Jon Vislie; September 2014
Hand-out ECON 4335 Economics of Banking

Supplement to Hellmann, Murdock and Stiglitz (2000)

A few remarks are provied as to how some of the results in the paper by Hellmann et al. are derived, as well as some explanations of the model not presented in class. Their story is about competition or financial market liberalization in the banking industry. If competition becomes too intense, as measured by some critical value of the elasticity to mobilize deposits, the banks will in a symmetric Nash equilibrium (in an infinitely repeated game) under moral hazard and limited liability, choose the risky asset portfolio, with no equity capital if they are free to choose, and offer a too high deposit rate at the deposit mobilization stage so as to capture higher market shares (market-stealing). If the bank should fail if the risky asset should be chosen the licence to run the bank is taken away by the regulator (Financial Supervisory Authority) at stage 4 along with losing the franchise or charter value. This market equilibrium is inefficient, because we can impose regulations so as to get a Pareto-improvement. The regulator needs to get the banks to choose the safe portfolio, by imposing some combination of capital requirement and deposit rate ceiling.

In section II of the paper a threshold rate of interest, given by $\hat{r}(k)$ is derived, from a "no-gambling condition", saying that the short-run or one-period gain from deviating to gambling from a strategy involving the choice of prudent (safe) asset at stage 3 of the game, where the symbols are defined in the paper, $\pi_G(r, r_{-i}, k) - \pi_P(r, r_{-i}, k)$ is below the present discounted value of future lost franchise value, $(1-\theta)\delta V_P(r, r_{-i}, k)$. The interpretation is that the current gain from deviating from the prudent strategy, is below the present value of the franchise value that is lost in the next period, that is why we discount V_p by the one-period discount factor $\delta \in \left(0,1\right]$, an event that occurs with probability $1-\theta$.

(A remark: Define the PDV of some time-independent (constant) future income stream, π_0, the number of NOK per unit of time, from today to infinity, where we use a discount factor $e^{-g\tau} \approx \delta$, where g is the instantaneous rate of interest (discount rate) under continuous compounding, and τ, here equal to one, is (in general) the length of the time period. We can then define the present discounted value of all these future income flows, as a stock or some wealth, the amount of NOK at t = 0, as:

$v = \int_0^\infty \pi_0 e^{-gt} dt = \pi_0 \left[-\frac{1}{g} e^{-gt} \right]_0^\infty = 0 + \frac{\pi_0}{g} = \frac{\pi_0}{g} \Leftrightarrow \pi_0 = gv$. The discrete version of this, when using the formula for an infinite geometric series, and take into account that it is a difference between continuous componding and a per period rate of interest, with a discount factor $\delta := \frac{1}{1+g}$, is then:

$v^* = \sum_{t=0}^{\infty} \delta^t \pi_0 = \sum_{t=0}^{\infty} \frac{\pi_0}{(1+g)^t} = \frac{\pi_0}{1-\delta} = \frac{\pi_0}{1-\frac{1}{1+g}} = \frac{1+g}{g}\pi_0$, which means that

$v^* = \pi_0 + \frac{\pi_0}{g}$. The difference between v and v^* is caused by the difference between the continuous and the discrete case. We get closer by looking at a case where the income flows start from $t = 1$. Then we have that the PDV is $v^{**} = v^* - \pi_0 = \frac{\pi_0}{g}$.)

To get the inequality $\pi_G(r, r_{-i}, k) - \pi_P(r, r_{-i}, k) \leq (1-\theta)\delta V_P(r, r_{-i}, k)$, we start with the *no-gambling condition* in equilibrium (at stage 3), with the effective discount factor for the gambling strategy being $\theta\delta$, when using that $V_G \leq V_P$, as given by

$$V_G \leq V_P \Leftrightarrow \frac{\pi_G}{1-\theta\delta} \leq \frac{\pi_P}{1-\delta} \Leftrightarrow \pi_G(1-\delta) \leq \pi_P(1-\delta\theta) \Leftrightarrow$$

$$\pi_G - \pi_P \leq \delta\pi_G - \theta\delta\pi_P = \delta V_G(1-\theta\delta) - \theta\delta V_P(1-\delta) \leq \delta V_P(1-\theta\delta) - \theta\delta V_P(1-\delta)$$

$$= \delta V_P\left[1 - \delta\theta - \theta(1-\delta)\right] = (1-\theta)\delta V_P(r, r_{-i}, k)$$

The one-period gain from deviating to gambling must be smaller than the opportunity cost; the lost return from the franchise value should the gamble fail. From this condition we can derive a threshold deposit interest rate, showing the border between gambling and no-gambling, with the property that for any deposit rate r **less** than (or equal to) this threshold value $\hat{r}(k)$, the bank will choose the safe or prudent asset at stage 3.

From $\pi_G - \pi_P \leq (1-\theta)\delta V_P(r, r_{-i}, k) = (1-\theta)\delta \frac{\pi_P}{1-\delta}$, we get:

$$\pi_G - \pi_P \leq (1-\theta)\delta \frac{\pi_P}{1-\delta} \Leftrightarrow \underbrace{\left[\theta(\gamma(1+k)-r)-\rho k\right]}_{\pi_G} \leq \underbrace{\left[\alpha(1+k)-\rho k - r\right]}_{\pi_P}\left[1+\frac{\delta(1-\theta)}{1-\delta}\right]$$

$$= \frac{1-\delta\theta}{1-\delta}\left[\alpha(1+k)-\rho k - r\right] \Leftrightarrow (1-\delta)\theta\gamma(1+k) - \theta(1-\delta)r - (1-\delta)\rho k$$

$$\leq (1-\delta\theta)\left[\alpha(1+k)-\rho k - r\right] \quad \Leftrightarrow$$

$$r\left[1-\delta\theta-\theta(1-\delta\right] \leq (1-\delta\theta)\left[\alpha(1+k)-\rho k\right] + (1-\delta)\rho k - (1-\delta)\theta\gamma(1+k) \Leftrightarrow$$

$$r(1-\theta) \leq \alpha(1+k) - \delta\theta\alpha(1+k) - \rho k + \delta\theta\rho k + (1-\delta)\rho k - (1-\delta)\theta\gamma(1+k)$$

$$= \alpha(1+k) - \delta\theta\alpha(1+k) - \rho k(1-\delta\theta-1+\delta) - (1-\delta)\theta\gamma(1+k) + (1-\delta)\alpha(1+k) - (1-\delta)\alpha(1+k)$$

On rearranging terms, we get:
$r(1-\theta) \leq (1-\delta)(1+k)(\alpha-\theta\gamma) - \delta(1-\theta)\rho k + \delta(1-\theta)\alpha(1+k)$; hence the no-gambling condition at stage 3 is reduced to:

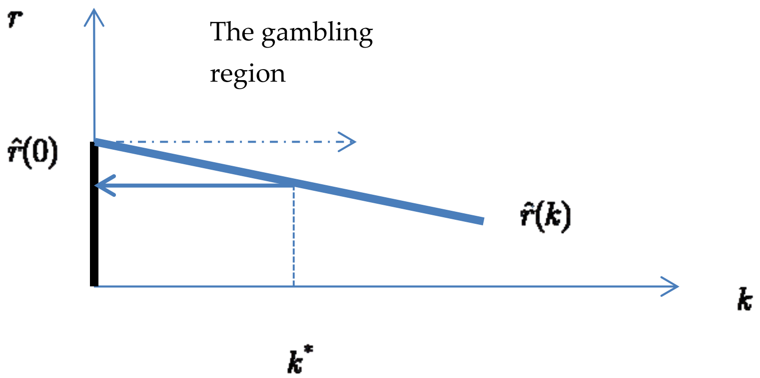
$$r \leq (1-\delta)[\frac{\alpha-\theta\gamma}{1-\theta}](1+k) + \delta\left[\alpha(1+k)-\rho k\right] := \hat{r}(k)$$

The function we have called $\hat{r}(k)$ is important for determining the set of Pareto-efficient outcomes. A Pareto-efficient outcome in this model, where no agent (depositor, bank or government – or the deposit insurance institution) can be made better off without harming other agents, is here translated to the minimal level of costly capital (with an exogenous required rate of return ρ) that for any deposit rate of interest is consistent with the banks choosing to invest in the prudent or safe asset. To get the bank not to play the risky strategy, the future gain or franchise (or charter) value has to be sufficiently high so that this franchise value that is at risk under gambling exceeds the short-run gain from gambling.

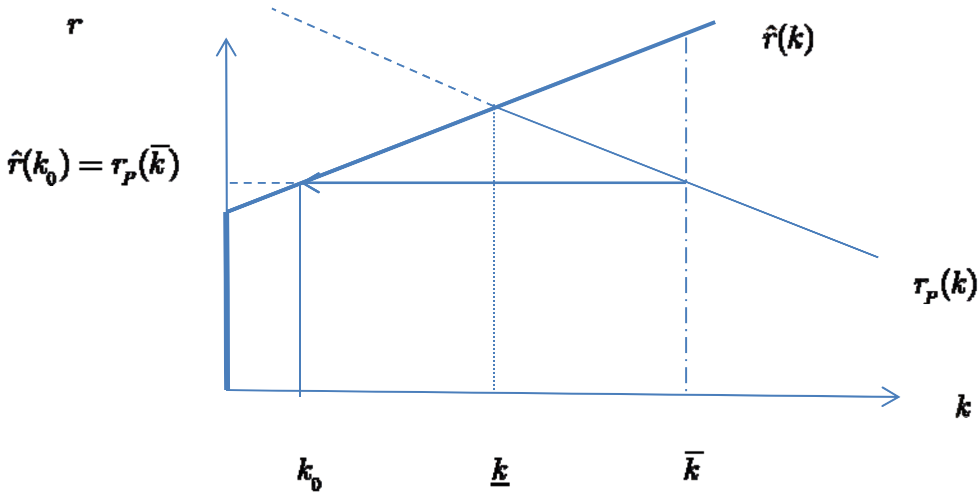
In an unregulated competitive equilibrium where banks play the prudent strategy, by choosing $k_P = 0$, and with a corresponding (symmetric) equilibrium deposit rate, derived from each bank solving the problem $Max_r V_P(r, r_P, k)$, under "rational expectations" about the best strategy choice by all competitors, by choosing $r_P(0) = \dfrac{\varepsilon\alpha}{1+\varepsilon}$, there is no need for government intervention. However, if the competitive pressure becomes that high so that $r_P(0) \geq \hat{r}(0)$, making the critical value of the elasticity of deposit mobilization with respect to own deposit rate, ε, equal to $\overline{\varepsilon}$, so that $\dfrac{\alpha\overline{\varepsilon}}{1+\overline{\varepsilon}} = \hat{r}(0) \Leftrightarrow \overline{\varepsilon} = \dfrac{\hat{r}(0)}{\alpha - \hat{r}(0)}$, then the banks will choose the risky strategy. (Proposition 1.) Then, if one were to rely completely on capital requirement alone as a measure against risk-taking, the requirement should have to be so strict, with capital being very expensive by assumption, that the resulting equilibrium outcome would be Pareto-inefficient. The argument is that the more capital, the "more skin in the game" or the more of the cost of gambling or downside risk is borne by the bank, inducing prudent behaviour. This capital-at-stake effect will therefore induce no-gambling. But because capital is costly ($\rho > \alpha$), a higher capital requirement will reduce current profits and hence the smaller is the increment to the franchise value. Then the future or long-run gain of running the bank goes down, which will increase the incentive for risk-taking or gambling, reinforcing the moral hazard problems or risk-taking behaviour. (A higher franchise value will make the loss if gambling fails higher.) Then to get the banks to choose a Pareto-efficient outcome as equilibrium outcome, the regulator has to impose a combination of deposit-rate control and capital requirement. (Capital requirement alone is not sufficient for Pareto-efficiency.) Imposing a ceiling on deposit rates is a way to restrict competition for market shares. Such a measure will therefore increase the profits per period (the profits will not be eroded as deposit rates cannot become too high), and hence the franchise value will be higher. Once the bank faces the risk of losing a high franchise value (above the expected one-period gain from gambling, as was shown earlier), it will choose a safe strategy.

To get the full picture of the set of Pareto-efficient outcomes (where no gambling takes place), we can first consider the set consisting of all deposit rates as given by $r \leq \hat{r}(0) = \dfrac{(1-\delta)(\alpha-\theta\gamma)}{1-\theta} + \delta\alpha$. Any deposit rate in this region is Pareto-efficient. A higher deposit rate will benefit depositors but will incur losses on the banks; a lower rate of interest will give the opposite result. (Remember that the equilibrium deposit rate under no-gambling, $r_P(0)$, is the one that maximizes $V_P(r, r_{-i}, 0)$; any deviation will therefore reduce the bank's payoff.) Hence, any deposit rate $r \leq \hat{r}(0)$, is Pareto-efficient. (See the vertical bold line segment in the figure below.)

Suppose then that the critical rate of interest $\hat{r}(k)$ is declining in k, with $\dfrac{\partial \hat{r}(k)}{\partial k} = (1-\delta)[\dfrac{\alpha-\theta\gamma}{1-\theta}] - \delta\left[\rho-\alpha\right] \leq 0$, which will occur if the banks are sufficiently patient or farsighted, with $\delta \geq \overline{\delta} := \dfrac{\alpha-\theta\gamma}{\alpha-\theta\gamma+(1-\theta)(\rho-\alpha)}$; hence $\delta \geq \overline{\delta} \Rightarrow \hat{r}(k)$ declining; not the other way! Now the negative franchise-value effect of a higher capital requirement is stronger than the positive capital-at-stake effect. (In the limit with $\delta \to 1$, we have $\hat{r}(k)_{\delta=1} = \alpha(1+k) - \rho k = \alpha - k(\rho-\alpha) = \alpha$ for $k = 0$.) If capital requirement is increased above zero, the banks' incentive to gamble increases; see the dotted arrow pointing to the right in the figure below into the gambling region, above $\hat{r}(k)$. The reason for this incentive is that costly capital will make the current profit smaller when k becomes higher; hence the increment to the franchise value becomes smaller. The second, dynamc effect dominates. Also, for any positive capital requirement, the deposit rate of interest realized along $\hat{r}(k)$ will fall below $\hat{r}(0)$. We can therefore achieve a Pareto improvement by reducing k to zero, while imposing a deposit-rate ceiling equal to $\hat{r}(k) \leq \hat{r}(0)$. Hence, if the banks are sufficiently patient or farsighted with $\delta \geq \overline{\delta}$, there exist deposit-rate ceilings that Pareto-dominate any capital requirement. (See the horzontal line with an arrow pointing towards the bold vertical line in the figure below, starting at $\hat{r}(k^*)$ and goes to the left.) Imposing a ceiling on the deposit rate equal to $\hat{r}(k^*)$, while combining this with *no* capital requirement, will leave the depositors with the same payoff as the deposit rate is the same, but the banks will have higher profits as they save expensive capital. If banks put sufficient weight on the future, as we perhaps should hope they do, any policy measure involving positive capital requirement is inferior, at least within the context of this model.



If on the other hand $\hat{r}(k)$ is increasing in *k*, as illustrated below, the banks can be viewed as being a bit more myopic as they do not put as much weight on the future as in the previous case, so that the capital-at-stake effect of a higher capital requirement dominates. Then, with more capital, the bank will to a larger extent internalize the cost of gambling. Under no regulation and given that the competitive pressure is above the critical value $\bar{\varepsilon}$, the banks will in this unregulated equilibrium choose no capital, gamble at stage 3 and offer deposit rate $r_G(0) = \dfrac{\varepsilon\gamma}{1+\varepsilon} > \hat{r}(0)$ at stage 1. This market equilibrium is Pareto-inefficient.



The government or regulator now wants the banks to behave prudently in equilibrium. (Any equilibrium in the gambling region is Pareto-inefficient; that is above $\hat{r}(k)$.) Suppose that the government, in the first place, attempts to rely solely on capital requirement. As indicated by Lemma 1, imposing a sufficiently high capital requirement on the banks, so that the capital-at-stake effect works in the direction of inducing prudent behaviour, the resulting equilibrium outcome is given by $r_P(\bar{k}) < \hat{r}(\bar{k})$, inside the Pareto-region, for the minimal effective capital requirement $\bar{k} > \underline{k}$ where $\underline{k}$ is defined so that $r_P(\underline{k}) = \hat{r}(\underline{k})$, with

$r_P(k) = \dfrac{\varepsilon}{1+\varepsilon}\left[\alpha(1+k) - \rho k\right]$ being declining in k because $\rho > \alpha$.

The story (or "proof") goes as follows: For any capital requirement $k < \underline{k}$, we have by assumption $r_P(k) > \hat{r}(k)$, and any bank, when deposit rates are freely set, will prefer to gamble. For $k = \underline{k}$, which obviously exists with our assumptions: $\hat{r}(k)$

increasing, $r_P(k)$ declining, with $r_P(0) > \hat{r}(0)$, we have by construction, that $V_G(r_P(\underline{k}), r_P(\underline{k}), \underline{k}) = V_P(r_P(\underline{k}), r_P(\underline{k}), \underline{k})$ for $r_P(\underline{k}) = \hat{r}(\underline{k})$. With deposits mobilized at stage 1, $D(r_P(\underline{k}), r_P(\underline{k}))$, which will be fixed at stage 3, we must then have

$$\frac{m_G(r_P(\underline{k}), \underline{k})}{1-\delta\theta} = \frac{m_P(r_P(\underline{k}), \underline{k})}{1-\delta} \Rightarrow m_G(r_P(\underline{k}), \underline{k}) = \frac{1-\delta\theta}{1-\delta} m_P(r_P(\underline{k}), \underline{k}) > m_P(r_P(\underline{k}), \underline{k}) > \theta m_P$$

saying that expected margin (or profit per unit deposit) from gambling exceeds the margin from playing safe. By definition we have $r_P(\underline{k}) = \arg\max_r V_P(r, r_P(\underline{k}), \underline{k})$, where a no-gambling equilibrium is characterized by $m_P(r_P(\underline{k}), \underline{k}) = \dfrac{D(r_P, r_P)}{\dfrac{\partial D(r_P, r_P)}{\partial r}}$. To check whether a deviation is profitable for a bank, if all other banks offer $r_P(\underline{k})$, "our" bank will consider whether the payoff can be increased by offering a higher deposit rate at stage 1, so as to capture more deposits on which the bank earns a higher margin at stage 3 when the risky lottery is chosen. If deviation is profitable, we have market-stealing through higher deposit rate offered by our bank, as can be seen from:

$$\frac{\partial V_G(r_P(\underline{k}), r_P(\underline{k}), \underline{k})}{\partial r} = \frac{1}{1-\delta\theta}[-\theta D(r_P(\underline{k}), r_P(\underline{k})) + m_G(r_P(\underline{k}), \underline{k})\frac{\partial D(r_P(\underline{k}), r_P(\underline{k}))}{\partial r}].$$

Use that $m_P(r_P(\underline{k}), \underline{k})\dfrac{\partial D(r_P, r_P)}{\partial r} = D(r_P, r_P)$ in this expression, to get:

$$\frac{\partial V_G(r_P(\underline{k}), r_P(\underline{k}), \underline{k})}{\partial r} = \frac{1}{1-\delta\theta}[-\theta m_P(r_P(\underline{k}), \underline{k}) + m_G(r_P(\underline{k}), \underline{k})]\frac{\partial D(r_P(\underline{k}), r_P(\underline{k}))}{\partial r} > 0.$$

Hence, a deviation by offering a higher deposit rate at stage 1, if all competitors offer $r_P(\underline{k})$, is profitable. The equilibrium outcome will be inefficient, located in the gambling region, with all banks offering deposit rates above $r_P(\underline{k}) = \hat{r}(\underline{k})$.

Therefore, imposing a sufficiently high capital requirement, without any deposit rate ceiling, no banks will find it profitable to deviate from the prudent strategy, because now the cost of gambling is fully borne by the bank. The smallest capital requirement that implements a no-gambling equilibrium is $\overline{k} > \underline{k}$, that must obey: $\max_r V_G(r, r_P(\overline{k}), \overline{k}) = V_P(r_P(\overline{k}), r_P(\overline{k}), \overline{k})$, with equilibrium deposit rate $r_P(\overline{k}) < \hat{r}(\overline{k})$, located in the no-gambling region.

But this equilibrium with a very strict capital requirement (and freely set deposit rates) alone is not Pareto-efficient. We can have a Pareto-improvement by imposing a combination of two measures or instruments; a deposit rate ceiling and a less strict capital requirement; as given by the pair $\left\{\hat{r}(k_0) = r_P(\overline{k}), k_0\right\}$, as indicated in the figure above by the horizontal line pointing (with arrow) to the left. The depositors are equally well off, whereas the banks are better off because they are required to keep less capital that is costly, with no loss to the government, because the safe portfolio strategy will be chosen by the banks; (see Proposition 4).