Seminar 5

Problem 1.

(i) Denote the projects:

Project A: the one with return G if success, zero otherwise
Project C: the one with return B if success, zero otherwise.

Then the variances of the projects are:

$$Var(A) = pG^2 - (pG)^2 = \mu G - \mu^2$$
$$Var(C) = qB^2 - (qB)^2 = \mu B - \mu^2$$

where I use the formula for variance, $Var(X) = E(X^2) - [E(X)]^2$

$Var(C) > Var(A)$, since $B > G$ by assumption. Hence, we can consider project C to be more risky than project A. (which is rather intuitive, since the probability of success is higher for project A than for project B)

(ii) The expected profits:

Project A

$$\pi^A = p[G - (1+R)I] - (1-p)K$$

Project C

$$\pi^C = q[B - (1+R)I] - (1-q)K$$

Which project is the most profitable?

$$\pi^A - \pi^C = p[G - (1+R)I] - (1-p)K - q[B - (1+R)I] + (1-q)K =$$
$$= \mu - p(1+R)I - K + pK - \mu + q(1+R)I + K - qK =$$
$$= (p-q)(K - (1+R)I) < 0$$

Hence, $\pi^A < \pi^C$, i.e. project C is the most profitable (the riskier project is more profitable).

(iii)
If an entrepreneur chooses to invest in the project, then she gets the expected profit. If not, then she gets zero. Thus the entrepreneur is indifferent between investing or not if:

$p[G - (1+R)I] - (1-p)K = 0$, for project A

and

$q[B - (1+R)I] - (1-q)K = 0$, for project C

Solving the equations above for R and defining critical values for projects A and C, R^A and R^C correspondingly, gives:

For project A:

$$p\left[G-(1+R^A)I\right]-(1-p)K=0 \Leftrightarrow pG-(1-p)K=p(1+R^A)I$$
$$\Rightarrow \frac{\mu}{pI}-\frac{(1-p)}{pI}K=1+R^A$$

$$\Rightarrow R^A=\frac{\mu}{pI}-\frac{(1-p)}{pI}K-1$$

Equivalently for project C:

$$R^C=\frac{\mu}{qI}-\frac{(1-q)}{qI}K-1$$

Thus we have:

$$R^C-R^A=\frac{\mu}{qI}-\frac{(1-q)}{qI}K-1-\frac{\mu}{pI}+\frac{(1-p)}{pI}K+1=\frac{\mu}{I}\left[\frac{p-q}{pq}\right]-\frac{K}{I}\left[\frac{p-q}{pq}\right]=$$
$$=\left[\frac{p-q}{pq}\right]\left(\frac{\mu}{I}-\frac{K}{I}\right)>0$$

So the critical rate of interest for project C is higher than for project A, i.e. $R^C>R^A$.

The equations for critical values define implicitly the demand for loan as a function of the interest rate, i.e.

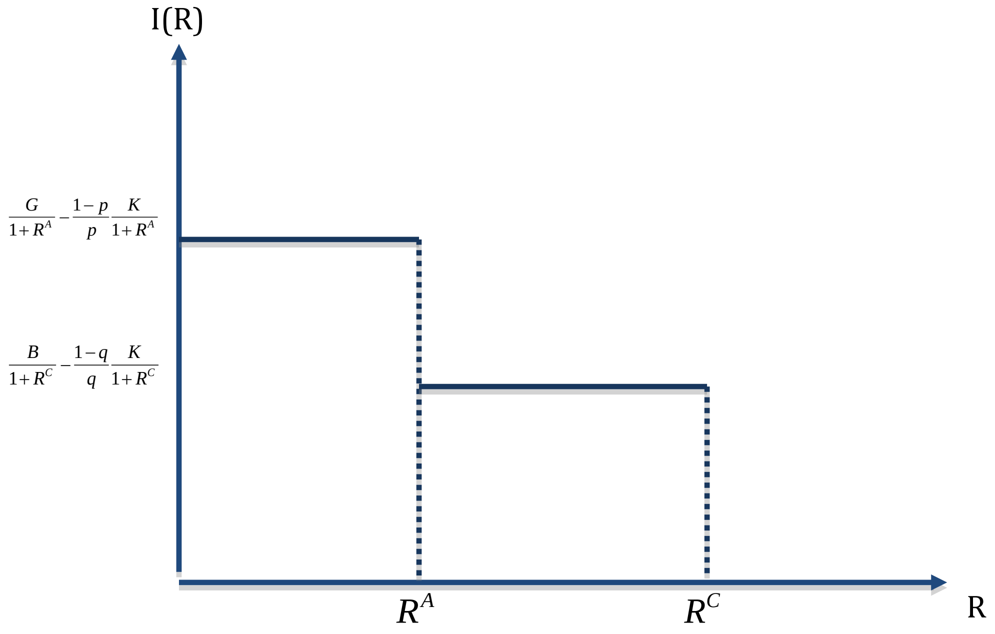
$$I^A=I(R^A)=\frac{G}{1+R^A}-\frac{1-p}{p}\frac{K}{1+R^A}$$
$$I^C=I(R^C)=\frac{B}{1+R^C}-\frac{1-q}{q}\frac{K}{1+R^C}$$

How will demand for loans vary with R?

If the bank sets the interest rate:

- $R\in\left[0,R^A\right]$, then both types of entrepreneurs will invest (take the loan from the bank)
- $R\in\left(R^A,R^C\right]$, then only the "risky" type will invest
- $R>R^C$, then no one will invest

Thus the demand for loans can be represented by the following diagram:



(iv) The expected profit of the bank is:

- *If both types invest*:

$$V = p(1+R)I + (1-p)K + q(1+R)I + (1-q)K - 2(1+r)I = I(1+R)(p+q) + K(2-p-q) - 2(1+r)I$$

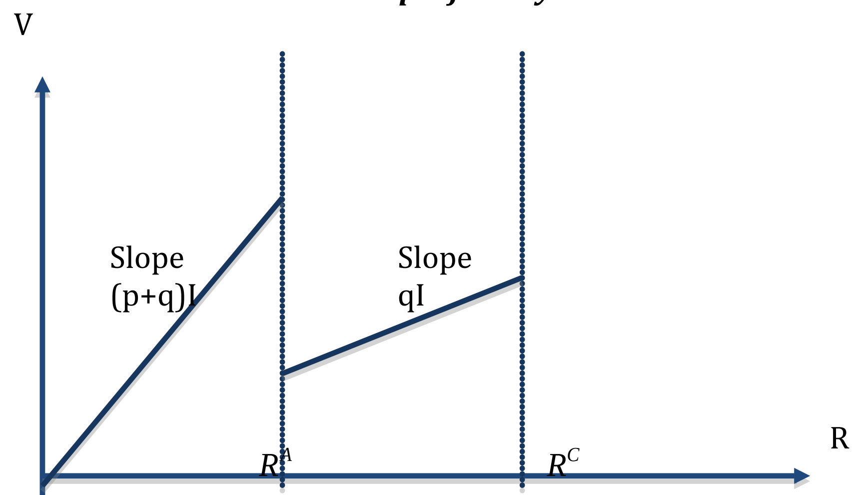
- *If only the risky type invests*:

$$V = q(1+R)I + (1-q)K - (1+r)I$$

- *If no one invests:*

$$V = 0$$

How will the bank's profit vary with R?



With the interest rate between zero and R^A, the bank's expeted profit is an increasing linear function with the slope (p+q)I. With the interest rate between R^A and R^C, the expected profit first falls (since the "safe" type drops out) and then increases with the slope qI < (p+q)I. The expected profit is zero for the interest rate above R^C, since no one is willing to take a loan from the bank.

The bank should finance the both projects if $V(R^A) > V(R^C) > 0$:

$$I(1+R^A)(p+q)+K(2-p-q)-2(1+r)I > q(1+R^C)I+(1-q)K-(1+r)I$$
$$\Leftrightarrow I(1+R^A)p+I(1+R^A)q+K(1-p) > q(1+R^C)I+(1+r)I$$
$$\Leftrightarrow I(1+R^A)p-Kp > q(R^C-R^A)I+(1+r)I-K$$
$$\Leftrightarrow p\left[I(1+R^A)-K\right] > q(R^C-R^A)I+(1+r)I-K$$
$$\Leftrightarrow p > \frac{q(R^C-R^A)I+(1+r)I-K}{I(1+R^A)-K}$$

Problem 2.

Parameters characterizing the return structure in the HMS-model are:

α, which is the return from the investment in the prudent asset;

γ, which is the return from the investment in the gambling asset, in case of success;

β, which is what the gambling asset yields in case of failure;

θ, which is the probability of success when gambling.

We know that business cycles are defined in terms of periodic fluctuations in economic activity. In the periods of expansion the economy is growing (measured by such macroeconomic variables as production and employment), while the recession periods are the periods when the economy is contracting. Investment is usually pro-cyclical and rather sensitive to the business cycle, since the earnings from investment tend to increase during booms and fall during periods of recession.

We would, for example, expect all the returns (both from prudent and risky investments) to increase in the periods of expansion. In addition we would expect (well, I would expect ;-)) the probability of success when gambling to become higher, making thus the expected return from gambling higher than the return from the prudent asset, i.e making gambling more attractive to the banks. To see that gambling becomes more attractive during economic booms we can look at the critical deposit interest rate in the HMS-model, which is:

$$\hat{r}(k) = (1-\delta)\left(\frac{\alpha - \theta\gamma}{1-\theta}\right)(1+k) + \delta\left[\alpha(1+k) - \rho k\right],$$

I assume further that the critical deposit rate is increasing in k, i.e. the discount factor is sufficiently small (banks put little weight on the future).

We know from the model that when the interest rate on deposits exceeds $\hat{r}(k)$, the bank will earn more from gambling, and thus will not be willing to invest in the prudent asset. Let, say, the deposit interest rate be just equal to the critical value before the expansion. Then in the period of expansion γ and α will increase (say, with equal amount), but θ will also increase, thus bringing the critical threshold down. As a result we would have the deposit rate higher than the threshold, which in turn would give the bank stronger incentive to gamble. In this case it would be more appropriate for the government to increase the couner-cyclical capital buffer rate, i.e. increase k in the modell (?), in order for the critical deposit rate value to increase.(?) We know, however, that strict capital requirements, though inducing prudent behavior, affect also the bank's charter value in a negative way, making the loss in case of failed gambling smaller. Hence the government should also use deposit-rate ceiling in order to prevent too high competition, and thus increase banks' profits. Only then will the banks' charter value matter (since it will be higher), when making decisions on whether to gamble or not.