(1+n)(1+g)} - k$$

So net foreign assets will grow at a constant rate. Only B/AN will be constant. This imlies either $CA > 0$ or $CA < 0$ **forever**, except if by coincidence $\bar{b} = 0$.

[2] This is equation (42) in O&R, page 158, if you set $g = 0$.

Application 3: Long-run levels of I/Y and CA/Y

Let us then look at what the steady state values of I/Y and CA/Y are. First, since capital grows at a constant rate $(1+n)(1+g)-1$:

$$I_t = [(1+n)(1+g)-1]K_t$$

Therefore, using that $rK = \alpha Y$ in steady state, investment relative to GDP is:

$$\frac{I}{Y} = [(1+n)(1+g)-1]\frac{\alpha}{r}$$

- The rate of investment is increasing in the growth (of either N or A) and the share of capital (α)
- and falls with the world interest rate.

Application 3: Long-run levels of I/Y and CA/Y II

We can then find the current account relative to GDP using $S/Y = CA/Y + I/Y$ and the expression for S/Y we've already derived. Hence

$$\begin{aligned}\frac{CA}{Y} &= (1-\alpha)\mu(1-\tau)\left(1-\frac{1}{(1+n)(1+g)}\right) - [(1+n)(1+g)-1]\frac{\alpha}{r} \\ &= [(1+n)(1+g)-1]\left(\frac{(1-\alpha)(1-\tau)\mu}{(1+n)(1+g)} - \frac{\alpha}{r}\right)\end{aligned}$$

- CA/Y will depend positively on r (as before), since that will make it more likely that you invest abroad rather than at home
- A higher savings rate and a larger wage share also increase CA/Y
- What is the net effect of an increase in n or g?
  - It will lead to a higher investment-to-output ratio
  - But it may also lead to higher savings (remember previous discussion). So net effect is not obvious

Application 3: Long-run levels of I/Y and CA/Y III

The Feldstein-Horioka puzzle:

- Under autarky we have $CA = 0$, and therefore $I/Y = S/Y$
- But perfect capital mobility should make I/Y and S/Y unrelated
- Feldstein and Horioka show that I/Y is *highly* correlated with S/Y in a cross-section
- Still an unresolved puzzle (see discussion in Obstfeld and Rogoff, 2000)

Application 3: Long-run levels of I/Y and CA/Y IV

Can we explain the puzzle using our life-cycle model? Assume that all variation among countries is due to differences in n and g. What would we observe?

- For I/Y to move one-for-one with S/Y, we need CA/Y to be fairly unresponsive to changes in n and g
- Numerical exercise does not indicate a constant CA/Y, so not possible to use the model to explain the puzzle completely (but common factor *do* drive investment and saving ratios!)

Outline

General equilibrium

Finally, let us consider two countries, to see what determines the world interest rate. Again: Only consider steady state. We assume that Home and Foreign have:

- The same production function (and α) $\Rightarrow$ Same steady state capital intensity k and efficient labor wage w
- Same utility function, discount factor, tax system, and in neither country do old people work $\Rightarrow$ Same savings rate μ

General equilibrium II

As in the two-perod model considered before, all we need to have a complete GE model, is to define all the equations that refer to Foreign, and then impose 'market clearing':

$$CA_t + CA_t^* = 0$$

Since $S_t = I_t + CA_t$ and $I_t = K_{t+1} - K_t$, this is the same as:

$$S_t^y + S_t^o + S_t^{y*} + S_t^{o*} = K_{t+1} - K_t + K_{t+1}^* - K_t^*$$

Two more steps. Since $S_t^o = -K_t - B_t$:

$$S_t^y - K_t - B_t + S_t^{y*} - K_t^* - B_t^* = K_{t+1} - K_t + K_{t+1}^* - K_t^*$$

and finally, getting rid of K_t, K_t^* and using that $B_t = -B_t^*$, we are left with

$$S_t^y + S_t^{y*} = K_{t+1} + K_{t+1}^*$$

General equilibrium III

Using

$$\begin{aligned} S_t^y &= N_t s_t^y = N_t \mu(1-\tau) w^* A_t \\ S_t^{*y} &= N_t^s s_t^{y*} = N_t^* \mu(1-\tau) w^* A_t^* \\ K_{t+1} &= (1+n)(1+g) A_t N_t k_{t+1} \\ K_{t+1}^* &= (1+n^*)(1+g^*) A_t^* N_t^* k_{t+1}^* \end{aligned}$$

and assuming $n = n^*$ and $g_a = g_a^*$, we have

$$[N_t A_t + N_t^* A_t^*](1-\tau)\mu w^* = [N_t A_t + N_t^* A_t^*](1+n)(1+g)k$$

So in equilibrium we know that:

$$(1-\tau)\mu w^* = (1+n)(1+g)k$$

General equilibrium IV

We can now look at two things. First, since the steady state (per efficient worker) wage is

$$w = (1-\alpha)k^{\alpha}$$

the equilibrium condition defines the steady state (common) capital intensity:

$$k = \left(\frac{(1-\tau)\mu(1-\alpha)}{(1+n)(1+g)}\right)^{\frac{1}{1-\alpha}}$$

General equilibrium V

In addition, since

$$k = \left(\frac{\alpha}{r}\right)^{1/(1-\alpha)}$$

we find the equilibrium world interest rate to be:

$$r = \alpha k^{\alpha-1} = \frac{\alpha}{1-\alpha}\frac{(1+n)(1+g)}{(1-\tau)\mu}$$

The interest rate is

- Increasing in g, n and α
- Decreasing in μ
- Increasing in the (global) tax rate

Similar to what we found in Lecture 1 ($r = \delta + \frac{1}{\sigma}r$).

Outline

Public debt

A model with government debt would be able to show how government transfers across generations affect the world interest rate. For instance:

- If it transfers resources to the current old by issuing debt
- and then increase future taxes enough to cover interest expences every period
- This reduces global supply of capital, and increases the real interest rate

Taxes and dynamic inefficiency

What about dynamic inefficiency? In an OLG model government transfer can also affect welfare.

- The global economy is dynamically inefficient if $1 + r < (1 + n)(1 + g)$ (same condition for the small open economy if the world interest rate is too low)
- Problem? Too much capital
- A higher tax rate will improve welfare in this case, since it leads to less investment in capital
- Why? Since a better pension system makes it less important for agents to save on their own

Unclear how important dynamic inefficiency is in practice.