ECON 4330
Seminar 2

1   Output growth and current account (Exam question from 2012)

Consider a two-period representative agent model for an open economy. Here we will look at the general equilibrium case, where we have two with identical CES-utility functions. In that case, the equilibrium world interest rate is given by

$$1 + r = \frac{1}{\beta} \left(\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*} \right)^{1/\sigma}$$

Here, r is the interest rate, while Y and Y^* are the output levels of Home and Foreign, respectively, for periods $i = 1, 2$. β is the (common) discount factor, and $1/\sigma$ is the (common) coefficient of relative risk aversion. An equivalent way of writing this is

$$1 + r = (1 + \delta)(1 + g)^{1/\sigma}$$

where $\delta = 1/\beta - 1$ is the discount *rate* and $g = (Y_2 + Y_2^*)/(Y_1 + Y_1^*) - 1$ is the global growth rate.

1. Interpret the effects δ and g have on the equilibrium interest rate r.

2. The countries are assumed to be identical in all respects, except for their output levels. Write g as a function of the two growth rates, $g^{home} = Y_2/Y_1 - 1$ and $g^{foreign} = Y_2^*/Y_1^* - 1$.

3. Let $\alpha = Y_1/(Y_1 + Y_1^*)$ be Home's share of world output in period 1. Use the result from (2) to explain how α matters for Home's influence on the world interest rate.

2 Short look at OLG

This might not be covered in class, but the model should be known so we do a simple, but instructive, exercise.

1. We use a simple OLG model where consumers live for two periods. They have exogenous labor income only in the first period and save for old age at a world interest rate r. The instantaneous utility function is log utility and the problem of generation t is given by:

$$
\begin{aligned}
\max_{c_t^y, c_{t+1}^o} \quad & U_t = log(c_t^y) + \beta log(c_{t+1}^o) \\
s.t \quad & c_t^y + b_{t+1} = y_t \\
& c_{t+1}^o = (1+r)b_{t+1}
\end{aligned}
\tag{1}
$$

   (a) Solve for the optimal savings for generation t.

   Assume that income grows by a fixed rate, g, every period such that $y_t = (1+g)^t y_0$. Solve for the net savings(savings of the young minus dis-savings of the old) in each period.

   (b) How does the time-profile of the net savings of the country depend on the growth rate?

   (c) How does this compare to the infinite horizon case from the last seminar?

3 Discussion

Please send us suggestions for discussion topics to nicolai.ellingsen@econ.uio.no or inga.heiland@econ.uio.no