Econ 4335: Seminar 4

Problem 1

a) Assume first complete information. Show then that the bank can extract all profits by extending loans at terms that depend on the type of the entrepreneur.

The expected profit of the good entrepreneur is:

$$\pi_{good} = p\left[G - \left(1 + r_{good}\right)I\right] - (1-p)0$$

$$\pi_{good} = p\left[G - (1 + r_{good})\right]$$

The expected profit of the bad entrepreneur is:

$$\pi_{bad} = q[B - (1 + r_{bad})I] - (1-q)0$$

$$\pi_{bad} = q[B - (1 + r_{bad})]$$

Because the bank is monopolistic, we know that the entrepreneurs only have the option of accepting the contract the bank offers, or not receiving a loan at all. The entrepreneurs will accept the contract the bank offers for any positive expected payoff and will be indifferent between accepting and not when the expected payoff is zero. The entrepreneurs will not accept the contract if their expected payoff is negative. This gives us the following participation conditions:

$$\pi_{good} = p\left[G - (1 + r_{good})\right] \geq 0$$

$$\left(1 + r_{good}\right) \leq G$$

$$\pi_{bad} = q[B - (1 + r_{bad})] \geq 0$$

$$(1 + r_{bad}) \leq B$$

The objective of the bank is to maximize profits, thus the conditions above must hold with equality. If this were not the case, the bank could increase its profits by increasing the interest rates. When the conditions hold with equality, we see that the expected profits of both the good and the bad entrepreneurs will be 0, while the monopolistic bank gets the entire surplus.

The profit of the bank is as follows:

$$\Pi = \alpha\left(p\left(1 + r_{good}\right) + (1-p)0 - (1+f)\right) + (1-\alpha)\left(q(1 + r_{bad}) + (1-q)0 - (1+f)\right)$$

$$\Pi = \alpha(pG - 1) + (1-\alpha)(qB - 1)$$

$$\Pi = \alpha(\mu - 1) + (1-\alpha)(\mu - 1)$$

$$\Pi = (\mu - 1)(\alpha + 1 - \alpha)$$

$$\Pi = (\mu - 1) > 0$$

b) Suppose next that only the entrepreneurs know their type. Illustrate how the bank's expected gross return will vary with $1 + r$, where r is a rate of interest paid on a loan, and explain how the mixture of loan applicants will change with the rate of interest.

When the bank does not know the types of the different entrepreneurs, it can no longer discriminate the interest rate between them. It must offer only one pooled interest rate. In problem a) we saw that:

$$(1 + r_{good}) = G$$

$$(1 + r_{bad}) = B$$

Because we know that $B > G$, we know that $r_{bad} > r_{good}$. When we remember that r_{good} is the interest rate that makes the good entrepreneurs indifferent between investing or not investing, we realize that if the bank sets an interest rate higher than $\hat{r} > r_{good}$, then none of the good entrepreneurs will be willing to invest.

However, as long as $\hat{r} < r_{bad}$, all the bad entrepreneurs will be willing to invest. However, the bank has no interest in offering an interest rate $r_{good} < \hat{r} < r_{bad}$ because with this intermediate interest rate, they both lose their good customers and they don't take the bad entrepreneurs' full profit. (With $\hat{r} < r_{bad}$ the bad entrepreneurs have a positive profit).

This means that the bank will only have incentive to offer either r_{good} or r_{bad}, and it will select the one that yields the highest profit to the bank.

The banks expected gross return will therefore vary as follows:

- Gross return is negative with an interest rate equal to 0 (?)
- Gross return increases linearly until the interest rate equals G
- At G the gross return drops because the good entrepreneurs drop out
- Increases linearly until the gross return equals B
- At B the gross return drops because now also the bad entrepreneurs drop out
- Gross return is 0 for all interest rates greater than B because there will be no willing borrowers.
- Bank will select an interest rate equal to B or G, so to maximize profits.

c) **What interest rate will a profit-maximizing monopolistic bank choose? What is critical to your answer?**

As noted above, the bank will either offer a pooled interest rate equal to r_{good} or r_{bad} and never an interest rate in between. The bank will offer the interest rate among the two that yields the highest return.

The profit to the bank when it offers a pooled interest rate $\hat{r} = r_{good}$

$$\Pi = \alpha\left(p\left(1 + r_{good}\right) - 1\right) + (1 - \alpha)\left(q\left(1 + r_{good}\right) - 1\right)$$

$$\Pi = \left(1 + r_{good}\right)\alpha p - \alpha + \left(1 + r_{good}\right)(1 - \alpha)q - (1 - \alpha)$$

$$\Pi = \left(1 + r_{good}\right)[\alpha p + (1 - \alpha)q] - \alpha - (1 - \alpha)$$

$$\Pi = \left(1 + r_{good}\right)[\alpha p + (1 - \alpha)q] - 1$$

$$\Pi = \left(1 + r_{good}\right)[m] - 1$$

$$\Pi = G[m] - 1$$

The profit to the bank when it offers a pooled interest rate $\hat{r} = r_{bad}$

$$\Pi = \alpha(p(1 + r_{bad}) - 1) + (1 - \alpha)(q(1 + r_{bad}) - 1)$$

$$\Pi = (1 - \alpha)(q(1 + r_{bad}) - 1)$$

The first term disappears because none of the good entrepreneurs will accept the bad interest rate as doing so would give them negative profits. The bank is thus indifferent between offering a pooled good interest rate and a pooled bad interest rate when:

$$\left(1 + r_{good}\right)[\alpha p + (1 - \alpha)q] - 1 = (1 - \alpha)(q(1 + r_{bad}) - 1)$$

$$G[\alpha p + (1 - \alpha)q] - 1 = (1 - \alpha)(qB - 1)$$

$$G[\alpha p + (1 - \alpha)q] - 1 = (1 - \alpha)qB - (1 - \alpha)$$

$$G\frac{[\alpha p + (1 - \alpha)q]}{(1 - \alpha)} - \frac{\alpha}{(1 - \alpha)} = qB$$

$$G\left[\frac{\alpha^0}{(1 - \alpha^0)}p + q\right] - \frac{\alpha^0}{(1 - \alpha^0)} = qB$$

$$\frac{\alpha^0}{(1 - \alpha^0)}[Gp - 1] + Gq = qB$$

When the relative fraction of good and bad is given by α^0 we see that the bank is indifferent between the two pooled rates. However, if $\alpha^0 < \alpha$, then the fraction of good investors is large enough so that the LHS is bigger than the RHS and therefor the bank earns a bigger profit by offering the good pooled interest rate, $\hat{r} = G$. On the other hand, if $\alpha^0 > \alpha$ then the share of bad investors is so large that the bank earns a bigger profit by offering a pooled bad interest rate $\hat{r} = B$.

A critical condition for this answer is that $qB > 1$(we assume that the good project is always creditworthy $pG > 1$). If the bad project is not creditworthy, then there will be no lending in the case where $\alpha^0 > \alpha$.

Problem 2:

1. **Provide a graphical illustration of how the payoffs to the firm may vary with R, and derive a critical value of R, denoted $\widehat{R}$, below (above) which the firm will choose the safe (risky) asset.**

The firm will select the safe project if the expected return of doing so is greater than the expected return of deviating and selecting the risky venture. The expected return of the safe project is as follows:

$$\pi_s = p(G - R) + (1 - p)0$$

$$\pi_s = p(G - R)$$

The expected net return of pursuing the risky project is:

$$\pi_r = q(B - R) + (1 - q)0$$

$$\pi_r = q(B - R)$$

This gives us the following incentive constraint:

$$\pi_s \geq \pi_r$$

$$p(G - R) \geq q(B - R)$$

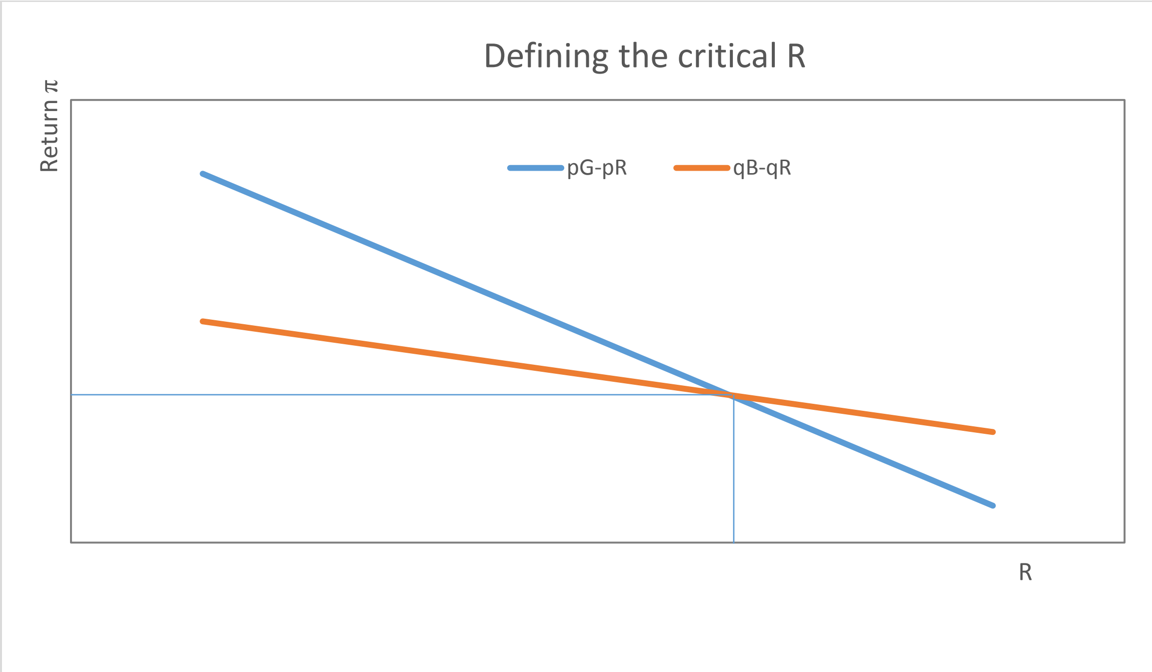
This tells us that the expected net return of investing in the safe asset must be at least as great as the expected net return from investing in the risky asset. Otherwise, the firm has incentive to deviate and invest in the risky asset. Assuming that when this holds with equality, the firms will invest in the safe asset, we can define a critical level of R that we call $\hat{R}$. $\hat{R}$ defines the highest repayment the bank can demand and at the same time ensure that the firms invest safely.

$$p(G - \hat{R}) \geq q(B - \hat{R})$$

$$pG - p\hat{R} \geq qB - q\hat{R}$$

$$\hat{R}(q - p) \geq qB - pG$$

$$\hat{R} \geq \frac{qB - pG}{(q - p)} = \frac{pG - qB}{p - q}$$



Because we know that $pG > qB$ we know that the return slope of the good firm will have a higher starting position than the return slope of the bad firm. Because $p > q$, we know that the return slope of the good firm is steeper than that of the bad firm. Thus, at some point they cross. This intersection defines the critical value of R and the return. We see that when R is above the critical value, the return to the firm is greatest when being “bad”.

2. **What conditions have to be satisfied in a (competitive) credit market equilibrium with only direct financing (no banks)?**

In a competitive credit market we know that the expected return from lending must be 0 because the competition will erode profits. The investors in this market can always get the return$1 + i$ from investing in the risk free asset. This means that the expected return of the project must be equal to $1 + i$ for the investors to be indifferent between the two. (But the profit is still 0).

The profits of the investors must be:

$$\Pi_{DI} = pR_p - 1 + i = 0$$

$$R_p = \frac{1}{p}$$

If we have that $R_p > \hat{R}$, then none of the firms will invest in the safe asset. All of the firms will invest in the risky asset. This means that in order to realize an equilibrium solution in which the firms invest in the good asset the conditions that must hold are:

$$R_p \leq \hat{R}$$

$$p(G - R_p) \geq 0$$

The first constraint is an incentive constraint that ensures that it is beneficial to invest in the safe asset rather than the risky asset. The second constraint is a participation constraint that must hold in order for the firms to be willing to invest at all.

3. **Derive the conditions for a competitive equilibrium with bank lending.**

The banking sector is competitive and therefore the profit to the bank must be 0 in equilibrium. This gives us the following condition:

$$\Pi_B = pR_B - (1 + i) - c = 0$$

$$R_B = \frac{1 + c}{p}$$

4. **For what parameter values of p will we have only direct finance, only bank lending and no lending, respectively?**

Investors are willing to lend when:

$$R_p \geq \frac{1}{p}$$

$$p \geq \frac{1}{R_p}$$

Banks are willing to lend when:

$$R_B \geq \frac{1 + c}{p}$$

$$p \geq \frac{1 + c}{R_B}$$

No one is willing to lend when:

$$p < \frac{1 + c}{R_B}$$