Econ4330 Seminar 6
Spring 2018

Solution proposal

Inflation targeting (partly based on Exam Question 2 from 2015)

During this exercise all variables are in logs.

- e is the nominal exchange rate
- p the price of home goods
- r the real exchange rate
- ρ the real interest rate
- y output
- g government expenditures on home goods

A bar above a variable distinguishes its value in a full equilibrium. A subscript $*$ means a foreign variable. Dots indicate time rates of change and subscript e an expectation.

1. Consider first real interest rate parity(RIP) in the sense that:

$$\rho - \bar{\rho} = (\rho_* - \bar{\rho}) + \dot{r}_e$$

   (a) Explain the conditions necessary for real interest rate parity to prevail.
   Solution
   Uncovered interest rate parity and consistent expectations is sufficient, see next question.

(b) Develop RIP using uncovered interest rate parity and the definition of the real exchange rate.
Solution
UIP:

$$\dot{e} = i - i_*$$

Real exchange rate:

$$r = e + p_* - p$$

Consistent expectations:

$$\dot{r} = \dot{e}_e + \dot{p}_{*e} - \dot{p}_e$$

Insert this in the UIP and solve using the definition of the real interest rate, $\rho = i - \dot{p}_e$.

$$\rho = \dot{r}_e + \rho_*$$

This condition together with four other equations characterize our model. In the reminder of this exercise we consider at small open economy where:

$$r = e + p_* - p \tag{1}$$
$$\rho - \bar{\rho} = (\rho_* - \bar{\rho}) + \dot{r}_e \tag{2}$$
$$\dot{r}_e = -\epsilon(r - \bar{r}) \tag{3}$$
$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} \tag{4}$$
$$\dot{p} = \dot{p}_e + \gamma(y - \bar{y}) + u_s \tag{5}$$

Where u_{y*} is a foreign demand shock and u_s is a cost push shock. The country's exchange rate is floating. The central bank practices strict inflation targeting. The aim is to keep home goods inflation $\dot{p}$, as close to the target, $\bar{\pi}$, as possible. The bank takes the private sector expectation $\dot{p}_e$ as given. Hence, in effect it decides the real interest rate $\rho = i - \dot{p}_e$, i, being the nominal interest rate. We can then proceed as if ρ is the instrument used by the bank.

2. Explain briefly the meaning of each equation (max 2-3 sentences per equation).
Solution
1. Definition of the real exchange rate, nominal pluss price difference.
2. RIP. Deviation from norm of the domestic interest rate equals the deviation from norm in the foreign interest real interest rate plus expected real depreciation.

3. Expected real depreciation. Regressive expectations with convergance speed ϵ.
4. IS-equation. Equilibrium in the goods market. Deviations from norm form.
5. Phillips-curve. Inflation will equal the expected inflation plus a term dependent on the output gap. plus a cost-push shock.

3. Derive the ISFX-equation(combinations of y and ρ yielding short run equilibrium in the goods market and the FX-market).
Solution
Combine 2,3 and 4:

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_\rho/\epsilon[(\rho_* - \bar{\rho}) - (\rho - \bar{\rho})] + \alpha_g(g - \bar{g}) + \alpha_* u_{y*}$$

4. Find the slope of the ISFX-curve, $\frac{\partial y}{\partial \rho}$, and interpret the expression.
Solution

$$\frac{\partial y}{\partial \rho} = -(\alpha_\rho + \alpha_r/\epsilon) < 0$$

If the effect of a change in the interest rate on gdp is strong, $(\alpha_\rho + \alpha_r/\epsilon)$ large, then the slope is steep. The central bank will no need to react very strongly to shocks as their tool is very effective.

5. Consider a positive shock to the foreign economy, $u_{y*} = \Delta > 0$

   (a) Show the response of the domestic Central Bank given strict inflation targeting. You can assume that the inflation target is credible.
   Solution
   Goal: $\dot{p} = \bar{\pi}$, assuming zero cost push shock, translates into keeping gdp at it's long run level.

$$\begin{aligned}
y - \bar{y} &= 0 \\
0 &= -\alpha_\rho(\rho - \bar{\rho}) + \alpha_\rho/\epsilon[(\rho_* - \bar{\rho}) - (\rho - \bar{\rho})] + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} \\
(\alpha_\rho + \alpha_r/\epsilon)(\rho - \bar{\rho}) &= \alpha_\rho/\epsilon(\rho_* - \bar{\rho}) + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} \\
\rho &= \bar{\rho} + \frac{1}{\alpha_\rho + \alpha_r/\epsilon}[\alpha_\rho/\epsilon(\rho_* - \bar{\rho}) + \alpha_g(g - \bar{g}) + \alpha_* u_{y*}] \\
\Delta\rho &= \bar{\rho} + \frac{1}{\alpha_\rho + \alpha_r/\epsilon}[\alpha_* \Delta u_{y*}] > 0
\end{aligned}$$

$$\frac{\partial \rho}{\partial u_{y*}} = \frac{\alpha_*}{\alpha_\rho + \alpha_r/\epsilon}$$

The central bank must increase the real interest rate to counteract the higher inflation coming from the positive export shock.

(b) Show how this will affect the real exchange rate both analytically and graphically.
Solution

$$r = \bar{r} - \frac{1}{\epsilon}[(\rho - \bar{\rho}) - (\rho_* - \bar{\rho})]$$

$$\frac{\partial r}{\partial u_{y*}} = \frac{\partial r}{\partial \rho}\frac{\partial \rho}{\partial u_{y*}}$$

$$\frac{\partial r}{\partial \rho} = -\frac{1}{\epsilon}$$

$$\frac{\partial r}{\partial u_{y*}} = -\frac{1}{\epsilon}\frac{\alpha_*}{\alpha_\rho + \alpha_r/\epsilon} < 0$$

The real exchange rate will go down, the local currency will appreciate. Graphically we can see this in a MP-FX diagram with r on the vertical axis and ρ on the horizontal. The MP curve will be upward sloping and the FX downward sloping. The MP curve will shift down, the FX curve stays constant. This will show and increase in the real interest rate and a decrease in the real exchange rate.

6. Discuss briefly how the answers to question 5 would change if the inflation target was not credible.
Solution
Here, anything goes. Just check that the student has a desent discussion of central bank credibility and/or expectations. Potential main points are how the central bank would need to compensate if their target was not credible, how that would affect the decission and economy, regressive vs extrapolative expectations, etc.

7. Consider a negative cost push shock, $u_s = -\Delta < 0$. What will the Central bank do and what is the effect on output. What are the direct effects of the shock and what are the effects through government intervention?
Solution
Now the central bank would have to set the interest rate such that

$y - \bar{y} = -\frac{1}{\gamma}u_s$.

$$
\begin{aligned}
y - \bar{y} &= -\frac{1}{\gamma}u_s \\
-\frac{1}{\gamma}u_s &= -\alpha_\rho(\rho - \bar{\rho}) + \alpha_\rho/\epsilon[(\rho_* - \bar{\rho}) - (\rho - \bar{\rho})] + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} \\
(\alpha_\rho + \alpha_r/\epsilon)(\rho - \bar{\rho}) &= \alpha_\rho/\epsilon(\rho_* - \bar{\rho}) + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} + \frac{1}{\gamma}u_s \\
\rho &= \bar{\rho} + \frac{1}{\alpha_\rho + \alpha_r/\epsilon}[\alpha_\rho/\epsilon(\rho_* - \bar{\rho}) + \alpha_g(g - \bar{g}) + \alpha_* u_{y*} + \frac{1}{\gamma}u_s] \\
\Delta\rho &= \bar{\rho} + \frac{1}{\alpha_\rho + \alpha_r/\epsilon}[\frac{1}{\gamma}\Delta u_s] < 0
\end{aligned}
$$

The central bank will lower the interest rate to counteract the negative cost push shock. This will bring the economy above it's trend level. The cost-push shock has no direct effect on gdp, but the effect comes through the central banks intervention.

8. Suppose there is a temporary cut in government expenditure, g. What will be the central bank's response? Discuss the impact of the expenditure cut on nominal and real exchange rates. How does the effect depend on the different elasticities in the demand function?
Solution
When inflation is expected to be on target, the CB aims at the interest rate that closes the output gap $y - bary$ With a temporary cut in g, demand falls. The CB cuts the interest rate to compensate for the demand shortfall. Substitution of eq (3) in (2) and solving for r gives

$$r = \frac{1}{\varepsilon}(\rho^* - \rho) + \bar{r}$$

Use this in the IS relation and differentiate to get

$$
\begin{aligned}
d\rho &= \frac{\alpha_g}{\alpha_\rho + \frac{\alpha_r}{\varepsilon}}dg \\
dg &< 0
\end{aligned}
$$

The denominator is the sum of the interest rate and exchange rate channel of monetary policy. The first channel is a direct effect on demand from lower RIR. The second works through a RER depreciation. When RIR is reduced, we need an expected appreciation to compensate for the interest rate differential. From eq (3) we see that agents

only expect an appreciation if r is above r, hence an immediate depreciation. When prices are sticky, RER jumps due to a corresponding nominal depreciation (cf. eq. 1). The change is

$$\begin{aligned} dr &= -\frac{\alpha_g}{\alpha_\rho + \frac{\alpha_r}{\varepsilon}} \frac{1}{\varepsilon} dg \\ &= -\frac{\alpha_g}{\varepsilon\alpha_\rho + \alpha_r} dg \end{aligned}$$

9. A permanent cut in government expenditures can be seen as a simultaneous reduction in g and $\bar{g}$ by the same amount. Explain why a permanent cut in g implies a depreciation of the equilibrium real exchange rate $\bar{r}$.
Solution

In the full equilibrium, output is determined by supply and the RIR is pinned down by equation 3. The full equilibrium IS relation is

$$\bar{y} = -\alpha_\rho \bar{\rho} + \alpha_r \bar{r} + \alpha_g \bar{g}$$

A lower g reduces full equilibrium demand. The RER must permanently depreciate to fill the demand gap caused by lower government spending. Why can't we have a reduction in the RIR? In the full equilibrium the real interest rate is the same home and abroad, cf eq (2). The required depreciation is

$$\begin{aligned} d\bar{r} &= -\frac{\alpha_g}{\alpha_r} d\bar{g} \\ d\bar{g} &< 0 \end{aligned} \tag{6}$$

Let $\bar{r}_0$ and $\bar{r}_1$ be the initial and new full equilibrium value, correspondingly

$$\bar{r}_1 = \bar{r}_0 - \frac{\alpha_g}{\alpha_r} d\bar{g}$$