Uncertainty and International Risk Sharing
Lectures 8 & 9, ECON 4330

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Uncertainty and International Risk Sharing

Central issues in this and the next lecture:

- Uncertainty
- International trade in risky assets

Why do we care?

- international financial markets have become increasingly integrated in the past 30 years
  - countries trade a variety of different assets; equity, currencies, derivatives, bonds
  - not only for intertemporal consumption smoothing (as considered sofar), but also to smooth consumption across uncertain future states
- trade in risky assets makes agents better off
  - e.g. insurance
- and changes agents’ response to shocks
  - e.g. the response of consumption and savings to an output shock
International financial openess, 1970–2004
(Domestic assets held by foreigners + Foreign assets held by domestic agents)/ GDP
source Lane and Milesi-Ferretti (2007)

Strong increase in international assets held in both groups
More so in industrialized countries (x7!) than in emerging and dev. countries (x3)
Financial openness

Financial openness (De Jure)
Chinn-Ito index based on IMF information on restrictions to capital movements

Note: Index between -2.5 and 2.5. -2.5 = Closed capital market; 2.5 = Fully opened

Source: Chinn and Ito, 2008
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Overview of lectures 8 & 9

We will discuss

- Gains from trade in risky assets
- How much insurance is optimal?
- What is the price of a risky asset?
- Implications for dynamics of the current account and investment

Central assumptions

- individuals need to have some idea about the possible states and their likelihoods
  - in fact, we will assume that all agents know and agree on the probabilities with which uncertain events occurs
- individuals are risk averse
- asset markets are complete (to be discussed)

We will also look into how well these models fare empirically

- international consumption correlations puzzle, home bias puzzle
- and possible explanations
Goals of this section:
- understand the role of risky assets and the concept of Arrow-Debreu securities
- optimal portfolio choice
- implications for the current account and the gains from trade in assets

Preliminaries:
- small open economy: prices exogenous
- single tradable good
- 2 periods, 2 states of nature $s = 1, 2$ in period 2
Arrow-Debreu securities

Arrow-Debreu securities (ADSs): analytical tool that simplifies analysis of asset markets

- uncertain states in period 2, $s = 1, 2$, happen with probability $\pi(s)$
- ADS for state $s$: pays 1 unit in state $s$ and 0 otherwise
- price of this ADS in units of today’s consumption: $\frac{p(s)}{1+r}$
- $B_2(s)$ net purchase of ADS for state $s$

Complete contingent claims market: for each $s \in S$ there exists an ADS

- we can replicate every risky as portfolio of ADSs
- no arbitrage condition: price of every risky asset $=$ price of corresponding ADS portfolio
- in a world with $S$ different states, knowing the prices of the $S$ ADSs is enough to know the price of every asset out of a potentially infinitely large set
Constructing a risk-free bond with ADSs

The following PF of ADSs replicates the risk-free bond: $B_2(1) = 1 + r; B_2(2) = 1 + r$
- expected payoff

$$\pi(1)B_2(1) + \pi(2)B_2(2) = \pi(1)(1 + r) + \pi(2)(1 + r) = 1 + r$$

- same as the risk-free bond
- bond market becomes redundant

No arbitrage condition: price of the safe ADS portfolio = price of risk-free bond

$$\left(1 + r\right)\frac{p(1)}{1 + r} + \left(1 + r\right)\frac{p(2)}{1 + r} = 1 \iff p(1) + p(2) = 1$$  \[1\]
Full insurance is possible...

Let $Y_2(s), C_2(s)$ denote period 2 state $s$ output and consumption.

Country can sell its risky output in both states $B_2(s) = -Y_2(s)$ for price $\frac{p(s)}{1+r}$ and use the revenue

$$\frac{p(1)}{1+r} Y_2(1) + \frac{p(2)}{1+r} Y_2(2)$$

to buy risk-free ADS portfolio with return $1 + r$.

• consumption in period 2 will be safe and equal to

$$C_2(s) = p(1) Y_2(1) + p(2) Y_2(2) \quad \text{for } s = 1, 2$$
What is the optimal consumption plan?

\[
\max_{B_2(1), B_2(2)} u(C_1) + \beta \pi(1) u(C_2(1)) + \beta \pi(2) u(C_2(2)) \quad \text{s.t.}
\]

Budget constraint in period 1:

\[
C_1 = Y_1 - \frac{p(1)}{1+r} B_2(1) - \frac{p(2)}{1+r} B_2(2)
\]

BC in period 2, state s:

\[
C_2(s) = Y_2(s) + B_2(s) \quad \text{for } s = 1, 2
\]
What’s the optimal consumption plan for different states tomorrow?

FOC for \( B_2(s) \):

\[
\frac{p(s)}{1 + r} = \beta \pi(s) \frac{u'(C_2(s))}{u'(C_1)}
\]  

(2)

Ratio of FOCs for \( s = 1 \) and \( s = 2 \)

\[
\frac{p(1)}{p(2)} = \frac{\pi(1)u'(C_2(1))}{\pi(2)u'(C_2(2))}
\]  

(3)

⇒ Risk-averse agent (\( u'' < 0 \)) chooses safe consumption \( C_2(1) = C_2(2) \) only if

\[
\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)} \text{ prices are actuarially fair}
\]

Generally, the agent chooses higher consumption in the state for which AD price is lower (adjusted for \( \pi(s) \))
What’s the optimal consumption plan for today?

Summing (2) over states and using no-arbitrage condition (1) yields *stochastic Euler equation*:

\[
\frac{1}{1 + r} = \beta \frac{\mathbb{E}_1 \{u'(C_2)\}}{u'(C_1)}
\]

(4)

where \( \mathbb{E}_1 \{u'(C_2)\} = \sum_s \pi(s) u'(C_2(s)) \)

\( C_1 \) and \( CA_1 = Y_1 - C_1 \) determined by (2), (4), and the intertemporal budget constraint

\[
C_1 + \sum_s \frac{p(s)}{1 + r} C_2(s) = Y_1 + \sum_s \frac{p(s)}{1 + r} Y_2(s)
\]

(5)
What determines the CA?

Alike the case with no uncertainty, the country’s comparative advantage or autarky prices determine the current account

- autarky prices (using $C_1^A = Y_1$, $C_2^A(s) = Y_2(s)$)

\[
\frac{p^A(s)}{1 + r^A} = \frac{\pi(s)u'(Y_2(s))}{u'(Y_1)}
\] (6)

- autarky IBC

\[
C_1^A + \sum_s \frac{p^A(s)}{1 + r^A} C_2^A(s) = Y_1 + \sum_s \frac{p^A(s)}{1 + r^A} Y_2(s)
\]

Gains from trade:

- trade consumption evaluated at autarky prices is more costly than autarky consumption
- otherwise, there would not have been gains from trade

\[
C_1 + \sum_s \frac{p^A(s)}{1 + r^A} C_2(s) \geq Y_1 + \sum_s \frac{p^A(s)}{1 + r^A} Y_2(s)
\] (7)
Weak law of comparative advantage (Deardorff 1980; Dixit & Norman 1980)

Subtract (5) from (7):

- Law of comparative advantage

\[
\sum_s \left( \frac{p^A(s)}{1+r^A} - \frac{p(s)}{1+r} \right) (C_2(s) - Y_2(s)) = \sum_s \left( \frac{p^A(s)}{1+r^A} - \frac{p(s)}{1+r} \right) B_2(s) \geq 0 \tag{8}
\]

- on average it must be true that \(B_2(s) > 0\) if \(\frac{p^A(s)}{1+r^A} > \frac{p(s)}{1+r}\)

Current account:

\[
CA_1 = Y_1 - C_1 = \sum_s \frac{p(s)}{1+r} B_2(s)
\]

- tends to be negative if on average \(\frac{p^A(s)}{1+r^A} < \frac{p(s)}{1+r}\)

- and positive if on average \(\frac{p^A(s)}{1+r^A} > \frac{p(s)}{1+r}\)
Weak law of comparative advantage

To summarize

- on average, the country imports state $s$ consumption ($B_2(s) > 0$) if
  \[ \frac{p^A(s)}{1+r^A} > \frac{p(s)}{1+r} \]
- the country runs a current account surplus in period 1 if, on average,
  \[ \frac{p^A(s)}{1+r^A} > \frac{p(s)}{1+r} \]
- and vice versa for $B_2(s) < 0$ and $CA_1 < 0$

This result is weaker than the deterministic case where

- if $CA_1 > (\ < \ ) 0$ if $1 + r^A < (\ > \ ) 1 + r$

It is still a very strong result because it does not require strong assumptions

- holds for any number of states
- holds if we have multiple goods
- only assumption on utility: more consumption is better
- weaker prediction is not due to uncertainty. The strong prediction also doesn’t hold in the deterministic case with more than one good consumed in both periods
Goals of this section:

- understand how $p(s)$ is determined in equilibrium
- country vs. global risk
- efficiency of the allocation

Preliminaries as before, but

- two countries: prices endogenous
Determination of prices

Prices for ADSs and consumption today are determined by FOCs and global market clearing
- we look at 2 countries and multiple states $s \in S$

FOCs according to (2)

$$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = \frac{p(s)}{1 + r} = \frac{\pi(s)\beta u'(C_2^*(s))}{u'(C_1^*)} \quad \forall s$$

- marginal rate of substitution of consumption today with consumption in any state $s$ equal across countries
- implies that marginal rates of substitution across states are also equalized

$$\frac{\pi(s)u'(C_2(s))}{\pi(s)u'(C_2(s'))} = \frac{p(s)}{p(s')} = \frac{\pi(s)u'(C_2^*(s))}{\pi(s')u'(C_2^*(s'))} \quad \forall s, s'$$

Global market clearing conditions (MCCs)
- period 1: $C_1 + C_1^* = Y_1 + Y_1^* = Y_1^W$
- period 2: $C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s) = Y_2^W(s)$ $\iff B_2(s) = -B_2^*(s) \quad \forall s$
Explicit solutions with identical CRRA

With \( u(C) = u^*(C) = \frac{C^{1-\rho}}{1-\rho} \), optimality condition (9) combined with MCC becomes

\[
\frac{p(s)}{p(s')} = \frac{\pi(s)}{\pi(s')} \left( \frac{Y^W_2(s)}{Y^W_2(s')} \right)^{-\rho} \quad \forall \ s, s'
\]  

(10)

- for risk averse agents \((\rho > 0)\), prices depend on aggregate risk, i.e. fluctuations in world output across states
- ADS prices will only be actuarially fair
  - if \( Y^W_2(s) = Y^W_2(s') \ \forall \ s, s' \), i.e. there is no aggregate risk
  - or agents are risk-neutral \( \rho = 0 \)
- generally, equilibrium prices of ADSs for states where consumption is abundant \((Y^W_2(s)\) large) are relatively low
- vice versa, ADSs for states where consumption is scarce will be expensive
- given \( Y^W_2(s)/Y^W_2(s') \ \forall \ s, s' \), fluctuations in country-specific output do not matter
Explicit solutions with identical CRRA

Using (10) and the no-arbitrage condition (1) we can solve for equilibrium ADSs prices

$$p(s') = \frac{\pi(s') Y^W_2(s')^{-\rho}}{\sum_s \pi(s) Y^W_2(s)^{-\rho}} \quad \forall s'$$

(11)

and $1 + r$

$$1 + r = \frac{Y^W_1^{-\rho}}{\beta \sum_s \pi(s) Y^W_2(s)^{-\rho}}$$

(12)

• in contrast to the deterministic model of OR 1 we have a weighted average of world output in the different states in the denominator
Efficient global risk sharing: Edgeworth Box
Goals of this section:

- trade in equity instead of AD securities
- financial market completeness and efficiency
- implications for dynamics of the current account
- optimal investment under uncertainty
- asset pricing

Preliminaries

- N countries
- 2 periods, S states of nature
- extension to multiple periods & investment
A model with trade in equity

Assets traded in period 1:

- claims on countries’ uncertain outputs
  - claim on country $m$’s output entitles owner to receive share $x^n_m$ of $Y^m_2(s)$ in state $s \in S$
  - similar to owning a share of country $m$’s mutual fund
    - $V_1^m$: period one market value of country $m$’s mutual fund in units of today’s consumption
    - $x^n_m$: country $n$’s net purchase of shares of country $m$’s mutual fund

- risk-free bond: $B_2^n$ net purchase by country $n$

Period 1 BC:

$$Y^n_1 + V^n_1 = C^n_1 + B^n_2 + \sum_{m=1}^{N} x^n_m V^m_1$$  \hspace{1cm} (13)

Period 2 BC:

$$C^n_2(s) = (1 + r)B^n_2 + \sum_{m=1}^{N} x^n_m Y^m_2(s)$$  \hspace{1cm} (14)
Optimal savings, portfolio choice, and consumption

FOC $B_2^n$:

$$u'(C_1^n) = \beta (1 + r) E_1 [u'(C_2^n)]$$  \hspace{0.5cm} (15)$$

FOC $x_m^n \forall m$:

$$u'(C_1^n) V_1^m = \beta E_1 [u'(C_2^n) Y_2^m]$$  \hspace{0.5cm} (16)$$

where $E_1 [x] = \sum_s \pi(s) x(s)$

Optimal consumption today and in every state $s$ tomorrow determined by (15), (16) and budget constraints (13), (14)
Explicit solution for CRRA utility

Solution method: conjecture equilibrium consumption and saving values, check that there exists a set of prices $r$, $V_1^m \forall m \in N$ so that equilibrium conditions (13)–(16) are fulfilled.

Conjecture: Optimally, every country $n$

- holds a constant share $\mu^n$ of every countries’ mutual fund: $x^n_m V_1^m = \mu^n V_1^m \forall m$
- consumes constant share $\mu^n$ of world output: $C_1^n = \mu_n Y_1^W$, $C_2^n(s) = \mu_n Y_2^W(s)$

where $\mu^n = \frac{Y_1^n + V_1^n}{\sum_m (Y_1^m + V_1^m)} = \text{share of country } n\text{’s wealth in total world wealth}$
Equilibrium with CRRA utility

FOC $B_2^n$:

$$1 + r = \frac{Y_1^{W-\rho}}{E_1 [Y_2^{W-\rho}]}$$  \hspace{1cm} (17)

FOC $x_m^n$:

$$V_1^m = \beta \sum_s \frac{\pi(s)u'(C_2^n(s))}{u'(C_1^n)} y_2^m(s) = E_1 \left[ \left( \frac{Y_2^W}{Y_1^W} \right)^{-\rho} y_2^m \right]$$  \hspace{1cm} (18)

⇒ there exist $N + 1$ prices satisfying the $N + 1$ FOCs of all $N$ countries

From the conjecture: all countries hold the same risky portfolio, i.e. $\frac{x_n^m}{x_{n'}^m} = \frac{x_k^m}{x_{k'}^m} \forall n, m, m', k \in N$

From the BC (13): $B_2^n = 0 \forall n$ no country holds the risk-free asset

- if not, some countries would have to short the risk-free asset → not consistent with $\rho_n = \rho_m$

- with CRRA utility and $\rho_n \neq \rho_m$, countries choose the same risky portfolio but more (less) risk averse countries buy (sell) more of the risk-free asset
Efficiency of the equilibrium

The equilibrium is efficient (MRS equalized across countries in every state) even though only shares of the $N$ mutual funds are traded

- in fact, the equilibrium is identical to case of complete markets for ADSs
- equilibrium interest rate is the same: cp. (17) and (12)
- also, recall from (11) that $p(s') = \frac{\pi(s') Y_2^W(s)^{-\rho}}{\sum_s \pi(s) Y_2^W(s)^{-\rho}}$. Then, (18) becomes

$$V_1^m = \frac{\sum_s p(s) Y_2^m(s)}{1 + r}$$
Efficiency of the equilibrium

Even absent contingent claims markets, equilibrium values of the mutual funds are determined as if contingent claims existed. Why?

- with CRRA, optimal portfolio composition independent of wealth
- facing identical prices, all countries hold the same risky portfolio
- all countries’ relative consumption across states is tied to world output in the same way
- no additional risk sharing is possible even if there were ADSs
Efficiency under more general utility functions

- previous results valid for broader class of utility functions (HARA class) without further assumption
- generally, optimal portfolio composition depends on the level of wealth. An efficient allocation can then always be reached if there is spanning
  - number of assets with linearly independent payoffs \( \geq \) number of states
Dynamic implications for the current account

Current account responses to shocks in setting with

- multiple periods, \( t \) initial period
- 2 countries, identical except for temporary output shock in \( t + 1 \), no investment

\[
Y_{t+1} = \bar{Y} + \varepsilon_{t+1} \\
Y^*_{t+1} = \bar{Y}^* + \varepsilon^*_{t+1}
\]

How CAs respond to shocks in \( t + 1 \) depends crucially on the presence of contingent claims markets
Dynamic implications for the current account

Suppose first, that there is a bond market but no market for contingent claims
- in \( t \): no intertemporal trade because countries are identical in expectation, \( CA_t = 0 \)
- in \( t + 1 \): \( \varepsilon_{t+1} > \varepsilon^*_{t+1} = 0 \)
  - home country is richer than expected
  - consumes a bit more and lends the rest to foreign: \( CA_{t+1} > 0 \)
  - now the two countries are different and hence there is a reason for intertemporal trade
- in \( s > t + 1 \) home receives interest on savings to finance continuously higher consumption
  - but no net savings/investment \( CA_s = 0 \) (absent further shocks)

Now suppose countries can trade claims on each others endowments
- in \( t \): countries buy (sell) 1/2 of the other country’s (their own) output: \( CA_t = 0 \)
  - both countries hold claims on 1/2 of world output in \( t + 1 \)
- in \( t + 1 \): \( \varepsilon_{t+1} > \varepsilon^*_{t+1} = 0 \)
  - both countries consume 1/2 of \( Y_{t+1}^W = Y_{t+1} + Y_{t+1}^* \)
  - home makes a net payment to foreign, but there are no net savings: \( CA_{t+1} = 0 \)
  - the countries are still identical, no incentives for intertemporal trade
What happens to the interest rate?

Bonds only:
- home’s savings ↑ while foreign’s savings are unchanged (ceteris paribus): \( r \downarrow \)

Bonds and contingent claims:
- both home and foreign want to save, net supply of world savings identical to the bonds-only case: \( r \downarrow \)

⇒ Contingent claims markets affect the distributional effects of temporary shocks, but not the price effects

This result, as well as the result on current account responses, hold in more general settings. But differences in preferences or initial endowments can lead to current account responses even in the presence of complete asset markets
What if shocks are permanent?

Bonds-only: \( CA_t = 0 \) and \( CA_s = 0 \) for \( s > t \)
- \( C_s = C_t + \varepsilon \)
- \( C^*_s = C^*_t \)
  - countries are no longer identical, but there is no reason for intertemporal trade because higher endowment levels are permanent

Bonds and contingent claims: \( CA_t = 0 \) and \( CA_s = 0 \) for \( s > t \)
- \( C_s = C_t + \frac{1}{2} \varepsilon = C^*_s \)
  - countries are still identical and don’t have a reason for intertemporal trade

In either case, there is no demand for savings, \( r \) is not affected.

Permanent shocks don’t matter neither for current accounts nor for the interest rate in either case (would be different if there was investment)
Let’s consider the case where period 2 output is endogenous: \( Y_2(s) = A(s)F(K_2) \)

- firms maximize NPV of profits \( \sum_s \frac{p(s)}{1+r} (A(s)F(K_2) + K_2) - K_2 \)
- FOC \( K_2 \):

\[
\sum_s \frac{p(s)}{1+r} (A(s)F'(K_2) + 1) = 1
\] (19)

- marginal return to investment, valued at ADS prices, equals marginal return to savings
Efficient investment under uncertainty

Foreign firms face similar prices $r, p(s)$, hence

$$\sum_s p(s) \left[ A(s)F'(K_2) + 1 \right] = 1 + r = \sum_s p(s) \left[ A^*(s)F'(K_2^*) + 1 \right]$$

- return to investment equalized across the world

Euler equation (2) implies

$$u'(C_1) = \beta \sum_s \pi(s)u'(C_2(s)) \left[ A(s)F'(K_2) + 1 \right] = \beta \sum_s \pi(s)u'(C_2(s)) \left[ A^*(s)F'(K_2^*) + 1 \right]$$

- marginal utility from investing anywhere in the world is equal to marginal utility of current consumption
International Portfolio Diversification

Asset pricing

As discussed above, prices of ADS are high for states where consumption is low ("bad states") and vice versa for "goods states"

What does this imply for asset prices in general?
  - e.g. a claim on a country’s GDP/mutual fund

\[
V^m_1 = \frac{\sum_s p(s) Y^m_2(s)}{1 + r} = E_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)} Y^m_2 \right] = \frac{E_1 \left[ Y^m_2 \right]}{1 + r} + Cov_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)}, Y^m_2 \right]
\]

- country m’s mutual fund price carries a risk premium \( Cov_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)}, Y^m_2 \right] \)
- risk premium increases the price if country m’s output is positively correlated with marginal utility
- i.e., negatively correlated with consumption
Asset pricing

- market value of firm $m$ which delivers output $\Pi^m(s) = A^m(s)F(K_2) + K_2$ in state $s$

\[
V_1^m = \frac{\sum_s p(s)\Pi^m(s)}{1 + r} = \frac{E_1[\Pi^m(s)]}{1 + r} + \text{Cov}_1\left[\frac{\beta u'(C_2)}{u'(C_1)}, \Pi^m(s)\right]
\]
Market incompleteness and empirical tests

Goals of this section:

- market incompleteness
- home-bias puzzle & international consumption correlation puzzle
- international portfolio diversification with non-traded goods
Market incompleteness

Market completeness means perfect risk sharing:

- MRS equalized across countries in every state: \( \frac{\beta u'(C_2(s))}{u'(C_1)} = \frac{\beta u'(C_2^*(s))}{u'(C_1^*)} \forall s \)

What if global risk sharing is incomplete?

- without complete asset markets, identical optimal portfolios, or spanning: MRS ex-post (when states are realized) not equalized across countries: \( \frac{\beta u'(C_2(s))}{u'(C_1)} \neq \frac{\beta u'(C_2^*(s))}{u'(C_1^*)} \)

- however, if there is trade in a risk-free bond: MRS equal in expectation

\[
E_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)} \right] = \frac{1}{1 + r} = E_1 \left[ \frac{\beta u'(C_2^*)}{u'(C_1^*)} \right]
\]
moreover, if there is also trade in some risky assets, asset pricing equation holds for the subset of traded assets, e.g. shares of firm \( m \)

\[
V_1^m = E_1 [\Pi^m] E_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)} \right] + Cov_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)}, \Pi^m \right]
\]

\[
= E_1 [\Pi^m] E_1 \left[ \frac{\beta u'(C_2^*)}{u'(C_1^*)} \right] + Cov_1 \left[ \frac{\beta u'(C_2^*)}{u'(C_1^*)}, \Pi^m \right]
\]

where \( \Pi^m(s) = A^m(s)F(K_2) + K_2 \)

risk premia for every traded asset are equal across countries

\[
Cov_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)}, A^m \right] = Cov_1 \left[ \frac{\beta u'(C_2^*)}{u'(C_1^*)}, A^m \right]
\]
Market incompleteness

What does this imply for a firm’s optimal investment decision (cp. 19)?

\[
\frac{d(V^m_1 - K^m_2)}{dK^m_2} = E_1 \left[ A^m F'(K^m_2) + 1 \right] E_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)} \right] + \text{Cov}_1 \left[ \frac{\beta u'(C_2)}{u'(C_1)}, A^m \right] F'(K_2) - 1 = 0
\]

- there is no conflict of interest among the firm’s investors
Testable implications of global risk sharing

Market completeness means perfect risk sharing

- MRS equalized across countries in every state: \( \frac{\beta u'(C_2(s))}{u'(C_1)} = \frac{\beta u'(C_2^*(s))}{u'(C_1^*)} \) \( \forall s \)
  - implies that consumption growth across is highly correlated across countries
  - and consumption growth more correlated across countries than output growth

- with CRRA utility, consumption growth perfectly correlated across countries and perfectly correlated with world output

Global risk sharing requires that individuals diversify their portfolio internationally

- with CRRA, countries hold a constant share of every other country’s total market portfolio
Consumption growth correlation puzzle

- first discussed by Backus, Kehoe, Kydland (1992)
- current view on data from Penn World Table:

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<thead>
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</thead>
<tbody>
<tr>
<td>OECD Corr(\hat{c}, \hat{c}^W)</td>
<td>.38</td>
<td>.12</td>
<td>.32</td>
<td>.38</td>
</tr>
<tr>
<td>OECD Corr(\hat{y}, \hat{y}^W)</td>
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<td>.21</td>
<td>.45</td>
<td>.69</td>
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<tr>
<td>Non-OECD Corr(\hat{c}, \hat{c}^W)</td>
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<td>-.05</td>
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<td>-.08</td>
</tr>
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<td>Non-OECD Corr(\hat{y}, \hat{y}^W)</td>
<td>-.03</td>
<td>-.02</td>
<td>.01</td>
<td>.19</td>
</tr>
</tbody>
</table>

Note: The table shows population-weighted average correlation coefficients between per capita consumption (output) with world per capita consumption (output), where world per-capita consumption (output) is the total of all 54 countries with non-missing values in the Penn World Table (version 8.1), excluding the country for which the correlation is computed.

- for OECD countries, consumption growth correlations are positive, but small and smaller than output correlations
- they have been increasing over time, but so have output correlations
Equity home bias puzzle

Figure: Home bias in equities across developed countries

Source: Figure 1 in Courdacier & Rey (2013, JEL). Home Bias in Equities measure equals 1 minus share of a country’s foreign equity holdings/share of foreign equity in world market portfolio
Equity home bias puzzle

Figure: Home bias in equities across emerging economies

Source: Figure 2 in Courdacier & Rey (2013, JEL). Home Bias in Equities measure equals 1 minus share of a country’s foreign equity holdings/share of foreign equity in world market portfolio
Equity home bias puzzle

- foreign equity shares in countries' portfolios are much smaller than the share of foreign equity in the world market portfolio
- investors hold mostly domestic equity

⇒ low consumption correlations are the other side of the same coin
How can we explain these puzzles?

Measurement of the home bias
- multinational firms: help explain a little bit, but not much (Mitra-Stiff, 1995)
- foreign direct investment (FDI) and foreign bonds: home bias exists for these assets as well, albeit to a smaller degree

Financial market incompleteness
- can explain both the home bias and the consumption correlation puzzle
- in particular if welfare gains from international risk sharing are actually small

Explanations that do not hinge on financial market incompleteness
- preference shocks
- non-tradable goods and real exchange rate risk
  - non-traded goods render part of the country risk non-insurable
Financial market incompleteness

Asset market incompleteness will lead to \( \frac{\beta u'(C^n_2(s))}{u'(\hat{C}^n_1)} \neq \frac{\beta u'(C^m_2(s))}{u'(\hat{C}^m_1)} \) ex-post

- no prediction on systematic relationship between countries’ consumption growth rates
- could be due to higher transaction or information cost for foreign investors and thus rationalize the home bias puzzle at the same time
  - higher transaction costs for foreign ownership: Tesar & Werner’s (1995) finding that higher turnover rate for non-resident holdings speaks against this
  - information friction: some support by Kang & Stulz (1995) who find that Japanese investors hold share of large firms if invested abroad
  - contractual frictions, limited enforcement of claims
Small gains from international risk sharing?

How large are the gains from international risk sharing? Disputed in the literature, e.g.

- Lucas (1987) finds that welfare cost of variability of US consumption between 1950–90 amounts to only 1/3% of annual consumption

- with different assumptions, Van Wincoop (1994) finds welfare gains from risk pooling among OECD countries amounting to 5.6% of annual GDP
Preference shocks

Differences in consumption growth across countries can also be caused by preference shocks $\varepsilon$, e.g. changes in health, weather, etc. which affect the marginal utility of consumption. Let 

$$U = u(C^n_1) + \beta \sum \pi(s)u(C^n_2(s), \varepsilon^n(s))$$

Then, efficient risk sharing requires

$$\frac{\beta u'(C^n_2(s),\varepsilon^n(s))}{u'(C^n_1)} = \frac{\beta u'(C^m_2(s),\varepsilon^m(s))}{u'(C^m_1)}$$

- optimal consumption growth rates depend on country-specific preference shocks
- no strict relationship between ex-post consumption growth rates across countries
- even if preference shocks are observable and claims contingent on preference shocks can be traded (which is both questionable)
International portfolio diversification with non-traded goods

Preliminaries
- 2 countries, 2 periods, $S$ states of nature
- 2 goods: one is non-tradable ($N$) and one is freely tradable ($T$)
- utility:
  \[ U = u(C^n_{T,1}, C^n_{N,1}) + \beta \sum_s \pi(s) u(C^n_{T,2}(s), C^n_{N,2}(s)) \]
- countries trade claims on each others’ mutual funds of total tradable $Y^m_{T,2}(s)$ and non-tradable $Y^m_{N,2}(s)$ production

Key difference to the model above: claims indexed to foreign non-tradable output can only be paid in units of the tradable good
- shocks to the price of non-tradable goods/the real exchange rate can not be insured
Consider the case where financial market is complete: claims on state contingent payments in tradable goods (ADSs) are also traded or there is spanning (\# independent assets > \# states)

- a *constraint-efficient* equilibrium is reached, where MRS of *tradables* consumption across time and states is identical across countries

\[
\frac{\pi(s)\beta \partial u(C_{T,2}^m(s), Y^{m}_{N,2}(s))/\partial C^m_T}{\partial u(C_{T,1}^m, Y^{m}_{T,1})/\partial C^m_T} = \frac{p(s)}{1 + r} = \frac{\pi(s)\beta \partial u(C_{T,2}^n(s), Y^{n}_{N,2}(s))/\partial C^n_T}{\partial u(C_{T,1}^n, Y^{n}_{N,1})/\partial C^n_T}
\]

(20)

- but the same is not true for non-tradables, where every country in every state and point in time optimally sets the MRS between non-tradables and tradables to

\[
\frac{\partial u(C^n_T, Y^n_N)/\partial C^n_N}{\partial u(C^n_T, Y^n_N)/\partial C^n_T} = p^n_N
\]

and \(p^n_{N,1}, p^n_{N,2}(s)\) is not equal across countries
International consumption correlations w/ non-tradable good (Lewis, 1996)

Equation (20) implies for any point in time $t$ that

$$\frac{\partial u(C^n_T, Y^n_N, t)/\partial C^n_T}{\partial u(C^n_T, Y^n_N, t-1)/\partial C^n_T} = \lambda_t$$

(21)

- $\lambda_t$ is a globally determined factor reflecting *global* scarcity of period $t$ consumption of tradables

Suppose per-period utility is CES-CRRA

$$u(C^n_T, C^n_N) = \frac{1}{1-\rho} \left\{ \left[ \gamma \frac{1}{\theta} C_T^{\theta-1} + (1-\gamma) \frac{1}{\theta} C_N^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \right\}^{1-\rho}$$

- $\rho$ risk aversion, $\theta$ elasticity of substitution between $C_N, C_T$
International consumption correlations w/ non-tradable good (Lewis, 1996)

Totally differentiating (21) gives

$$\hat{C}_T^n = \frac{-\theta}{1 - \phi(1 - \theta \rho)} \hat{\lambda}_t + \frac{(1 - \phi)(1 - \theta \rho)}{1 - \phi(1 - \theta \rho)} \hat{Y}_N^n$$

where $\phi < 1$ (22)

- tradable consumption growth depends on global factor and non-tradable output
- special case: $\theta \rho = 1$ (utility from tradables and non-tradables is additive) implies that every country’s tradable consumption is perfectly correlated with the world factor (and with every other country)
International consumption correlations w/ non-tradable good (Lewis, 1996)

Test for constraint-efficient global risk sharing: $\psi_3 = 0$ in

$$
\Delta \log(C^n_{T,t} - Y^n_{D,t}) = \nu_t + \psi_1 \Delta \log Y^n_{N,t} + \psi_2 \Delta \log Y^n_{D,t} + \psi_3 \Delta \log(Y^n_{T,t} - Y^n_{D,t}) + \varepsilon^n_t,
$$

country-panel regression where

- $\nu_t$ global factor
- $Y^n_{D,t}$ durables consumption. Durable goods themselves are tradable, but per-period usage most often is not

In country panel spanning 1970–1985, Lewis finds

- significantly positive $\psi_3$: reject global constraint-efficient risk sharing
- but for group of countries without severe foreign exchange restrictions, $\psi_3$ insignificant


For all other papers cited see reference list in Obstfeld, M. & K. Rogoff 1996, Foundations of International Macroeconomics. MIT Press