Jon Vislie; September 2014
Hand-out ECON 4335 Economics of Banking

Supplement to Holmström & Tirole: Market equilibrium

The model outlined in Holmström and Tirole (1997) illustrates the role of capital, both among entrepreneurs, uninformed investors and banks (or monitoring institutions), for the undertaking of investment projects under market failure caused by moral hazard, both among entrepreneurs and banks. (The first group – the entreprenuers – should, irrespective of how the project is financed, be induced to choose the "good" lottery – the only one being socially beneficial – with a high probability of return and with no private benefit, B, that is reaped only if the "bad" lottery is chosen), whereas the banks should, under indirect finance be induced to monitor (at some cost c) and also get the entreprenuer to choose the "good" lottery with no private benefit as given in the monitor case by $b < B$, and also $b + c < B$.

In the model, some critical values are derived (as done in the article), from which we can divide the borrowers into three groups: The capital-rich entrepreneurs – with a good record – who solely use direct finance, entrepreneurs with medium-size capital get indirect finance (from banks acting also on behalf of uninformed investors), and capital-poor entrepreneurs – young firms without any record – will not undertake the investment (but will make their funds available in the market). The member of this last group has too little capital to put into the project so as to convince external actors that they will behave diligently or being sufficiently responsible for choosing the good lottery. (A borrower has to have enough skin or stake in the game in order to have any incentive to increase the value of the project; this incentive is related to own capital put into the project.)

From the relevant incentive and individual rationality constraints we get to the upper critical value, denoted $\overline{A}(r)$, with r as the required rate of return to uninformed investors, as the lowest value of capital that will entitle an entrepreneur with direct finance. This critical value is shown in the paper (not necessary to define the remaining symbols) to be given by:

The project is externally financed if and only if: $A + I_i \geq I$ or

$$(1) \quad A \geq I - I_i \geq I - \frac{p_H}{1+r}[R - \frac{B}{\Delta p}] := \overline{A}(r)$$

where $\overline{A}(r) > 0$ by assumption, with I_i being the amount invested by outside investors, increasing in r and increasing in the *agency cost* $\frac{B}{\frac{\Delta p}{p_H}} = \frac{B}{\frac{p_H - p_L}{p_H}}$ as well.

(This agency cost is higher the higher is the opportunity cost to the entreprenuer to work hard, B, as well the lower is the likelihood ratio $\frac{\Delta p}{p_H}$ which signals the informativeness of observing high return. The higher is this likelihhood ratio, the more likely is it that the entreprenuer has worked or not shirked (chosen the good lottery), when observing high return, and the smaller is the agency cost. (An entrepreneur with capital below this critical value will not be granted credit without banks. A firm with equity or wealth above this critical value, will put her entire capital into the project, whereas the uninformed investor will at least put in $I - A$.)

An underlying assumption is that $p_H R - (1+r)I > 0 > p_L R - (1+r)I + B$; only projects with no shirking will be socially desirable to undertake. Entreprenuers that are hit by the constraint (1), will be constrained even though the good project should be undertaken. There is credit rationing in equilibrium; a market failure. The question then is: Can this market failure be alleviated by introducing banks?

By introducing banks, that can monitor and get the entreprenurs to work (with a lower opportunity cost or smaller private benefit from choosing the "bad" lottery, b), the capital-constrained entrepreneurs can now be financed and therefore undertake their projects. For such indirect finance to take place, incentive as well as individual rationality constraints have to be satisfied, when the banks' required rate of return is λ (which must exceed r because of monitoring cost c, and that banks can act like uninformed investors without undertaking the monitoring task, by requiring a rate of return r). We then find that a bank will go in with an investment

$I_b \leq I_b(\lambda) := \frac{p_H c}{(1+\lambda)\Delta p}$, decreasing in λ, showing the smallest amount an

entrepreneur will borrow from a bank. (Indirect finance is more expensive than direct finance.) We then have that if $A + I_i + I_b(\lambda) \geq I$, a project will be financed, which can be translated to a lower critical value on own capital, below which no entreprenuer will get her project financed:

$$(2) \quad \underline{A}(r,\lambda) = I - I_b(\lambda) - I_i = I - \frac{p_H c}{(1+\lambda)\Delta p} - \frac{p_H}{1+r}(R - \frac{b+c}{\Delta p})$$

This lower critical value is increasing in both rates of return, r and λ. (It is assumed that $\underline{A}(r,\lambda) < \overline{A}(r)$.)

We then have: Firms with capital $A \in \left[\underline{A}(r,\lambda), \overline{A}(r)\right]$, in "number", $G(\overline{A}(r)) - G(\underline{A}(r,\lambda))$, each being financed by banks with borrowing $I_b(\lambda)$, total demand for bank credit is:

$$(3) \quad D_b(r,\lambda) := \left[G(\overline{A}(r)) - G(\underline{A}(r,\lambda)\right] \cdot I_b(\lambda)$$

Note that for a given r, the lower critical value will be increasing in λ; hence the number of firms being granted bank credit will go down. Also, because $I_b(\lambda)$ is decreasing in λ, a higher required rate of return from banks, for a given required rate of return from uninformed investors, will reduce total demand for bank credit. Hence we have $\frac{\partial D_b}{\partial \lambda} < 0$. Because both critical values are increasing in *r*, we cannot state precisely the sign of $\frac{\partial D_b}{\partial r}$; it will depend on properties of the distribution $G(A)$.

In equilibrium, with an exogenous bank capital, that is put into the projects (showing the banks' *lending capacity*), as given by K_b, the equilbrium condition for *bank credit* is:

$$(4) \quad K_b = \left[G(\bar{A}(r)) - G(\underline{A}(r,\lambda)\right] \cdot I_b(\lambda)$$

For any *r* and K_b, this market clearing condition will determine a unique rate of interest for bank credit (or informed capital, the term used by H&T), as given by $\lambda(r, K_b)$. Note that for a smaller value of K_b, λ must be higher. Why? Suppose the opposite, claiming that λ will go down. Then $I_b(\lambda)$ will increase, and because the lower critical value will be lower, we get that more firms being granted a higher loan from the banks. This is not compatible with a lower bank capital. Hence; the rate of interest for bank credit must increase as K_b becomes smaller; i.e.; $\dfrac{\partial\lambda(r,K_b)}{\partial K_b} < 0$.

To go a step further, we can look at the full market equilibrium, where we consider the pair of interest rates (r,λ) that will simultaneously clear the market for bank credit as well as the market for uninformed capital.

Let household saving be given by an increasing saving function $S(r)$, and let this be part of the supply of uninformed credit, along with supply of funds or capital by entrepreneurs with capital/cash below the lower critical value $\underline{A}(r,\lambda)$. Hence the supply of uninformed capital is

$$(5) \quad S(r) + \int_0^{\underline{A}(r,\lambda)} A g(A) dA$$

whereas demand is given by

$$(6) \quad D_i(r,\lambda) = \int_{\underline{A}(r,\lambda)}^{\bar{A}(r)} \left[I - A - I_b(\lambda)\right] g(A) dA + \int_{\bar{A}(r)}^{\hat{A}} \left[I - A\right] g(A) dA = \int_{\underline{A}(r,\lambda)}^{\hat{A}} [I - A] g(A) dA - K_b$$

where we have used (4).

Hence we have the final equilbrium condition, requiring that total supply of external capital; LHS of (7) below is equal to aggregate demand for capital; the RHS of (7), where the distribution of A is defined on $\left[0, \hat{A}\right]$:

$$(7) \quad S(r) + \int_{0}^{\underline{A}(r,\lambda)} A g(A) dA + K_b = \int_{\underline{A}(r,\lambda)}^{\hat{A}} [I - A] g(A) dA$$

What is now the impact on equilbrium of changes in exogenous variables, related either to (negative) shift in demand for capital, or shift in supply of capital. A negative shift in demand can be traced back to a "collateral squeeze" as firm values are reduced in a recession (the balance-sheet channel, as the distribution G is shifted to left; according to first-order stochastic dominance. The supply can go down, either because K_b can be reduced (through the lending channel; a credit chrunch), so that the banks' lending capacity is reduced (due to high losses on loans), or savings among households are subject to a negative shift ("hoarding liquidity"). What is the impact on the equilibrium rates of interest due to these changes?

Comparative statics

Suppose the supply of capital will go down; the LHS of (7) is reduced. To restore equality, the RHS must of course go down. One outcome is that the lower critical value must increase. Then previously bank-financed firms close to this critical value will now be denied finance. If the reduced supply is the outcome of a reduction in K_b, λ will increase. Hence, $\underline{A}(r,\lambda)$ will go up; entrepreneurs with capital close to this critcial value will now be moved from undertaking the project to become suppliers of capital in the uninformed segment, whereas entrepreneurs close to the upper critical value are unaffected. However, because the supply of uninformed capital now is increased, r might go down, giving a benefit to the capital-rich entrepreneurs. (However, this lower r will reduce saving from households.) At least one of the interest rates must increase under a capital squeeze. Now banks demand a higher rate of interest whereas bond owners may require a lower rate of interest.

If households reduce their saving, through a negative shift in the supply function $S(r)$, while bank capital is kept fixed, the first effect is, I guess, that $\underline{A}(r,\lambda)$ must increase, as r is increased in the first step. Without further information about $G(A)$, we cannot predict the impact on λ. (What we can say, because of (4), λ and r must change so that (4) is obeyed, with K_b kept unchanged.)

At last, if a shock is explained by a "collateral squeeze", when the distribution G moves to the left according to first-order stochastic dominance, then from (7), and with $K_e = \int_0^{\hat{A}} A g(A) dA$, which is reduced due to the squeeze (the avergae entrepreneur has less capital), we have:

$$(7)' \quad S(r) + K_e + K_b = I \int_{\underline{A}(r,\lambda)}^{\hat{A}} g(A) dA = I\left[1 - G(\underline{A}(r,\lambda))\right]$$

If the LHS of (7)' is reduced, when average firm value declines during a recession, the RHS must decline as well, through an increase in the lower critical value. (Here we have a fixed scale of investment.) At least one of the interest rates must increase so as to increase the lower critical value. The "weakest" entrepreneurs, those entrepreneurs with capital close to $\underline{A}(r,\lambda)$ are hurt. Then we have the following: Lower values of capital among entrepreneurs (as will happen during a recession) will reduce the financing options for undertaking profitable investment projects, and hence amplifying the initial recession. (This is sometimes called "an accelerator effect".)