

Question 1 (Week 38)

An entrepreneur with own cash or internal funds A can undertake a project at investment cost $I > A$, so that $I - A$ must be externally funded or borrowed from outside banks or investors. (All parties are risk neutral and protected by limited liability; hence no payment is made if failure.) The financial contracts are signed and investment decisions are made in the first period. In the second period the return from the project is realized and claims are settled. If the project is undertaken and succeeds, it yields a verifiable gross return $R > 0$; and zero if it fails. The probability of success is p , but because the project is subject to moral hazard, the entrepreneur can either work hard (exert effort) or relax (shirk), after having got external finance. These options are translated into variable probability of success: Working hard yields a probability of success $p = p_H$, whereas shirking yields the lower success probability $p = p_L$, but with a private benefit B (measuring the savings in effort by not taking the other option). Let $\Delta p := p_H - p_L > 0$. Assume that the rate of return expected by investors (equal the riskless rate of interest) is zero, and that only the “good” project is economically viable; $p_H R - I > 0 > p_L R + B - I$.

Now we restrict attention to direct finance. (In another problem, banks as monitors are introduced.) Direct finance is possible if external lenders do not make a loss. The rate of interest on a loan, if the project is financed, $1 + i$ per unit, if success. The expected profit for the borrower with initial cash A is then $V = p[R - (1 + i)(I - A)]$. (For the project to be an interesting option, one must have $V \geq A$.)

- a) Suppose that the lender wants to induce the entrepreneur to work hard. Show and explain that the incentive constraint for the entrepreneur is:

$$p_H[R - (1 + i)(I - A)] \geq p_L[R - (1 + i)(I - A)] + B.$$

- b) What is the competitive interest rate on loans?
 c) What conditions must be satisfied for direct finance to take place?
 d) Suppose that $p_H R - I < \frac{p_H}{\Delta p} B$. Explain and interpret this condition.
 e) Derive a critical value for the amount of cash below which no borrower will obtain credit. Why do you think that some projects with positive net present value will now be rejected credit?