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The Commercial Banking Firm: A Simple Model

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Abstract

A commercial bank must decide its volume of illiquid loans and investments without full knowledge of its deposits. In case withdrawals exceed its defensive position in liquid assets, the bank incurs extra costs in meeting reserve requirements, penalties in borrowing or losses in disposing of illiquid assets. In case of good luck on deposits, an unnecessarily large defensive position sacrifices profits from more remunerative but less liquid assets. The paper presents a simple model of this precautionary portfolio decision by a profit-maximizing bank. The effects of variations in monetary policy, banking regulations, and competitive structure are traced: reserve requirements, access to central bank credit, deposit interest rate ceilings, monopolistic power in loan and deposit markets, relative importance of capital and time deposit liabilities, size and nature of illiquidity penalties, and degree of deposit uncertainty.

Commercial banks are the most important kind of financial intermediary. Their liabilities are the closest privately issued substitutes for government currency. Demand deposits serve as means of payment generally acceptable for most transactions. For several reasons, depositors bear very little of the risks of the loans the banks make. There are economies of scale in pooling of default risks specific to borrowers and in specialized administration and appraisal of the loans. The banks' shareholders assume the residual risk; only after their equity is wiped out would depositors' claims be jeopardized. Finally, the government stands behind bank deposit liabilities, both as »lender of last resort» to tide banks over crises of illiquidity and as insurer of deposits against the contingency of insolvency.

Government monetary and credit policy operates mainly through the commercial banking system, and this is the main reason today to give special attention to commercial banks. Historically the first reason for government intervention in the banking business was to protect depositors (or in older times bank note holders) against the risks of bank illiquidity and insolvency. But the public regulations and institutions established for this purpose can also be used to regulate credit markets in the interests of economic stabilization, maintenance of the value of the currency, or other government objectives. As the protective purpose of government interven-

tion has been achieved, these other objectives have come to dominate relations of government to the commercial banking system.

This paper presents a primitive theory of a single commercial banking enterprise. It refers principally to the traditional distinctive function of banks, to buy and hold assets of longer maturity and less liquidity than their liabilities. In recent years banks, large banks especially, have increasingly become brokers buying and selling marketable assets and liabilities more closely matched in maturity and liquidity. The simple model of this paper does not apply to that business. Moreover, the model refers principally to the United States. Because of legal limitations on branch banking, there are many more banking firms and the system is more decentralized and competitive—monopolistically competitive in Chamberlin's sense—than in other economies. The legal framework and central banking institutions assumed also conform to American practice. Nevertheless, I believe and hope that the model has wider applicability.

I. The Portfolio Choices of a Bank

In a first approximation to understanding the nature of the business decisions confronting a commercial banker, the assets of a commercial bank may be divided into two categories: *loans and investments* and *defensive assets*. *Loans and investments* are in the short run either illiquid or unpredictable in value. To be sure of realizing their full value, the bank must hold them to maturity. Consequently, these assets can be available for meeting deposit withdrawals only at some risk of loss. It may indeed be impossible to sell or to borrow against certain loans. *Defensive assets* are, in contrast, assets of very high liquidity. The bank knows that they can be sold, or borrowed against, without loss or delay. *Defensive assets* include currency, deposits in the central bank, deposits in other banks, overnight loans to other banks (known as »federal funds» in the U.S.), well-secured call loans, Treasury bills, and other paper of equivalent quality and eligibility as collateral. In this usage the term covers both primary and secondary reserves, i.e., both those assets that qualify as legal reserves, and those that are so readily convertible into legal reserves that the banks can regard them as the virtual equivalent.

Law or convention generally requires the bank to hold a certain quantity of defensive assets, the bank's required reserves. When the Monetary Control Act of 1980 becomes fully operative in 1987, depository institutions in the United States will be required to hold in currency or on deposit in a Federal Reserve Bank about 12 percent of demand deposits and other checkable accounts and 3 percent of non-personal savings deposits and time deposits of original maturity shorter than 3½ years.

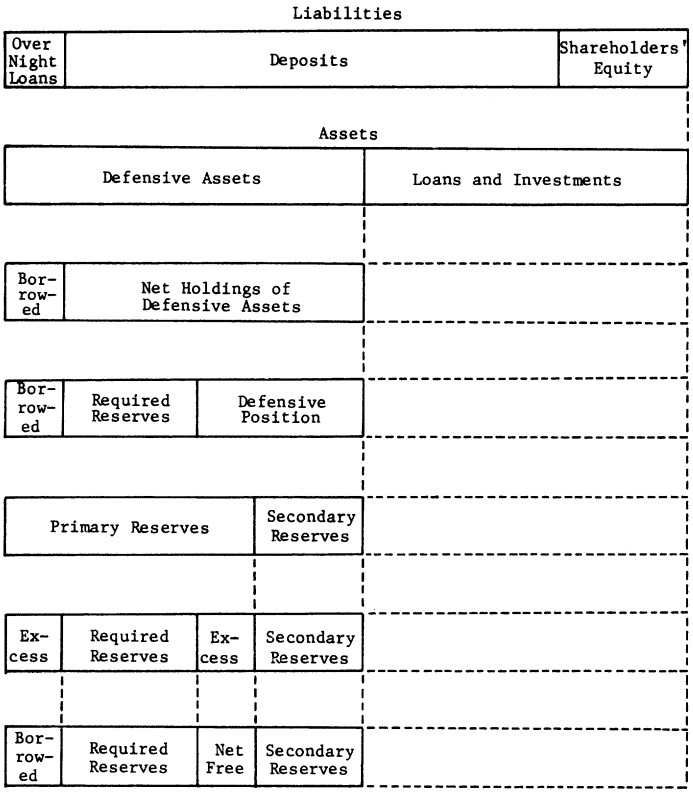

Fig. 1 Schematic representation of bank balance sheet.

A bank's holdings of defensive assets will almost always exceed its required reserves. But some of these holdings may be offset by short term or overnight borrowing from other banks or from the central bank. The amount by which the bank's *net* holdings of defensive assets exceed its required reserves will be called its *defensive position*. For it is this margin which measures the bank's ability to meet reserve requirements if it should be confronted with unusual deposit withdrawals or extraordinary demands for loans. In this contingency the bank can draw down its deposits in other banks; or fail to renew overnight loans of »federal funds» to other banks; or present maturing Treasury bills for cash instead of new bills; or borrow money on the collateral of Treasure bills; or sell bills either outright or with agreement to repurchase them.

The basic accounting identity for the commercial bank (abstracting from its physical assets and other inessential accounts) is:

Deposits+Shareholders' equity=Required reserves+Defensive position+ Loans and investments

Figure 1 displays this accounting identity graphically, and also sets forth some other definitions and classifications that will be useful.

It is a convenient simplification to distinguish three kinds of bank decisions: (1) the sizes of the two broad asset categories, the *loan and investment* portfolio and the net *defensive position*, (2) the management of the defensive position: the kinds and amounts of primary or secondary reserve assets held, and the amount of short-term borrowing, and (3) the composition of the loan and investment portfolio. This paper concerns mainly the first decision, discusses some aspects of the second, and does not treat the third.

II. The Bank's Deposits

The volume of deposits in a bank is partly within and partly outside its control. The location of a bank, in both a geographic and an economic sense, gives it a natural clientele of depositors. The bank can seek to increase its attractiveness to depositors by the interest rates it offers on deposits, by the quality and costs of its services, and by the usual media of indirect competition—the splendor of the building, the organ music and the lollipops for children, the advertising, the cultivated identification with community service, and so on. However, some of these forms of competition for deposits are limited by law or by explicit or tacit convention.

In the United States today, banks are prohibited from paying interest on demand deposits of corporations and from exceeding legal ceilings for interest on other checkable deposits, passbook savings accounts, and certain time deposits.

There is another and more direct way by which a bank may be able to influence the level of its own deposits. When a bank makes a loan to one of its customers it simply credits the amount to the borrower's account. In the first instance, therefore, the bank's deposits are increased dollar for dollar with its loans. As the borrower spends the proceeds by check, some of the recipients will leave the money on deposit with the lending bank, while others will deposit their receipts in other banks or convert them into currency. As these recipients spend their balances and in succeeding generations of transactions, the lending bank will lose more and more of the deposit created by its initial loan. On the other hand, a borrower may build up his deposit in the lending bank towards the end of the term of his loan, in preparation for repaying it.

Sometimes a bank requires the borrower to hold a certain fraction of the loan on deposit during the term of the loan; in effect, the bank has lent less than the nominal amount of the loan. This practice should be distinguished from the "you lend to me now, I'll lend to you later" bargain that charac-

terizes, with varying degrees of formal and explicit understanding, the relations of a bank and a continuing customer. By the terms of such an understanding, a borrower compensates for his loan with deposits in past or future, not by holding some of the loan itself on deposit. Through these continuing customer relations, a bank's loans affect its deposits, but at some future time. It is equally true that deposits today entail some commitment to provide future loan accommodation, Indeed, one way in which banks compete for deposits with each other, and with other short-term borrowers, is by offering to regular depositors the reciprocal assurance of credit on favorable terms when they want to borrow rather than lend.

The degree to which a bank can expect to retain deposits resulting from its own loans depends on its size relative to other banks. It a bank is the only one in its community, it will retain more than if the local payees of the borrower's checks are scattered among several banks. But even a local monopoly bank will lose deposits as transactions spread to other areas of the economy. And a national monopoly bank, if one existed, would still face leakages to currency holding, foreign balances, and other assets. The restrictions on branch banking in the United States make the leakages from an individual bank typically much larger than in the banking systems of most other countries, where a handful of banks, each with many branches, dominates the scene. Where in England or Canada or Sweden a loan by one branch increases deposits in another branch of the same bank, in the U.S. a loan by one bank increases deposits in another bank.

The leakage is naturally greater and quicker for bank investments in nationally marketable securities than for loans to local businessmen. An individual bank, even a large one, can be assumed to retain virtually nothing at all of amounts placed in defensive assets. A bank's decision to hold currency, or deposits in the central bank, or the debts of other banks, or Treasury securities does not lead to any significant increase in the amounts that others choose to leave on deposit with the bank.

III. Bank Portfolios and Profits

In considering the bank's broad portfolio choice, between *loans and investments* and *defensive position,* it will be simplest to begin with a bank whose deposits are costless and independent of the bank's own portfolio decisions. Moreover, the bank is assumed to know with certainty what its deposits will be over the period of time that its funds are committed. This assumption abstracts, of course, from the essential problem of banking, the unpredictability of deposits, and it will be removed shortly. Beginning with the unrealistic case of certainty is an analytic and expository convenience.

In Figure 2, assets are measured horizontally and deposits vertically. The

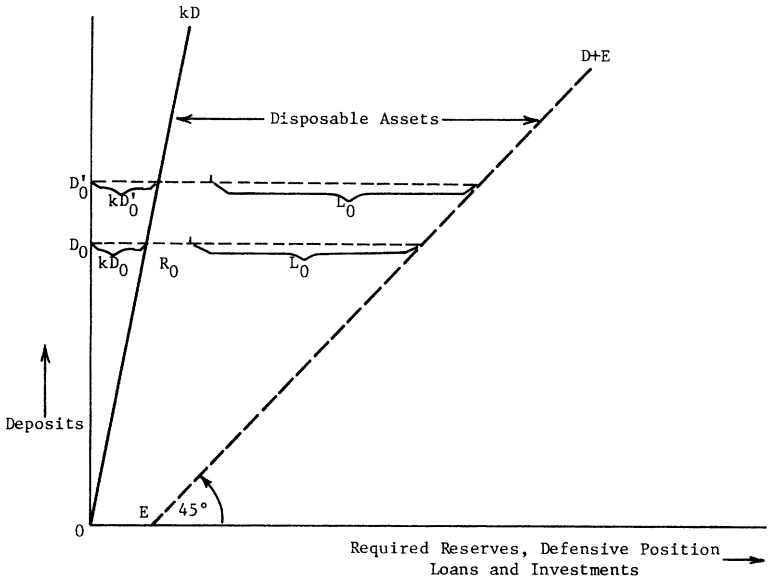

Fig. 2

basic accounting identity behind the diagram is the same one given above. translated into symbols as follows:

$$D+E=kD+R+L$$

Thus the horizontal distance to the dashed line $D+E$ represents total assets, and the horizontal distance to the solid line kD represents required reserves. The horizontal distance between these lines represents "disposable assets," the amount that can be divided between defensive position R and loans and investments L. If loans and investments exceeded disposable assets, the defensive position would be negative.

Now suppose that the bank earns a rate of interest r on its defensive position R (or pays interest at this rate on a negative position), and earns on a loan and investment portfolio of size L a total revenue $P(L)$ (net of administrative costs and actuarial allowance for default). Its deposits are a given amount beyond its control, D_0, and its equity is fixed at E. The bank will seek to maximize the total net revenue on its portfolio, $P(L)+Rr$, subject to the balance sheet constraint $L+R=E+(1-k)D_0$. If the marginal revenue from loans $P'(L)$ is constant and greater than r, this maximization puts no limit on the loan and investment portfolio other than the bank's ability to borrow to finance a negative defensive position. If the marginal revenue from loans $P'(L)$ is always smaller than r, the bank will simply hold all its deposits and equity in defensive assets.

A situation in which declining marginal revenue sets a positive limit on

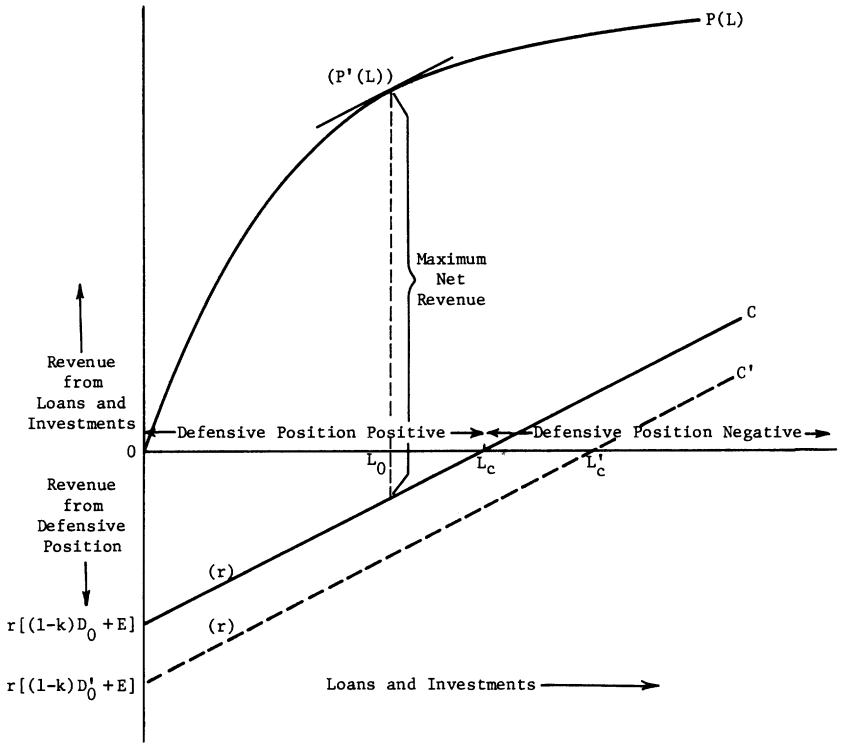

Fig. 3

the loan and investment portfolio is shown in Figure 3. Here loans and investments L are measured horizontally. L_c is the point at which loans and investments exhaust disposable assets: that is, $L_c=E+(1-k)D_0$. To the left of L_c, defensive position R is positive; to the right, negative. Total net revenue from loans and investments is measured vertically upward from the origin, and total revenue from defensive position is measured in the opposite direction, downward from the origin. A negative defensive position means, of course, negative revenue from this source; in Figure 3 this is shown when the lower revenue curve rises above the horizontal axis. Thus total revenue from the two sources combined appears in Figure 3 as the difference between the two curves. Indeed the lower line—revenue from defensive position— may be regarded as the cost curve, albeit opportunity cost, to the loans and investments revenue curve. In Figure 3 a revenue maximum occurs at L_0, where marginal revenue is the same from the two sources. The slope of a line or curve is indicated in the diagram in parentheses; for example, the slope of line C is r.

A higher level of deposits will enable the bank to enjoy a stronger defensive position at any given volume of loans and investments. Suppose,

for example, that deposits rise from D_0 in Figure 2 to D_0', so that the bank now has, after meeting reserve requirements of kD_0', disposable deposits of $D_0'(1-k)$. Then the point of zero defensive position L_c in Figure 3 will shift to the right, e.g. to L_c', by the amount $(1-k)(D_0'-D_0)$. There will be a parallel shift to the right, from C to C' in the line representing the revenue from the defensive position in Figure 3. The bank will have no reason to change its volume of loans and investments. The volume that equated the marginal revenue and marginal opportunity cost of lending still does so. Given the required reserve ratio k, a dollar increase in deposits means an increase of required reserves of k and an increase in defensive position of $1-k$.

III.1 *Penalties for Negative Defensive Position*

So far it has been assumed that the bank can always borrow short term at the same rate r that positive holdings of defensive assets yield. If so, the marginal opportunity cost of lending is the same for negative defensive positions as for positive. But more likely the bank must pay a higher rate to borrow, or—what amounts to the same thing—must liquidate assets that would have yielded a higher rate than r. Let the effective rate for negative positions be $r+b$. In addition, there may be a fixed cost of net borrowing, independent of the amount borrowed, attributable to the costs and inconveniences of arranging a loan or to the loss of prestige involved in displaying a shortage of owned reserves. In Figures 4*a*, 4*b* and 4*c*, the curve C representing revenue from defensive position increases in slope at L_c from r to $r+b$. In Figure 4*a* there is no fixed cost involved in having a negative defensive position. But in Figures 4*b* and 4*c* the existence and amount of a fixed cost a are shown by the upward jump in the curve C.

These changes in the opportunity cost curve at L_c have no effect on the bank's decision if the decision is in any case to lend an amount less than or equal to L_c, i.e., to hold a positive or zero defensive position. This will be true whenever $P'(L_c)$, marginal revenue from loans at the point L_c, is smaller than or equal to r. But the extra costs of a negative position are relevant if $P'(L_c)$ exceeds r. Here there are several possibilities:

(a) $r<P'(L_c)\leq r+b$. (Not diagrammed) The best portfolio for the bank is the one corresponding to L_c, a zero defensive position.

(b) $r+b<P'(L_c)$, no fixed cost (Figure 4*a*). Here the bank will choose a volume of loans and investments higher than L_c and a negative defensive position.

(c) $r+b<P'(L_c)$, positive fixed cost (Figures 4*b* and 4*c*). Here the bank may, as in case (b) proceed to a point beyond L_c where $P'(L)=r+b$. This is illustrated in Figure 4*b*. However, it is also possible that the revenue at such a point is smaller than the revenue at L_c because of the fixed cost of borrowing. This outcome is illustrated in Figure 4*c*.

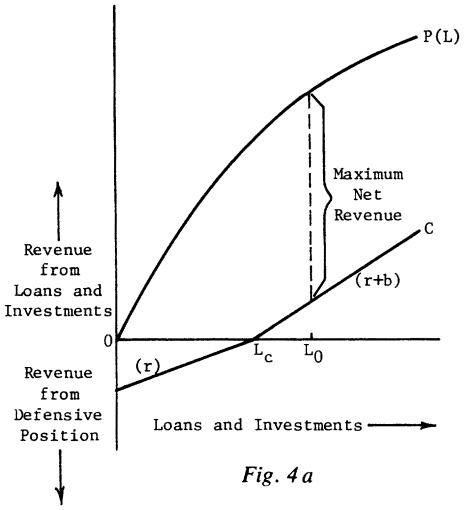

Fig. 4 a

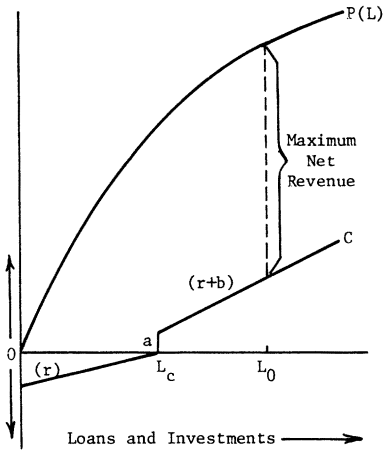

Fig. 4 b

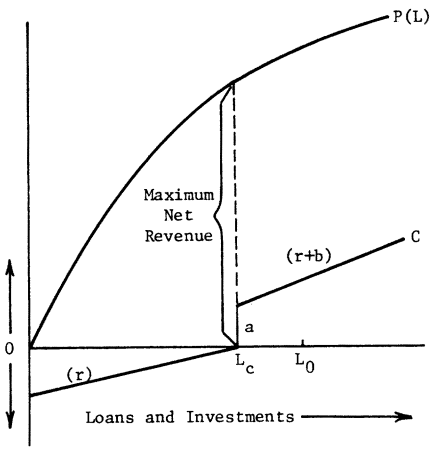

Fig. 4 c

The preceding analysis has assumed that the bank neither earns interest on required reserves nor pays interest on its deposits. These assumptions can be relaxed somewhat without altering the analysis.

Central banking systems differ in respect to the payment of interest to banks on required reserves. In the U.S. Federal Reserve System, member bank reserve balances earn no interest; banks can enjoy the prevailing yield r on defensive assets only to the extent that their defensive positions exceeds requirements. Currently, however, there is considerable agitation for payment of interest on reserves, because banks are increasingly competing for deposits with money market funds and other institutions not

required to hold reserves. In other banking systems, e.g. the United Kingdom and Canada, two reserve requirements or conventions have sometimes been in effect simultaneously, a liquid assets convention and within that a cash requirement. In the U.K., for example, banks could earn the prevailing rate r on all their defensive assets including those qualified for the 30 percent liquid-assets convention, except for the 8 percent of deposits required to be held in cash or with the Bank of England.

If interest is earned on required reserves, these earnings add a constant amount to the bank's revenue from its given volume of deposits. This addition does not affect the calculations that determine the optimal allocation of those deposits between loans and investments and defensive assets.

Likewise, the analysis applies when the bank must pay interest to its depositors so long as the deposit interest rate is regulated and fixed below what the bank can earn on a dollar of deposits. It is still in the bank's interest to accept as many deposits as are available to it. The interest outlay is then a fixed cost, the same whatever the bank's portfolio.

Although these interest receipts and outlays do not affect the maximum-profit portfolio, they do affect the bank's profits. They may be relevant, therefore, to the shareowners' decision whether to stay in the business or not.

III.2. *The Value and Cost of Equity*

How much is a dollar of new equity worth to the bank? More precisely, how much additional profit will the bank earn if an additional dollar of capital is subscribed? With the exception of the case illustrated in Figure 4c, additional capital will simply be invested in an increased defensive position and will thus yield either r or $r+b$. Since r or $r+b$ is also the marginal revenue from lending, it is equally true to say that the value of new capital is $P'(L)$. In the exceptional case, the new equity will go into loans and investments; it will yield the marginal revenue $P'(L)$, which exceeds $r+b$. If the existing shareholders of the bank can attract a new stock subscription of a dollar by the prospect of a dividend yield no larger than these marginal contributions to profit, it will be worth their while to do so. In the long-run equilibrium of the banking firm, the marginal cost of equity capital will just equal the marginal profit. The bank may nonetheless be making monopoly profits which would attract new firms if free entry into the industry were possible.

III.3. *The Value and Cost of Deposits*

How much is a dollar increase in deposits worth to the bank? As noted above, an increase in deposits of \$1 will normally mean an increase of \$$1-k$ in the defensive position and an increase of \$$k$ in required reserves. Therefore, assuming no interest is paid on required reserves, a dollar more in

deposits will permit the bank to add $(1-k)r$ to its earnings if its defensive position is positive, or $(1-k)(r+b)$ if its defensive position is negative. Since in these situations $P'(L)$ is equal to r or $r+b$, the value of deposits can also be expressed as $(1-k)P'(L)$.

The situation pictured in Figure 4*c* is again something of an exception. There an influx of deposits will push the jump in the opportunity cost curve to the right. It will still be profitable for the bank to lend up to the jump, the point of zero defensive position. Hence an increase in deposits will increase loans and investments rather than defensive position. It will, as in the other cases, add $(1-k)$ times the marginal revenue of lending, $(1-k)P'(L_c)$, to the bank's revenue. But this is greater than $(1-k)(r+b)$.

The marginal value of deposits to the bank is smaller than the marginal value of equity in the proportion $(1-k)$, because of the reserve requirement against deposits. But of course it is generally easier to attract deposits than to attract capital.

If interest is paid at rate r' on required reserves, then the marginal cost of deposits will be equated to a weighted average of r' and the marginal return on disposable assets, with weights k and $1-k$ respectively. So long as r' is the smaller, one effect of an increase in the required reserve ratio k is to lower the value of deposits to the bank.

Banks generally accept more than one kind of deposit. Suppose l is the reserve requirement on a particular category of time deposits T. Then if r' is zero, the additional earnings made possibly by a dollar additional of time deposits is $(1-l)P'(L)$. Assuming l is smaller than the reserve ratio k required for demand deposits, the gross marginal value of time deposits lies between the corresponding values of equity and demand deposits. But of course the net value of time deposits depends also on the interest that must be paid to attract them.

III.4. *Unrestricted Competition for Deposits*

So long as competition for deposits is effectively limited by laws or gentlemen's agreements prohibiting or limiting interest payments to depositors, then the firm and the industry are in disequilibrium. These devices permit the banking industry as a whole to receive rents which under effective competition would be paid to depositors. In the long run the competition of new banking firms could reduce the marginal revenue from lending until the value of deposits is brought down to their cost. But in practice entry is not free. In the United States entry is regulated by federal and state chartering authorities.

For many years the legal ceilings on interest rates payable on time and savings accounts in the United States were so low as to prevent effective rate competition for deposits. Raising or lifting of ceilings since 1962 paved the way for vigorous and open rate competition for time deposits. As would

be expected, the rates banks offer for time deposits, which have small or zero reserve requirements, are close to the yields they can earn on defensive assets. Although effective ceilings, zero to 5½ percent, apply to demand deposits, other checkable deposits, and passbook savings accounts, these will be gradually raised under existing legislation. By the end of the decade it is quite possible that the system will be transformed into one without effective regulation of deposit interest rates.

In a full equilibrium, without deposit rate ceilings, the banking firm will equate the marginal cost of each class of deposits to its marginal value, i.e., to the addition to earnings on assets which an additional dollar of deposits will permit. Moreover, the optimal portfolio will generally be such that this addition to earnings will be the same no matter in which asset the new deposit is invested. If the value of an additional deposit exceeds its cost, the bank will seek to attract deposits by raising the rate of interest paid depositors or by other competitive devices.

The marginal cost of attracting deposits may exceed the average interest rate on deposits and may rise with the bank's volume of deposits. For one thing, some of the costs of attracting deposits are not payments to depositors at all but diffuse costs of administration, promotion, advertising, and atmosphere. In paying depositors, moreover, the bank may be a monopsonist, just as it has some monopoly power in selling loans to borrowers. No doubt the bank can to some extent discriminate among depositors, as it can among borrowers. Thus if it takes 12 percent to attract a new depositor, the bank is not necessarily forced to pay 12 percent to all its depositors. As noted above there are indirect ways of giving a depositor special remuneration, e.g. in ancillary services or in promises of preferred treatment when the depositor wishes to borrow.

IV. Uncertainty about Deposits

IV.1. *The Function of Reserves and Defensive Assets*

So far it has been assumed that the bank knows for sure what its deposits will be. The bank holds defensive assets beyond its reserve requirements only to the extent that their yield is competitive with the marginal revenue from lending. In this analysis the properties of defensive assets—highly liquid and predictable in value—were quite inessential.Any other assets with comparable yields might find their way into bank portfolios with loans and investments.

The principal reason that banks hold defensive assets—generally at lower yields than the marginal returns from lending—is to defend themselves against deposit withdrawals which they cannot perfectly foresee. Many factors outside the bank's power either to control or to predict can change the bank's deposits from week to week, and even from day to day. For

example, expansion or contraction of the lending of other banks will spill deposits and reserves into a bank, or suck them from it. In deciding the volume of its loans and investments, the bank must commit itself to illiquid assets *before* it knows what its deposits will be, and must consider the consequences of large if improbable withdrawals.

At one time the consequence might have been a literal inability to honor demand obligations. The bank would "fail" not because its loans and investments were bad but simply because they were illiquid. The original and historic purpose of reserves was to protect the bank and its depositors against this kind of failure.

Modern financial institutions have virtually eliminated this danger. As a "lender of last resort" the central bank—the Federal Reserve System in the U.S.—can prevent banks from failing simply from lack of liquid funds to meet deposit withdrawals. The possibility of a contagious loss of confidence in a particular bank or in banks in general is greatly reduced both by the availability of a lender of last resort and by government guarantees of deposits. A bank may fail because of poor management or extraordinarily bad luck in making loans and investments, not because these assets are illiquid but because they are of insufficient value even if held to maturity and beyond. The stockholders' capital in the enterprise is the depositors', or deposit insurer's, cushion against this kind of misfortune.

The historic function of reserves as a cushion against insolvency due simply to illiquidity has been rendered obsolete. The modern function of reserves is to provide a mechanism of monetary control over the economy by the central bank. Why do banks hold reserves? They hold reserves because they are required to do so by law or by convention with the virtual force of law. These required reserves are—paradoxically in view of the original function of reserves and indeed of reserve requirements—unavailable to meet deposit withdrawals. Why do banks maintain a net defensive position in excess of reserve requirements? They hold secondary reserves for fear that they might not pass the required reserve test without incurring the special costs of borrowing or of liquidating high-yielding investments. The consequence of deposit withdrawals, against which the bank protects itself by excess reserves, is not the disaster of insolvency but the additional cost, including perhaps inconvenience and damage to prestige, involved in meeting the reserve test. Given these costs, uncertainty about the future level of deposits may lead to a lower volume of lending and to a higher defensive position than the profit maximization discussed above.

The analysis will assume that only demand deposits are subject to uncertainty. If the bank also has time deposits, these are assumed to be no less illiquid than loans. Thus the making of loans need not be deterred by fear of losing time deposits, for the loans will be repaid soon enough to meet such withdrawals. This means that time deposits play much the same role as

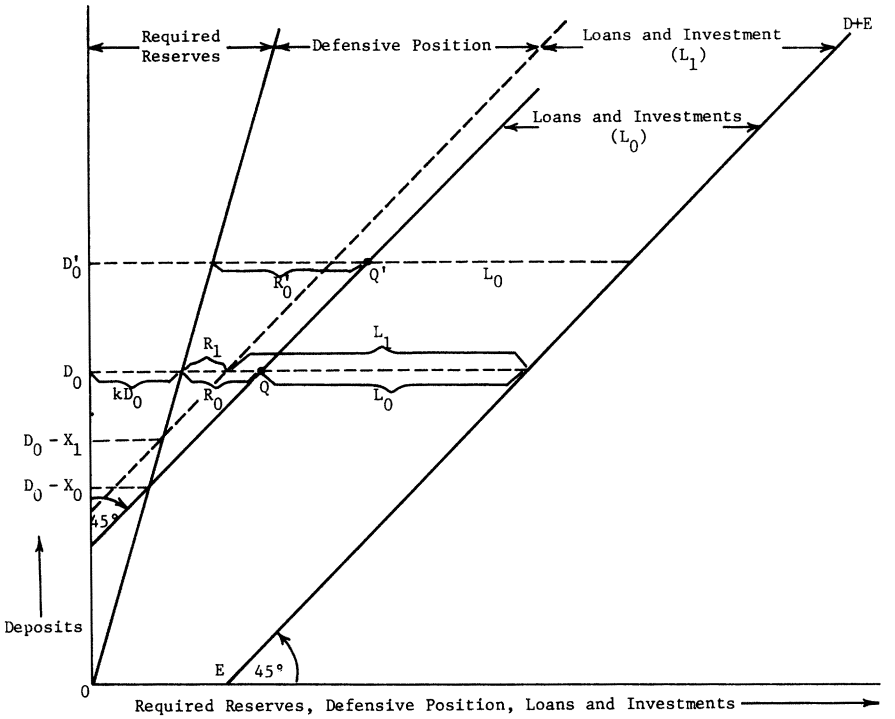

Fig. 5

equity, with the exception that they entail interest payments and possibly reserve requirements. This assumption is unrealistically extreme, but it is qualitatively in the right direction. Time deposits are less volatile and unpredictable than demand deposits, and a bank with a high proportion of time deposits has less to fear from unanticipated withdrawals.

IV.2. *The Portfolio that Maximizes Expected Profit*

Once the bank has chosen a volume of loans and investments, a dollar change in deposits will mean a $\$(1-k)$ change in defensive position. An influx of deposits will increase the defensive position at least temporarily; if the bank becomes convinced it is a permanent gain presumably it will choose a new and higher volume of loans and investments. A loss of deposits will lower and perhaps wipe out the defensive position. Again, this outcome may be only temporary. If the loss of deposits proves to be permanent, the bank will in time lower its volume of loans and investments in order to reconstitute its defensive position.

In Figure 5 let L_0 represent, as in Figure 2, the volume of loans and investments chosen by the bank. Let D_0 be the expected volume of deposits, in the probability sense, the next week. In Figure 2 the bank was

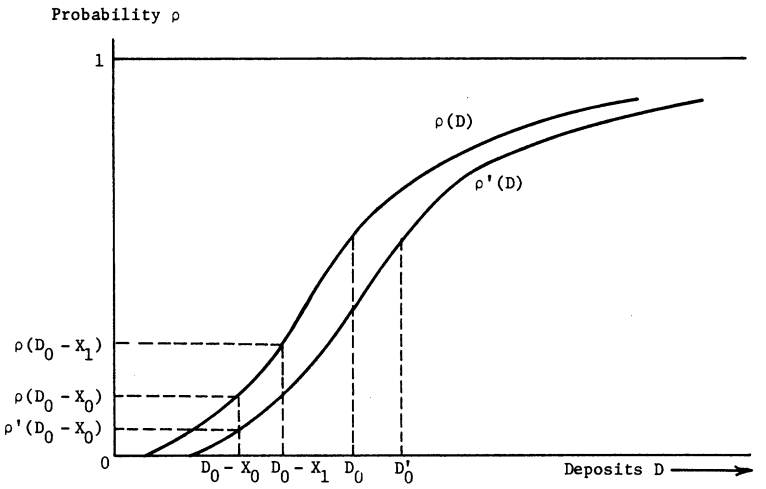

Fig. 6

sure of D_0; now various deposit levels both higher and lower are possible, and D_0 is just the mean of a probability distribution. Correspondingly R_0 is in Figure 5 the expected defensive position. Through point Q in Figure 5 a 45° line has been drawn, parallel to the line $D+E$. Given the volume of loans and investments L_0, the actual outcome for deposits and defensive position will be somewhere on this line. If more deposits come to the bank, the outcome will be on this line above and to the right of Q. If there are deposit withdrawals beyond expectation, the outcome will be to the left of and below Q. A reduction of deposits by X_0, from D_0 to D_0-X_0, would wipe out the defensive position.

The relevant probabilities of future deposits are not objective ones, if indeed these exist, but probabilities estimated by the bank in the light of its past experience. In Figure 6, curve $\varrho(D)$ represents the cumulative probability distribution of deposits. Any point on the curve is to be read as follows: the ordinate gives the probability that the future deposits will not exceed the abscissa; for example, the probability is $\varrho(D_0-X_0)$ that deposits will not be greater than D_0-X_0. As in Figure 5, D_0-X_0 represents the deposit level at which defensive position is zero; therefore $\varrho(D_0-X_0$ is the probability that the bank will have to borrow or liquidate investments. Suppose there is an increase in expected deposits, say from D_0 to D_0' in Figures 5 and 6 because the bank has become through external circumstances or through its own competitive measures a more attractive depository. This is assumed to shift the entire cumulative distribution to the right, from $\varrho(D)$ to $\varrho'(D)$ in Figure 6. The expected defensive position is increased from R_0 to R_0' in Figure 5. The deposit level at which the defensive position

is zero remains the same, D_0-X_0, but the probability of encountering a negative defensive position is reduced to $\varrho'(D_0-X_0$.

It is assumed that the probability distribution shifts in a special way when expected deposits change, namely that the probability that actual deposits will not exceed any given fraction or multiple of expected deposits remains unchanged. Thus, in the example above, $\varrho(xD_0)=\varrho'(xD_0')$ whether x is 1/10 or 1/2 or 1 or 3 or any other positive number.

Given the original probability distribution of deposits, $\varrho(D)$, an increase in loans and investments will lower the expected defensive position, reduce the safety margin it provides against losses of deposits, and increase the probability of a negative defensive position. Consider, for example, an increase in loans and investments from L_0 to L_1 in Figure 5. This lowers the expected defensive position from R_0 to R_1, reduces the safety margin from X_0 to X_1, and increases the probability of a negative defensive position to $\varrho(D_0-X_1)$.

Every increase in loans thus increases the probability that the bank will be subject to the special costs of meeting a negative defensive position. The opportunity costs of making loans and investments must be reckoned as the *expected* costs of providing the funds, taking into account the reduction in expected defensive position and safety margin. Therefore the expected special costs must be included. They contribute to marginal as well as to total expected costs.

IV.3. *Effects of Uncertainty*

The nature of the change in opportunity costs introduced by uncertainty about deposits is indicated in Figures 7, which should be compared to Figures 4. In Figure 4*a* L_c (equal to $D_0(1-k)+E$) is shown as the volume of loans and investments corresponding to a zero defensive position. At this critical volume the opportunity cost curve increases in slope from r to $r+b$. And in Figures 4*b* and 4*c* the cost curve also jumps at this point, reflecting a once-for-all cost of borrowing a. In Figures 7 the same volume of loans and investments L_0 now corresponds to an *expected* defensive position of zero. But the probability of a negative defensive position is no longer zero to the left of this point and one to its right. Rather this probability grows continuously as L increases. Consequently, the expected opportunity cost for every level of L must include an allowance for the probability that the defensive position will turn negative and impose special costs on the bank—an allowance which is greater the higher the chosen volume of loans and investments. This allowance has the effect of smoothing the cost curve C', as illustrated in Figures 7

Formally, as shown in the Appendix, marginal expected cost is

$$r+bF(X)+\frac{a}{(1-k)D_0}f(X)$$

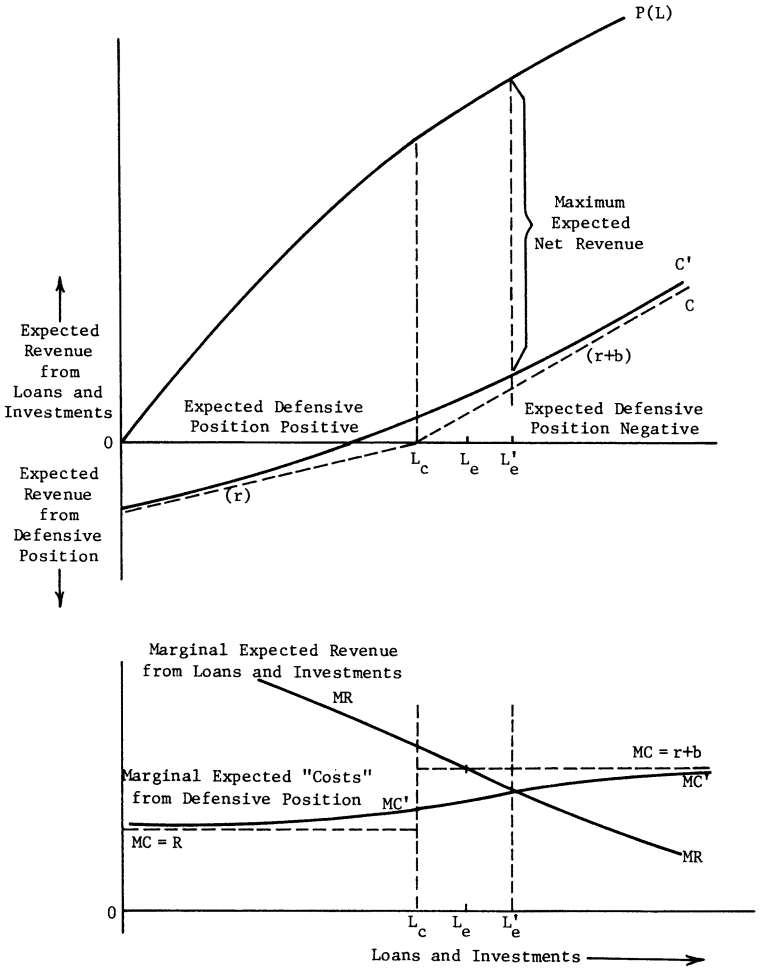

Fig. 7a

Here X is a function of L such that $D_0(1-X)$ is the level of deposits, $(L-E)/(1-k)$, at which the defensive position would be zero, $F(X)$ is the probability that deposits will not exceed this crucial level, and $f(X)=dF(X)/dX$ is the corresponding probability density. This marginal cost the bank equates to the marginal revenue of lending $P'(L)$.

How does uncertainty affect the volume of loans and investments?

In Figure 7*a*, there is no fixed cost of borrowing, no jump in the cost curve under certainty (curve C in Figure 7*a*). Uncertainty results in an expected total cost curve like C' in Figure 7*a* above C everywhere, asymptotic to C at both ends. The corresponding change in marginal cost is shown in the lower panel. The jump in marginal cost MC, at L_c, is now distributed throughout the new marginal cost curve MC'. With uncertainty

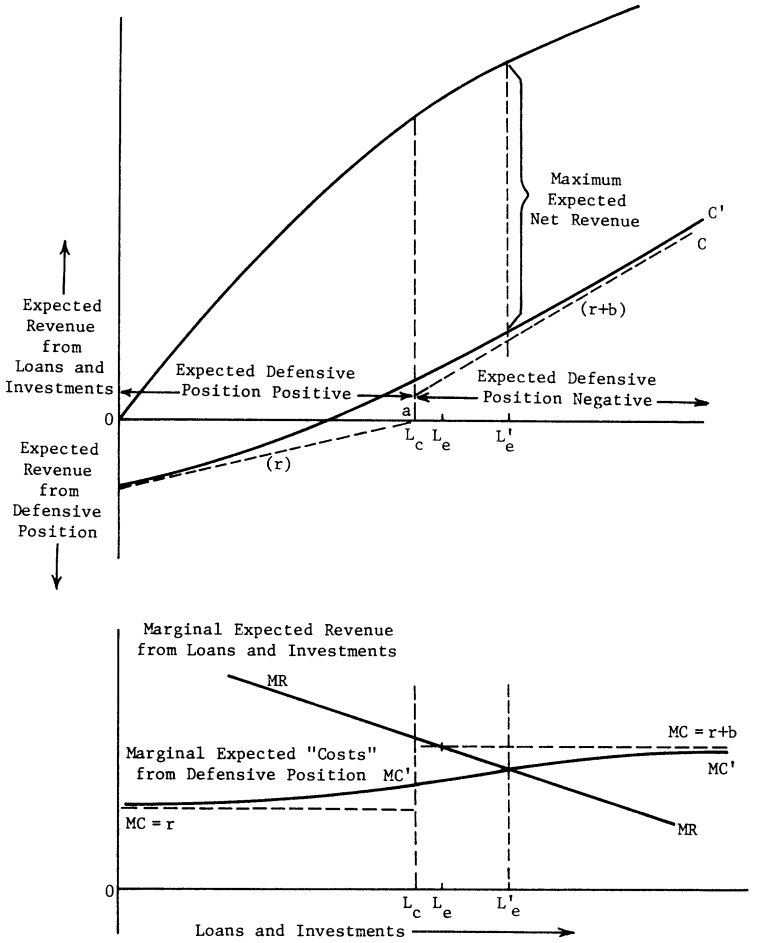

Fig. 7b

about deposits, marginal cost rises continuously, and always lies between r and $r+b$. Therefore, if the maximum-profit volume of loans and investments were previously below L_c, implying a positive defensive position, deposit uncertainty leads to a *lower* volume of lending and a *larger* expected defensive position. But if the maximum-profit volume of loans and investments under certainty were greater than L_c, implying a zero or negative defensive position, uncertainty leads to a *higher* volume of lending and a *smaller* defensive position, i.e., more expected borrowing. This second result is the one illustrated in Figure 7*a* by the shift in equilibrium lending from L_e to L'_e. The third possibility is that the equilibrium under certainty is exactly at L_c—the marginal revenue curve goes through the jump in MC at that point. In this event uncertainty may either increase or

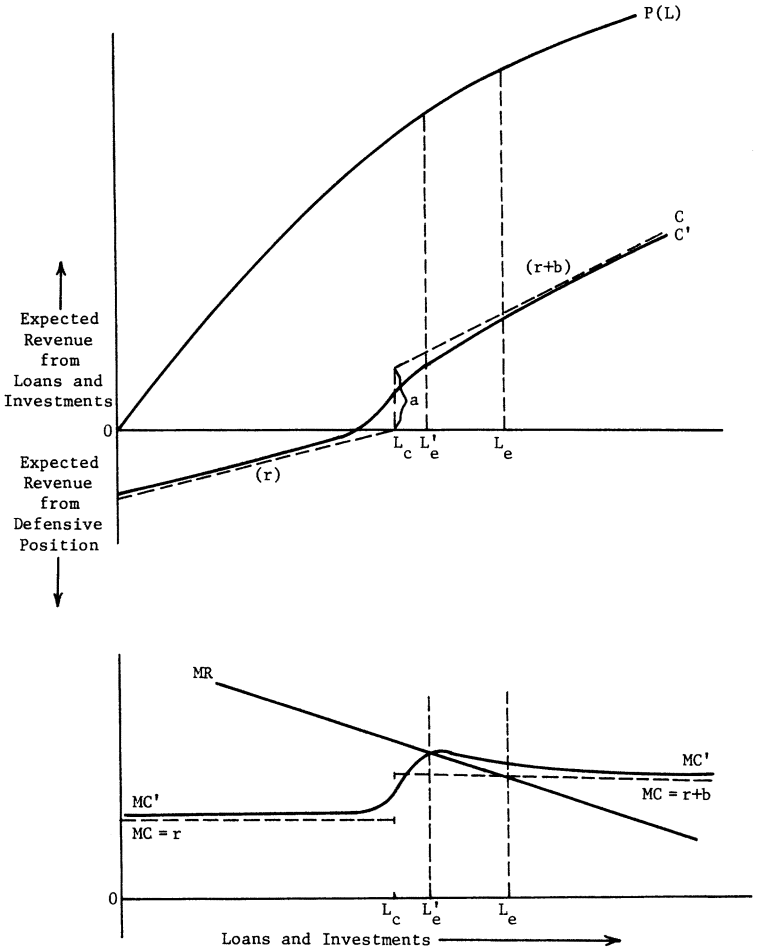

Fig. 7c

decrease lending, depending on whether marginal revenue is greater or less than the new marginal cost MC' at that point.

In Figure 7*b* there is a jump in total cost C at L_c, a once-for-all cost of borrowing. But this cost is small relative to the additional interest b incurred on a negative defensive position. The results are qualitatively the same as in the previous case. If the equilibrium under certainty were at L_c but marginal revenue $P'(L)$ exceeded $r+b$, as illustrated in Figure 4*c*, then of course the introduction of uncertainty will lead to increased lending and to a negative defensive position.

In Figure 7*c*, however, the fixed cost of borrowing, a, is large relative to the special interest charge b. As a result, the total cost curve under uncertainty, C', lies below the corresponding curve under certainty, C, for

Table 1. *Effect of uncertainty about deposits on volume of loans and investments and expected defensive position*

	Equilibrium under certainty		
	Positive defensive position	Zero defensive position	Negative defensive position
a) Fixed cost of borrowing zero or small (Figures 7*a* and 7*b*)	Lending diminished Defensive position increased	Result uncertain	Lending increased Defensive position reduced
b) Fixed cost of borrowing large (Figure 7*c*)	Lending diminished; defensive position increased	Result uncertain	Result uncertain

values of L above L_c, i.e., for negative expected defensive positions. Also, the marginal cost curve MC' eventually rises above $(r+b)$, and then approaches $r+b$, the marginal cost under conditions of certainty. The new marginal cost MC' is smooth, but it is not continuously rising. If the defensive position would be positive under certainty, then it will be increased—and lending curtailed—under uncertainty. This is the same conclusion as in the two previous cases. For negative defensive positions, the situation is more complicated than in the previous two cases. Even there the introduction of uncertainty *may* so increase marginal cost as to reduce lending and increase (algebraically) the defensive position. This is the possibility illustrated in Figure 7*c*.

These results may be summarized as shown in Table 1.

The possibility that uncertainty increases the bank's loans and investments seems surprising, and the cases in which this result occurs are probably not of great impirical importance. These are cases where the defensive position is negative or zero. Ordinarily banks in the United States are not in this situation. These analytical possibilities may have greater relevance in certain foreign banking systems, where banks customarily lend and invest funds borrowed from the central bank.

IV.4. *Value and Cost of Deposits*

What are the disposition and the value of an increase in expected deposits, as from D_0 to D_0' in Figures 5 and 6? A shift of this kind moves L_c to the right in Figure 7. At any given volume of loans and investments, say L_e' in the diagrams, the penalties of a negative defensive position are less probable and have less weight in the calculation of marginal cost. Therefore the

marginal cost curve is moved to the right and generally down. The result is an increase in the equilibrium volume of loans and investments. It is not true, as it was with certainty about deposits, that an influx of deposits goes entirely into defensive assets.

Correspondingly, the value in added expected earnings of an addition to expected deposits is $(1-k)$ times the marginal revenue of lending, or, what is in equilibrium the same thing, $(1-k)$ times the marginal expected revenue from defensive position. Even when the expected defensive position is positive, deposits are worth more than $r(1-k)$. An increment in expected deposits acquires additional value by lowering the probability that random deposit loss will inflict on the bank the penalty costs of a negative defensive position.

An exception may occur when the marginal cost curve has the hump shape of Figure 7*c*. Sliding this curve to the right will raise it at certain points. If the previous equilibrium had occurred on a declining stretch of the curve (not as depicted in Figure 7*c*), then the new equilibrium will involve a lower volume of loans and investments. The increase in expected deposits derives considerable value from lowering the probability of incurring the fixed penalty cost, but to exploit this improvement the bank must plan a much higher defensive position than before.

As in the case of certainty, two situations may be distinguished with respect to the supply of deposits to the individual bank:

(a) The expected volume of deposits is externally determined, beyond the bank's control. A market interest rate may have to be paid on deposits; but given the limited supply of deposits available to the bank, their value exceeds this interest rate. In these circumstances the cost of deposits affects the expected profits of the bank but not its optimal portfolio.

(b) The bank can influence its expected volume of deposits by its interest payments and other outlays. Total costs of deposits are uncertain because deposits themselves are uncertain. A simple and natural assumption is that the average cost of deposits is a function of expected deposits, generally an increasing function. Given this average cost, actually realized total costs depend on the random element in deposits. For example, the bank sets an interest rate but is uncertain what volume of deposits it will attract. In these circumstances, the bank will equate the marginal cost of expected deposits to their value. Decisions about deposit levels and about portfolio composition are intertwined. For example, if the marginal costs of attracting a given volume of expected deposits decline, the bank will accept more deposits and make more loans. But as the marginal revenue of loans declines, the bank will shift relatively more of any new funds into defensive assets.

In Figure 8 this complete equilibrium is illustrated. The horizontal axis measures either the bank's expected disposable funds $E+(1-k)D_0$ or their balance sheet equivalent $L+R$. Curve *MCD* represents the marginal cost of

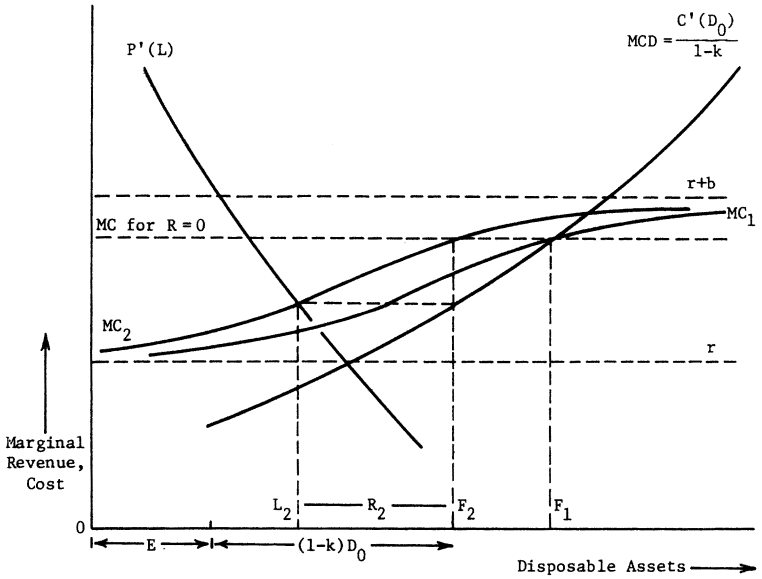

Fig. 8. Full equilibrium of bank.

the disposable funds; if $C'(D_0)$ is the marginal cost of a dollar of expected deposits, then $C'(D_0)/1-k$ is the marginal cost of a dollar available for lending or placing in defensive assets. MC represents, as in previous diagrams, the marginal opportunity cost of lending, measured by the expected return from defensive position. But in this diagram the level of expected deposits is not fixed. Instead the MC curve slides laterally, with the point of zero defensive position L_c always corresponding to the abscissa $E+D_0(1-k)$. Thus if disposable funds are F_1 the proper MC curve is MC_1, whereas if disposable funds are F_2, the applicable curve is MC_2. Now F_1, with zero defensive position, is clearly not an equilibrium. Suppose lending is set equal to F_1 so that defensive position is zero. Although the value of defensive assets equals the cost of deposits, the marginal revenue of lending falls short of both of them. There is too much lending; and in order to cut it back deposits should be lowered and defensive position increased. This leads towards F_2, which is an equilibrium, with loans of L_2 and defensive position of R_2.

V. The Bank's Response to External Changes

V.1. *Exogenous Changes in Expected Deposits*

What is the bank's response to an exogenous decrease in expected deposits D_0? For a reference point, consider the following first approximation: The bank places all of its capital funds in loans and investments, meets its

Table 2*a*

Assets		Liabilities	
Expected required reserves	10	Expected deposits	100
Loans, etc.	64	Equity	10
Expected defensive position	36		

Table 2*b*

Assets		Liabilities	
Required reserves	6	Deposits	60
Loans, etc.	64	Equity	10
Defensive position	0		

expected reserve requirements, and divides the remainder of its expected deposits in constant proportions between loans and expected defensive position. A bank following this policy would respond to a 10% cut in expected deposits by a 10% cut in $L-E$, the amount of deposits placed in loans. The logic of the policy is that the bank is still protected against the same percentage downward deviation of deposits from expectation—the probability of a negative defensive position remains the same. This may be illustrated by a numerical example, showing expected and contingent balance sheets before and after the change in expected deposits. Here the required reserve ratio is assumed to be 10% (Table 2*a*). Committed to loans of 64, the bank could lose 40% of its expected deposits before running out of defensive position (Table 2*b*). Now the bank's expected deposits are cut by 10%, and the first approximation policy indicates a proportionate cut in $L-E$, originally 54, and in expected defensive position, originally 36 (Table 3*a*). The bank is still protected against a 40% "loss" of deposits, i.e., its defensive position will be positive unless deposits fall as low as 54 (Table 3*b*).

What might cause the bank to deviate from this policy? For one thing, the marginal revenue of lending $P'(L)$ might rise as L is reduced. By itself, this would lead the bank to a smaller curtailment of lending than the first approximation strategy suggests.

On the other hand, the marginal value of defensive assets may also rise. This marginal value depends on r, b, and a, all of which are constant. It depends also on X, the crucial percentage downward deviation of deposits at which the defensive position would be negative. This is also constant under the assumed policy. But it depends further on the level of expected

Table 3 *a*

Assets		Liabilities	
Expected required reserves	9	Expected deposits	90
Loans, etc.	58.6	Equity	10
Expected defensive position	32.4		

Table 3 *b*

Assets		Liabilities	
Required reserves	5.4	Deposits	54
Loans, etc.	58.6	Equity	10
Defensive position	0		

deposits D_0 itself, provided the penalty a is positive and constant. The reason is that each dollar increase in expected defensive position lowers the critial deposit level the same dollar amount whether expected deposits are high or low. (In the tabular example above, reducing loans by 9 and correspondingly increasing the expected defensive position lowers the critical deposit level by 10, to 50 and 44 respectively, whether expected deposits are 100 or 90.) But the same dollar amount is a larger percentage when deposits are low, and therefore a greater reduction in the probability of trouble. (In the example, it was equally likely that with expected deposits of 100 actual deposits would be as low as 60 and that with expected deposits of 90 actual deposits would be as low as 54. But it is less likely that 90 will turn out to be 44 or lower than that 100 will turn out to be 50 or lower.) The presence of a fixed penalty leads to a kind of an economy of scale of deposits: the marginal opportunity cost of lending is higher when expected deposits are lower.

The first approximation policy would be optimal if the marginal revenue of lending $P'(L)$ were constant and if there were no fixed penalty ($a=0$). Otherwise, following this policy in the face of a decline in expected deposits will raise both the marginal revenue and the marginal cost of lending. The optimal cut in lending may be either smaller or greater than indicated by a *pro rata* reduction in $L-E$.

When deposits are endogenously responsive to the bank's own payments to attract them, any rise in the marginal value of assets will lead the bank to "buy" more deposits. The bank will not fully acquiesce in an unfavorable shift in expected deposits but will partially offset it by incurring greater average and marginal costs to attract deposits.

V.2. *Other Changes in Available Funds*

If additional funds become available as equity or time deposits, they can be placed in loans and investments without altering the bank's margin of safety. The first approximation, therefore, is that all of a dollar of additional equity, and $\$(1-l)$ of an additional dollar of time deposits, will go into loans. This rule leaves the size and marginal value of defensive position unchanged. But unless the bank is operating in a purely competitive loan market, the marginal revenue of lending will decline. Thus the first approximation overstates the loan response. To keep its alternative investments equally valuable on the margin, the bank will have to use some of its additional funds to improve its expected defensive position.

V.3. *The Yield of Defensive Assets*

With given expected deposits, (Figure 7) an increase in r will raise the marginal opportunity cost of landing. The bank will substitute defensive assets for loans and investments. When expected deposits and the costs of attracting them are endogenous (Figure 8) along a given supply curve of deposits to the bank, an increase in r will also induce the bank to seek and accept more deposits, at higher marginal cost. Defensive position gains from new deposits as well as from curtailment of loans. Reductions in r have the opposite effects.

One source of variation in r is monetary policy, and indeed it is largely through changes in interest rates on defensive assets in the "money market" that the individual bank feels the impact of monetary policy.

V.4. *Penalties for Negative Defensive Position*

An increase in the variable penalty b will raise both the level and the slope of the marginal cost curve, with consequences similar to those of an increase in r as just discussed. The same is true of an increase in the fixed penalty a—except that for high values of L relative to D_0 this lowers the slope of the MC curve.

For the individual bank increases in penalties may arise from a number of sources: greater expectation of risk of a future rise in interest rates, meaning greater losses if investments must be liquidated to cover a negative defensive position; greater interest charges and transactions costs in arranging loans from other banks or the central bank; greater estimate of damage to future credit-worthiness involved in near-term borrowing.

V.5. *Required Reserve Ratio*

One very direct way in which the monetary authorities affect the bank is by setting the legal reserve ratio k. With given expected deposits a rise in k means that the bank must provide for higher expected required reserves, by

Table 4*a*

Assets		Liabilities	
Expected required reserves	20	Expected deposits	100
Loans, etc.	58	Equity	10
Expected defensive position	32		

Table 4*b*

Assets		Liabilities	
Required reserves	12	Deposits	60
Loans, etc.	58	Equity	10
Defensive position	0		

curtailing either loans or expected defensive position or, most likely, both. How much will loans be curtailed?

Consider, to begin with, the same first approximation used in Section V.1 in connection with deposit inflows. This suggests the following reaction to a change in reserve requirement: the bank adjusts its loans so as to maintain unchanged the probability of a negative defensive position. This means that $(L-E)/(1-k)$ remains constant; the elasticity of $L-E$ with respect to $1-k$ is one. For example, suppose that the bank had chosen the balance sheet shown in Table 2*a* above when the required reserve ratio was 10%. The bank is protected for deposits as low as 60 as shown in Table 2*b*. Now let the required reserve ratio be raised to 20%. The following plan will maintain the same protection as before (Table 4*a*). Once again the bank is protected for deposits as low as 60 (Table 4*b*). The bank lends out its capital (10) and beyond that maintains the same relative dispositions of its disposable deposits. In Table 2*a* the expected disposable deposits of 90 are divided 3/5 loans (54), 2/5 defensive position (36). In Table 4*a* the expected disposable deposits, now 80, are divided in the same proportions: 3/5 loans (48), 2/5 defensive position (32).

This approximation would be exact if it maintained equality of marginal revenue from lending and expected marginal revenue from defensive position. But both of the elements of this equality will, in general, be changed. Reduction in the volume of loans L will raise its marginal revenue $P'(L)$ unless the bank is operating in a purely competitive loan market. The second element, the marginal revenue from defensive position, is equal to

$$r+bF(X)+\frac{a}{(1-k)D_0}f(X), \quad \text{where } X=\frac{L-E}{(1-k)D_0}-1$$

is the percentage deviation from expected deposits at which defensive position would be zero (-40 in the tabular illustration above), $F(X)$ is the probability of deviations as bad as that or worse, and $f(X)$ is $dF(X)/(dX)$, the corresponding probability density function at X.

The approximation keeps X constant, and r, b, a, and D_0 are all constant. But unless a is zero, the rise in k increases the third term of this marginal opportunity cost of lending. When jumping over the line to a negative defensive position entails a fixed cost, the fact that a dollar increase in loans and investments increases the probability of crossing the line must be charged against it. Furthermore, the amount by which an additional dollar of lending raises this probability is greater when the reserve requirement is higher. Each dollar increase in L raises the critical level of deposits by $1/(1-k)$, e.g. by 10/8 when the required reserve ratio is 20% compared to 10/9 when it is 10%. With both the marginal revenue and marginal cost of lending higher, it is not possible to say in general in which direction the first approximation errs. The first approximation—the unitary elasticity of $L-E$ with respect to $1-k$—is exact if marginal revenue from lending $P'(L)$ is constant and if the fixed penalty a is zero.

The foregoing analysis of changes in reserve requirement refers to the case of exogenous deposits. What of the other case, where the bank can determine its own volume of expected deposits? Clearly an increase in k raises the cost of obtaining, via additional deposits, funds to place in loans or in defensive assets. The bank will seek a smaller volume of expected deposits, and therefore curtail its loans even more than in the first case.

VI. Retention of Deposits

The preceding analysis has assumed that the bank's volume of deposits is independent of the size of its loans and investments. Deposits may depend on the interest rates and services the bank offers depositors and on its advertising and public relations expenditures. But so far they do not depend on the bank's portfolio choice. For reasons outlined in Section II above this is an extreme assumption, and it is time to examine the implications of its removal.

Suppose instead that, given the attractions the bank offers potential depositors, the volume of its deposits is the sum of an autonomous element D_0 and an induced element $\varphi(L)$ which depends on its loans and investments. With zero loans and investments deposits are just D_0; that is, $\varphi(0)=0$. The function $\varphi(L)$ may be called the bank's deposit retention function, and its slope $\varphi'(L)$, which lies between zero and one, its marginal retention ratio.

Note that the bank is assumed to be unable to affect its deposits by acquiring defensive assets; its marginal retention ratio from this part of its

portfolio is zero. Indeed, it is only because its deposits depend on how the bank divides its portfolio among alternative assets that deposit retention may become a relevant factor in its portfolio choices. The fact that loans are more likely to be "redeposited" than other bank asset purchases may dispose the bank to favor loans.

The advantage which deposit retention gives to loans may be illustrated most clearly in the first deterministic model, where deposits are exogenous, costless, and foreseen with certainty. An expansion of \$1 of loans and investments now increases deposits by $\$\varphi'(L)$ and lowers the defensive position not by \$1 but only by $\$(1-\varphi')$. Required reserves increase by $\$k\varphi'$. Defensive position is reduced by $\$(1-\varphi'+k\varphi')$. Thus the marginal opportunity cost of lending \$1 is $r[1-\varphi'\cdot(1-k)]$ instead of r for defensive position positive, or $(r+b)[1-\varphi'\cdot(1-k)]$ instead of $r+b$ for defensive position negative. This marginal cost is lower, the higher the marginal retention ratio. Therefore, the volume of loans and deposits will be higher, and the defensive position lower, the higher the retention ratio.

The previous diagrammatic analysis can be easily reinterpreted. Take D_0—the wholly autonomous volume deposits assumed in Figures 2, 3, and 4—to be the volume of deposits the bank can have without making any loans at all. Then the effect of deposit retention is to rotate downward the cost curve of Figures 3 and 4, pivoted on its intersection with the vertical axis, and to give it a lower slope, still positive but not necessarily constant. With this amendment, the previous analysis and conclusions still hold. Note, however, that the size of the required reserve ratio k now affects the marginal opportunity cost of lending, which is higher the larger the fraction of induced deposits that must be placed in interest-barren required reserves.

However, the advantages of deposit retention are partially or even wholly lost if the "retained" deposits impose interest or other costs on the bank. For example, if an interest rate $C'(D)$ is paid to depositors the marginal cost of lending is raised by $\varphi'(L)C'(D)$. If the bank is in complete equilibrium, and if "retained" deposits cost as much as any other deposits, their marginal cost is equal to their value in any investment use. Hence $C'(D)$ is equal to $r(1-k)$—or $(r+b)(1-k)$—and the retained deposit has no net value. The marginal cost of lending reduces once more to r, or $r+b$, and portfolio choice is independent of deposit retention.

Deposit retention entails similar modification and reinterpretation of the analysis of the effects of uncertainty about future deposits. But the conclusions, including the variety of possible cases, remain qualitatively the same.

VII. Risk Neutrality or Risk Aversion?

The analysis above assumes that the bank is risk-neutral. The firm maximizes expected profits, and this also maximizes its expected utility of

profits. Of course the hazards of deposit fluctuation affect the bank's decision. But they do so through their calculable effect on expected costs rather than through any assumed disutility of variance of profits. To some degree the two approaches are interchangeable. The fixed cost a of a negative defensive position has been treated above as a definite pecuniary cost. It might alternatively be regarded as a psychological cost, i.e., an approximate pecuniary equivalent of the disutility of adverse fluctuations of deposits. In any case, superimposing risk aversion on the model as presented would tilt the bank's decision further toward a conservative portfolio policy, reducing illiquid loans and investments relative to defensive assets.

Risk-neutrality seems the appropriate assumption for the firm. A bank is managed by specialists engaged in taking a long sequence of risks of deposit fluctuation and can expect bad luck and good luck to "average out." That is, the long-run variance of the profits associated with any given policy is much smaller than the short-run variance. It is true that the firm might not survive a sequence of heavy losses. In the model above this danger enters through the size of the parameters a and b; if they are large enough relative to loan and deposit opportunities the firm will follow a cautious policy, perhaps accepting no illiquidity risks at all.

The variance of profits of a risk-neutral bank will be taken into account by the shareowners, who may be risk-averse individuals. In a system without deposit insurance, the risks of illiquidity taken by a bank would also be considered by depositors gauging the chances that the bank will be unable to meet its liabilities. The portfolio choices of the bank would then affect its supply of deposits in a manner that works in a direction opposite to the loan retention mechanism discussed above: a more conservative balance sheet would attract more deposits. Deposit insurance has largely eliminated this consideration. It has not done so entirely, because it covers only the first $100,000 of a deposit and because there may be delays in receiving payment. Moreover, the surveillance formerly exercised by depositors is in some degree replaced by the surveillance of the insurer.

VIII. Concluding Remarks

The simple model of a commercial banking firm presented in this paper examines the choice between reserves and other defensive assets, on the one hand, and less liquid loans and investments, on the other. The bank must make this choice before it knows what its volume of deposits will be. A drain of deposits to other banks, e.g. an adverse balance in interbank check clearing, has to be met by disposal of assets of one kind or the other or by borrowing. The same problem arises for banks in the aggregate if the public withdraws deposits in order to hold currency.

Meeting a drain by selling loans and investments or by borrowing is

typically more expensive than drawing on reserves or selling liquid defensive assets. The additional expense of a negative defensive position is modeled to consist of a fixed cost plus a penalty proportional to the size of the shortfall. The bank weighs the probability of incurring such expense against the probability of foregoing profit by holding less remunerative defensive assets in redundant amounts if deposit experience turns out to be favorable.

The behavior of the bank in this model is a prime example of what Keynes called the *precautionary motive* for holding money and other liquid assets. Other economic agents face similar problems and exhibit similar behavior. Thus the model, in spirit though not in institutional detail, applies generally to households, businesses, financial institutions, and other agents who make illiquid commitments in the face of uncertainties about their future streams of receipts.

In its simplicity the model ignores several important aspects of the decision problems of real-world banks: (1) It is a static model and does not deal with the intertemporal structure of bank liabilities and assets. Their maturities, for example, will affect the bank's defensive positions for many—in principle for all—future dates. (2) Assets, liabilities, and borrowing capabilities are not so neatly divisible into two categories but constitute a spectral menu of liquidity characteristics. Consequently the model's separation of the bank's decision on defensive position versus loans and investments from portfolio choices within those categories is somewhat artificial. (3) The model does not explicitly handle uncertainties other than those connected with deposits. These include: calls on lines of credit committed to long-term customers who use the bank both as a depository and as a contingent source of credit, fluctuations in interest rates and prices of marketable securities, and possible defaults on commercial loans.

Nevertheless, I believe, the model captures in essence the distinctive features of bank portfolio choice most relevant to banks' crucial role in the monetary system. It provides therefore a *micro*-economic foundation for analysis of the *macro*-economic effects of monetary policy instruments under various institutional and regulatory regimes. Among the policy instruments and structural variations of interest are the procedures and targets of central bank open market operations, the nature and level of reserve requirements, the payment or non-payment of interest on reserve assets, the costs and terms of central bank lending to banks, the existence and levels of legal ceilings on deposit interest rates, and the number and competitive structure of firms in the banking industry. A tractable model of the banking firm is a prerequisite to economy-wide analysis of financial and monetary systems that rely heavily on intermediation and "inside" money.

Appendix

A.1. *Certainty about Deposits*

A.1.1. *Deposits Exogenous and Costless.* In the first model deposits are assumed to be known with certainty, but possibly to depend on the loans and investments of the bank. They are assumed "exogenous" in the sense that the bank cannot influence the quantity of deposits by varying its outlays for deposit interest, depositor services, or advertising.

The balance sheet identity for the bank is:

$$(D_0+\varphi(L))(1-k)+E-R-L=0 \tag{1}$$

The meanings of the symbols are as follows:

$D_0+\varphi(L)$	deposits
k	the required reserve ratio
E	shareholders' equity
R	defensive position
L	loans and investments
$\varphi(L)$	the deposit retention function: $\varphi(0)=0$, $0\leqslant\varphi'(L)<1$

The profits per year of the bank are:

$$\Pi=P(L)+Y(R) \tag{2}$$

where $P(L)$ is the net revenue from loans and investments and $Y(R)$ the net revenue from the defensive position. Marginal revenue from lending $P'(L)$ is assumed positive but declining with L.

$$Y(R)=\begin{cases} rR & R\geqslant 0 \\ (r+b)R-a & R<0 \end{cases} \tag{3}$$

Here r is the interest rate earned on a positive defensive position; $r+b$, with $b\geqslant 0$, the interest rate paid on a negative defensive position; and $a\geqslant 0$ the fixed penalty cost per annum of a negative defensive position.

It is assumed, in conformity with U.S. institutions, that no interest is earned on required reserves.

Using (1) and (3), (2) can be rewritten:

$$\begin{cases} \Pi_1(L)=P(L)+r\{(D_0+\varphi(L))(1-k)+E-L\} & (R\geqslant 0) \\ \Pi_2(L)=P(L)+(r+b)\{(D_0+\varphi(L))(1-k)+E-L\} & (R<0) \end{cases} \tag{4}$$

The problem is to maximize (4) with respect to L.

Let L_c be the value of L such that $R=(D_0+\varphi(L))(1-k)+E-L=0$.

Let L^* be the value of L, if one exists, such that

$$P'(L^*) = -Y'(R)\frac{\partial R}{\partial L} = Y'(R)(1-\varphi'(L^*)(1-k))$$

$$= \begin{cases} r(1-\varphi'(L^*)(1-k)) & (L^* \leqslant L_c) \\ (r+b)(1-\varphi'(L^*)(1-k)) & (L^* > L_c) \end{cases} \tag{5}$$

There are several cases:

(a) $L^* \leqslant L_c$; L^* is the solution.

(b) $L^* > L_c$ and $\Pi_2(L^*) > \Pi_1(L_c)$; L^* is the solution.

(c) $L^* > L_c$ but $\Pi_2(L^*) < \Pi_1(L_c)$; L_c is the solution.

(bc) $L^* > L_c$ but $\Pi_2(L^*) = \Pi_1(L_c)$; both L_c and L^* are solutions.

(d) $r(1-\varphi'(L_c(1-k)) < P'(L_c) \leqslant (r+b)(1-\varphi'(L_c)(1-k))$; L_c is the solution and profits are $\Pi_1(L_c)$.

When the maximizing value of L is L^*, it is not altered by exogenous changes in E or D_0. The marginal values of such changes reflect simply their investment in defensive assets. $\partial\Pi/\partial D_0 = (1-k)\,\partial\Pi/\partial E$ because k of every incremental deposit must be placed in interest-barren reserves.

$$\frac{\partial \Pi}{\partial E} = \begin{cases} r & (L^* \leqslant L_c) \\ r+b & (L^* > L_c) \end{cases} \tag{6}$$

When the maximizing value of L is L_c, then

$$\frac{\partial \Pi}{\partial E} = \frac{\partial \Pi_1}{\partial L_c} \cdot \frac{\partial L_c}{\partial E} = \frac{P'(L_c)}{1-\varphi'(L_c)(1-k)} \tag{7}$$

Again $\partial\Pi/\partial D_0$ is $(1-k)$ times this quantity.

A.1.2. *Deposits Exogenous at a Given Cost.* A variant on this model would require the bank to pay a fixed interest rate on deposits, though at this rate it could not obtain more deposits than $D_o + \varphi(L)$. Assume this rate d to be smaller than $r(1-k)$. Then (5) must be revised to reflect the smaller value of retained deposits:

$$P'(L^*) = Y'(R) - \varphi'(L^*)(Y'(R)(1-k+) - d) \tag{5'}$$

If d exceeds $r(1-k)$ but does not exceed $(r+b)(1-k)$ it is clearly not to the bank's interest to accept deposits and invest them in defensive assets. Instead the bank will be fully loaned up, with $R=0$, and with $D \leqslant D_o + \varphi(L)$. The volume of loans and investments will be such that

$$P'(L^*)(1-k) = d \qquad \left(D = \frac{L^*-E}{1-k} \leqslant D_0 + \varphi(L^*)\right) \tag{5''}$$

If $P'(L_c)(1-k)>d$, the bank may find it profitable to accept all the deposits available at d and obtain additional lendable funds by borrowing at rate $(r+b)$. Then equation (5′) applies once more.

If d exceeds $(r+b)(1-k)$, the "bank" will accept no deposits and simply borrow at rate $(r+b)$ to finance any lending beyond its capital:

$$P'(L^*)=r+b \qquad (R=L^*-E) \qquad (5'')$$

A model in which the several margins are met simultaneously is given in the next section.

A.1.3. *Deposits Endogenous*. The second model assumes that the bank can choose its volume of deposits by incurring a cost $C(D)$ of a total volume of deposits $D=D_0+\varphi(L)$. The bank may now be regarded as having two decision variables D_0 and L. Then (4) becomes

$$\begin{cases} \Pi_1(L, D_0)=P(L)+rR-C(D) & (R\geqslant 0) \\ \Pi_2(L, D_0)=P(L)+(r+b)R-C(D)-a & (R<0) \end{cases} \qquad (8)$$

At $R=0$, Π_1 can be written solely as a function of L, namely:

$$\Pi_1=P(L)-C\left(\frac{L-E}{1-k}\right) \qquad (R=0) \qquad (9)$$

Given $R=0$, the maximizing L, call it L_c, is such that:

$$(1-k)P'(L_c=C'\left(\frac{L_c-E}{1-k}\right) \qquad (10)$$

As before, L^*, if it exists, will be a solution to one of the following two pairs of equations:

$$\begin{cases} \dfrac{\partial \Pi_1}{\partial L}=P'(L)+\{r(1-k)-C'(D)\}\varphi'(L)-r=0 \\ \dfrac{\partial \Pi_1}{\partial D_0}=r(1-k)-C'(D_0+\varphi(L))=0 \end{cases} \qquad (L\leqslant L_c) \qquad (11)$$

$$\begin{cases} \dfrac{\partial \Pi_2}{\partial L}=P'(L)+\{(r+b)(1-k)-C'(D)\}\,\varphi'(L)-(r+b)=0 \\ \dfrac{\partial \Pi_2}{\partial D_0}=(r+b)(1-k)-C'(D_0+\varphi(L))=0 \end{cases} \qquad (L>L_c) \qquad (12)$$

These may be rewritten as follows:

$$\begin{cases} P'(L^*)=r \\ e(1-k)=C'(D_0^*+\varphi(L^*)) \end{cases} \qquad (L^*<L_c \qquad (11')$$

$$\begin{cases} P'(L^*) = r+b \\ (r+b)(1-\mathrm{k}) = C'(D_0^*+\varphi(L^*)) \end{cases} \qquad (L^* > L_c) \qquad (12')$$

There are several cases, as above:

(a) $L^* \leqslant L_c$; L^*, D_0^* is the solution.

(b) $L^* > L_c$ and $\Pi_2(L^*, D_0^* > \Pi_1(L_c)$; L^*, D_0^* is the solution.

(c) $L^* > L_c$ but $\Pi_2(L^*, D_0^*) < \Pi_1(L_c)$; L_c is the solution.

(bc) $L^* > L_c$ but $\Pi_2(L^*, D_0^*) = \Pi_1(L_c)$; both L_c and (L^*, D_0^*) are solutions.

(d) $r < P'(L_c) < r+b$; L_c is the solution and profits are $\Pi_1(L_c)$.

Note that when deposits are endogenous the degree of loan retention is irrelevant. On the margin a deposit costs as much as it earns. Hence the bank does not gain by having part of a loan end up as a deposit for which it must pay. (This would not be true if the marginal cost of a deposit obtained from loan retention were lower than the marginal cost of other deposits.)

In general,

$$P'(L^*) = Y'(R)\left(1-\varphi'(L^*)\frac{\partial \Pi}{\partial D_0}\right) \qquad (13)$$

In the first model $\partial\Pi/\partial D_0$ is $Y'(R)(1-k)$ or $Y'(R)(1-k)-d$ and this leads to (5) or (5′). In the second model $\partial\Pi/\partial D_0$ is zero, and this leads to (11′), (12′).

A.2. *Uncertainty about Deposits*

A.2.1. *Deposits Exogenous but Random.* Now assume that deposits are $D_0(1+x)+\varphi(L)$ where $x \geqslant -1$ is a random variable with mean zero. Let $F(X)$ be the probability that $x \leqslant X$, and let $f(x)$ be the corresponding probability density function. The value of L is set prior to the deposit outcome. Therefore, defensive position R is also a random variable, equal to $(D_0(1+x)+\varphi(L))(1-k)+E-L$. For given L the critical value X at which R is zero is given by:

$$XD_0 = \frac{L-E}{(1-k)} - \Sigma(D) = -\frac{\Sigma(R)}{(1-k)} \qquad (14)$$

where $\Sigma(R)$ is expected reserves and $\Sigma(D) = D_0+\varphi(L)$ expected deposits. Every dollar increase in lending raises XD_0 by $1/(1-k)-\varphi'(L)$.

Profits are $\Pi(L, x) = P(L)+Y(R)$:

$$\begin{cases} \Pi_1(L, x) = P(L)+rR & (x \geqslant X) \\ \Pi_2(L, x) = P(L)+(r+b)R-a & (x < X) \end{cases} \qquad (15)$$

Therefore:

$$\Sigma(\Pi) = P(L) - r(1-k)XD_0 - b(1-k)XD_0F(X) + (1-k)bD_0\int_0^X xf(x)\,dx - aF(X) \tag{16}$$

$$\frac{\partial\Sigma(\Pi)}{\partial L} = P'(L) - r[1-(1-k)\varphi'(L)] - b[1-(1-k)\varphi'(L)]F(X) + \frac{\partial X}{\partial L}[-b(1-k)XD_0f(X) + b(1-k)XD_0f(X) - af(X)] \tag{17}$$

Maximum expected profits are obtained when

$$P'(L) = [1-(1-k)\varphi'(L)]\left[r + bF(X) + \frac{a}{(1-k)D_0}f(X)\right] \tag{18}$$

The term on the right is the marginal expected revenue from defensive position, the marginal opportunity cost of making loans. Since both $F(X)$ and $f(X)$ will generally be rising in the relevant range, marginal cost is increasing. But for high L and X, $f(X)$ declines with X, and from this arises the possibility that marginal cost declines in some range. Assuming that $f(X)$ goes to zero at both extremes of X, marginal cost approaches r as L goes to zero, and approaches $r+b$, from either above or below, as L and X become indefinitely large.

The value of additional equity E is

$$\frac{\partial\Sigma(\Pi)}{\partial E} = P'(L)\frac{\partial L}{\partial E} + \frac{\partial\Sigma Y(R)}{\partial L}\frac{\partial L}{\partial E} + \frac{\partial\Sigma Y(R)}{\partial E} \tag{19}$$

From (18) we know that the first two terms add to zero. To evaluate the third term, differentiate $\Sigma(\Pi)-P(L)$ in (16) with respect to E, noting that

$$\frac{\partial X}{\partial E} = -\frac{1}{(1-k)D_0}.$$

$$\frac{\partial\Sigma(\Pi)}{\partial E} = r + bF(X) + \frac{a}{(1-k)D_0}f(X) \tag{20}$$

Similarly,

$$\frac{\partial\Sigma(\Pi)}{\partial D_0} = r(1-k) + b(1-k)F(X) + \frac{a}{D_0}F(X)(1+X) \tag{21}$$

A.2.2. *Deposits Endogenous*. As in the case of deposit certainty, an alternative model would permit the bank to influence its own deposits by means other than making loans. In this case interest payments or other

outlays by the bank shift the central value D_0 around which the probability distribution $f(X)$ pivots. But the total cost of deposits is, like the volume itself, a random variable. Suppose that cost of deposits satisfies:

$$C = C(D_0+\varphi(L))+\frac{C(D_0+\varphi(L))}{D_0+\varphi(L)}xD_0 \tag{22}$$

The bank, by setting L and D_0, establishes an average cost of deposits, e.g. an interest rate. The total volume of deposits bearing this established rate then depends on the random element x. Since $D_0+\varphi(L)$ and $C(D_0+\varphi(L))$ are non-stochastic, the expected cost of deposits is just $C(D_0+\varphi(L))$ and the marginal expected cost is as before $C'(D_0+\varphi(L))$. When this is taken into account the term involving $\varphi'(L)$ drops out of (18) and the conditions of maximum profit become:

$$P'(L) = r+bF(X)+\frac{a}{(1-k)D_0}f(X) = \frac{C'(D_0+\varphi(L))}{1-k} \tag{23}$$