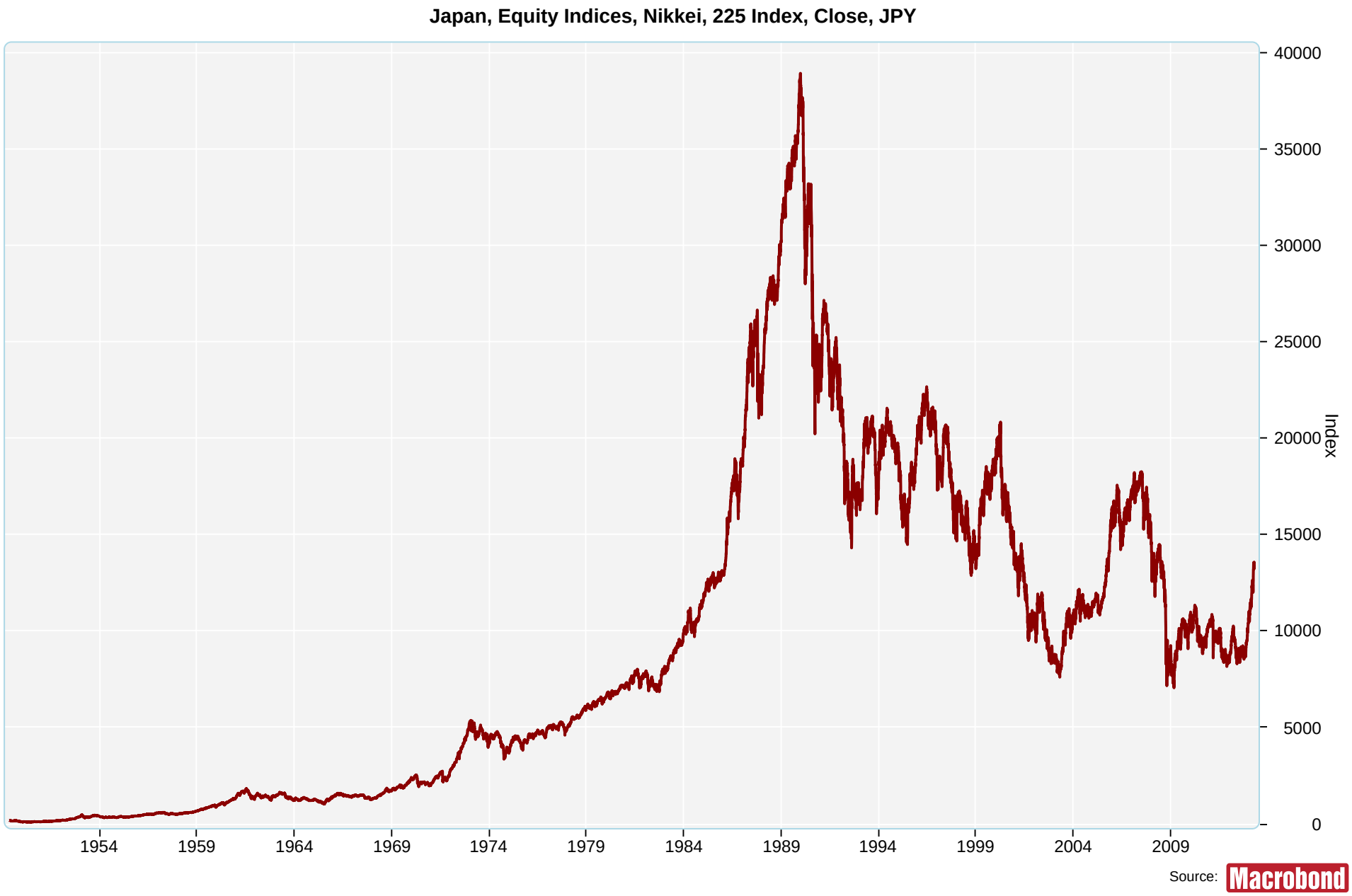

Japan, Equity Indices, Nikkei, 225 Index, Close, JPY
40000
35000
30000
25000
20000
15000
10000
5000
0
Index
1954
1959
1964
1969
1974
1979
1984
1989
1994
1999
2004
2009
Source: Macrobond

Bubbles - definition

- Asset price that deviates from fundamental values
- Share price that exceed the present value of future expected dividends
- House price that exceeds the expected present value of future rents
- Bubbles may burst even if no new information has arrived
- Assets may show bubble-like movements without actually being out-of-touch with fundamentals

Rational bubbles: Definition

- Investors know the fundamental value and are fully conscious that they buy into a bubble
- Rational return expectations: Mathematical expectations taking full account of all available information and the correct model
- Sole reason to invest in the bubble is the belief that it will continue and get bigger
- Multiple equilibria

A generic example

Asset price today is

$$V_t = R^{-1}[d_{t+1} + E_t V_{t+1}] \tag{1}$$

where R is risk-free interest rate and d_t is dividend. Fundamental solution is

$$V_t^* = \sum_{i=1}^{\infty} R^{-i} E_t d_{t+i} \tag{2}$$

More solutions:

$$V_t = V_t^* + b_t \quad b_{t+j} = R^j b_t \tag{3}$$

with bubble obeying

$$b_{t+1} = R b_t$$

Bursting bubble

p = probability of bursting
b_t^c = value if continuing
Solves (1) if

$$b_t^c = R^{-1}(1-p)b_{t+1}^c \Leftrightarrow b_{t+1}^c = Rb_t^c/(1-p) \tag{4}$$

If bubble does not burst, it grows faster than interest rate.
Note that both p and the starting value for b are arbitrary

Rational bubbles: when can they occur

Consider only objects with intrinsic value, paper money another story

- Has to be infinitely many investors that the bubble can be passed on to
- Expected return must be equal to or exceed the risk-free interest rate
- Investable funds cannot grow faster than economy forever
- Expected return must *not* exceed the growth rate of the economy
- Rational bubbles can only exist if interest rate is below growth rate
- Bubbles cannot be negative (assuming free disposal)
- The asset must not be easily reproducible

Overlapping generations

- Young and old
- Saving available at end of period, s fraction of the wages of the young w.

In equilibrium

$$k_{t+1} + b_t = s_t \leq w_t$$

- Crowding out
- Limited capacity for absorbing bubbles
- If b^c grows faster than wages, bubble bursts in finite time
- Bubble then bursts immediately

Rational bubbles and financial frictions

Allen and Gale Chapter 9.1

- Investors with limited liability and default risk
- Bubble defined as difference between expected value of cash flow from un-leveraged and leveraged investment
- Depositors are without access to safe asset the risky investments.
- Banks are inadequately paid for risk
- Depositors pick up the bill

Rational bubbles and financial frictions

Ventura and Martin sections 1 and 2

- Standard definition of rational bubble
- OLG: Bubbles crowd out productive investment
- V and M Expansionary bubbles possible
- Financial friction: Borrowing constraint
- Bubbles relaxes constraint, more efficient investment
- Possible even if continuation value grows faster than interest rate.

Keynes on asset prices

A conventional valuation which is established as the outcome of the mass psychology of a large number of ignorant individuals is liable to change violently as the result of a sudden fluctuation of opinion due to factors which do not really make much difference ...the market will be subject to waves of optimistic and pessimistic sentiment, which are unreasoning and yet in a sense legitimate where no solid basis exists for a reasonable calculation.

It might have been supposed that competition between expert professionals,..., would correct the vagaries of the ignorant individual left to himself. ...however..most of these persons are,in fact, largely concerned not with making superior long-term forecasts of the probable yield of an investment over its whole life, but in foreseeing changes in the conventional basis of valuation a short time ahead of the general public."

Keynes: *The General Theory of Employment, Interest and Money*, p. 154

Irrational exuberance

Behavioral theories:

- Extrapolation bias
- Herding
- This time is different

Replicating actions that have been successful for others is often a good strategy outside of financial markets
Experience from stock markets

- Momentum trading can be profitable
- Buying value stocks can also be profitable

Natural expectations

Fuster, Laibson and Mendel: Natural Expectations and Macroeconomic Fluctuations

Intuitive expectations = simple regression:

$$\Delta x_{t+1} = \phi \Delta x_t + \epsilon_{t+1}$$

True model:

$$x_{t+1} = \alpha x_t + \beta x_{t-1} + \eta_{t+1}$$

Natural expectations = average of true and misspecified model

Produce hump-shaped responses to shocks

Figure 1
Real S&P 500 index and present value of subsequent dividends

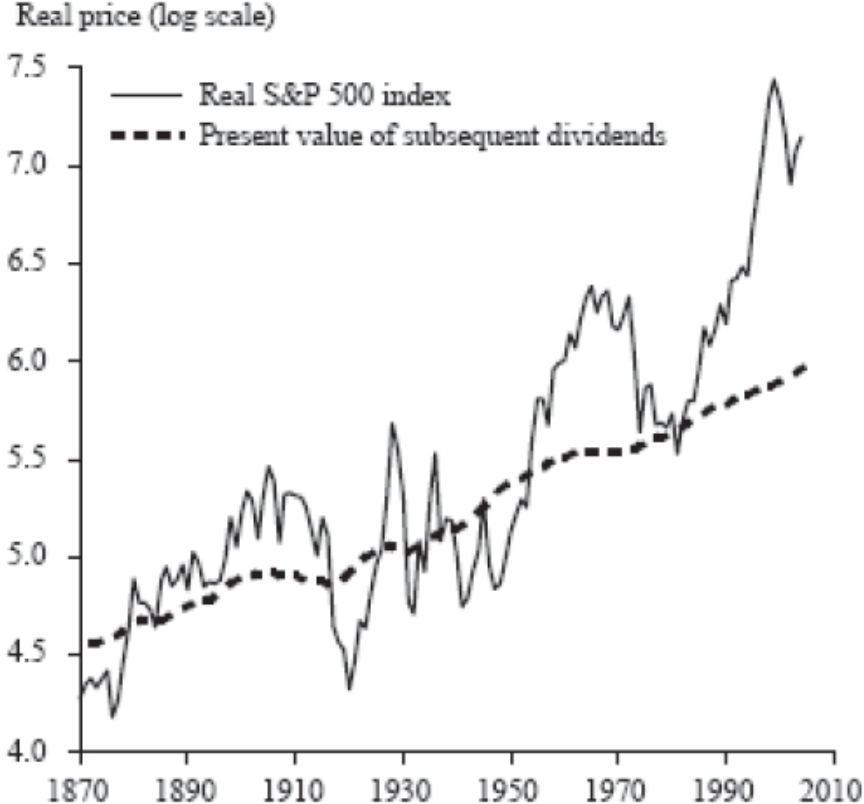
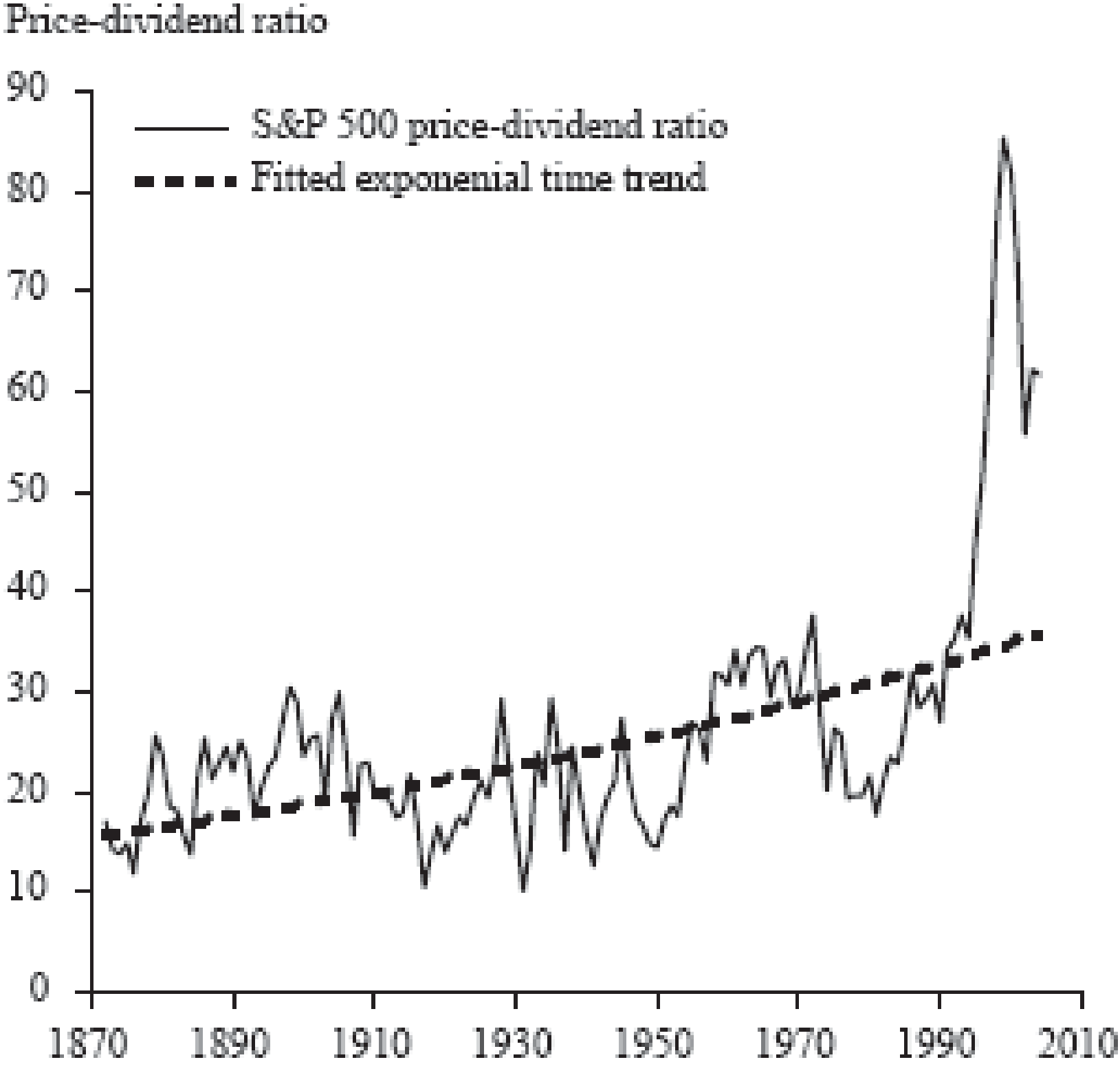


Figure 2
U.S. price-dividend ratio



Ventura and Martin Highlights

ECON4335

Asbjørn Rødseth

April 24, 2013

Consumers

- Live for two periods
- Each supplies one unit of labor in the first period
- Save all wage income for consumption in the second period
- Are risk neutral
- Share ε are entrepreneurs

New and old firms

- Entrepreneurs: Invest in new firms, borrow
- Non-Entrepreneurs: Invest in old firms, lend to entrepreneurs
- Entrepreneurs are more efficient in investment

A real investment $z_{j,t}$, yields

- for a non-entrepreneur: $z_{j,t}$ units of capital
- for an entrepreneur: $\pi_t z_{j,t}$ units of capital where $\pi_t > 1$

Capital accumulates according to

$$k_{j,t+1} = z_{j,t} + (1-\delta)k_{j,t}, \quad \text{for old firm} \tag{1}$$

Only old firms produce final goods.

The fundamental value of a firm

- The fundamental value of a firm is the value of the capital stock that belongs to the firm

$$V_{j,t} = (1 - \delta)k_{j,t} \tag{2}$$

- $V_{j,t}$ is the price of an old firm in period t after production has been carried out and depreciation has taken place, but before the new owner has added his new real investments to the capital stock

Some macro relations

Per worker production function in macro

$$y_t = \ell_t^{1-\alpha} k_t^{\alpha} = k_t^{\alpha} \tag{3}$$

y_t, k_t and ℓ_t are per worker, $\ell_t = 1$
Since non-entrepreneurs demand the same gross return, R_{t+1} ,from buying shares in old firms and lending to entrepreneurs;

$$R_{t+1} = 1 + \alpha k_{t+1}^{\alpha-1} - \delta = \alpha k_{t+1}^{\alpha-1} + 1 - \delta \tag{4}$$

Labor is paid its marginal product

$$w_t = (1-\alpha) k_t^{\alpha} \tag{5}$$

Entrepreneurs: The Credit Constraints

Entrepreneur is allowed to borrow at most:

$$f_{j,t} = \frac{\Phi\pi}{1-\Phi\pi} w_t > 0 \quad for \quad \Phi\pi < 1 \tag{6}$$

- Borrowing limit is proportional to wage income
- Multiplier because more investment yields access to more loans
- Since $\pi > 1$ entrepreneur can always get higher return than non-entrepreneur $\Longrightarrow$Entrepreneur always borrow as much as possible.

Dynamics of the economy's capital stock.

$$k_{t+1} = w_t + \varepsilon(\pi - 1)z_{j,t} \tag{7}$$

k_{t+1} =wages (savings)
+ gain from entrepreneurs doing part of the investment

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi - 1)}{1 - \Phi\pi}\right](1 - \alpha)k_t^{\alpha} \tag{8}$$

Bubbles

Value of firm with bubble:

$$V_{j,t} = (1-\delta)k_{j,t} + b_{j,t} \tag{9}$$

$(1-\delta)k_{j,t}$ = fundamental, $b_{j,t}$ =bubble

- Non-entrepreneurs have to pay more for firms
- Entrepreneurs get more for new firms and get more credit
- Some savings are diverted from real investment to consumption of the old
- More real investment is undertaken by the most efficient investors
- Total effect on real investment ambiguous

Requirements for a rational bubble

- Bubble must have expected return equal to interest rate

$$\frac{E_t b_{j,t+1}}{b_{j,t}} = E_t R_{t+1} \tag{10}$$

- Bubble should never become too large for the young to purchase

(11)

Bubbles relax the credit constraint

New firm at t, value at $t+1$:

$$V_{j,t+1} = (1-\delta)k_{j,t+1} + E_t b^N_{j,t+1} \tag{12}$$

The basis for getting loans is augmented by the present value of the bubble:

$$f_{j,t} = \frac{\Phi\pi}{1-\Phi\pi}\left[w_t + \frac{E_t b^N_{j,t+1}}{R_{t+1}}\right] \tag{13}$$

Two opposing effects on capital accumulation

$$k_{t+1} = w_t \underbrace{-b_t - b_t^N}_{a} + \frac{\varepsilon(\pi-1)}{1-\Phi\pi} w_t + \underbrace{\frac{\varepsilon(\pi-1)}{1-\Phi\pi} \cdot \frac{E_t b_{j,t+1}^N}{R_{t+1}}}_{b} \tag{14}$$

a: savings go to buy bubbles instead of real investment

b: entrepreneurs get to do more of the investment

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi-1)}{1-\Phi\pi}\right](1-\alpha)k_t^{\alpha} + \frac{\varepsilon(\pi-1)}{1-\Phi\pi_t} \cdot \frac{E_t b_{j,t+1}^N}{\alpha k_{t+1}^{\alpha-1} + 1 - \delta} - b_t - b_t^N \tag{15}$$

Bubbly episodes - an example

- Probability of bubble ending in period t constant equal to p
- Bubble starts with $b_t^N = b^N > 0$
- While bubble goes on $b_t^N = nb_t, \quad n > 0$
- Probability of a second bubble starting is negligible
- Auxiliary assumption: $\delta = 1$

The dynamics of the aggregate bubble

Expected growth in aggregate bubble comes both from old and new firms:

$$E_t b_{t+1} = R_{t+1}(1+n)b_t \tag{16}$$

Since bubble breaks with probability p

$$E_t b_{t+1} = p \cdot 0 + (1-p)b^c_{t+1} \tag{17}$$

where b^c_{t+1} is value of bubble if it continues.
Combining the two equations gives

$$b^c_{t+1} = \left[R_{t+1}(1+n)/(1-p)\right] b_t \tag{18}$$

- A bubble that continues grows faster than the interest rate

Contractionary and expansionary bubbles

Define share of bubble in savings

$$x_t = b_t / w_t$$

V and M shows that x_t evolves independently of k_t.
Capital stock evolves according to

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi - 1)}{1 - \Phi\pi} - (1 - \frac{\Phi(\pi - 1)n}{1 - \Phi\pi})(1 + n)x_t\right](1 - \alpha)k_t^{\alpha} \tag{19}$$

- Contractionary bubbles

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} < 1$$

  Bubbles reduce capital stock and raise interest rates
- Expansionary bubble

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} > 1$$

  Strong increase in loans to entrepreneurs, many new bubbles.
  Bubbles raise capital stock, lowers interest rates.

Expansionary bubble

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} > 1$$

Bubbles lead to strong credit expansion

- Made possible by financial frictions
- A stable stationary equilibrium for x_t with $0 < x_* < 1$ may exist.
- Bubbles may start low and grow for a long time
- AS the bubbles grow, interest rate goes down

More relevant for current crisis