Seminar # 5 7/10

(1) RN entrepreneurs, $E_1 = 1 = E_2$
entrepreneurs in economy = 2

Share $E_1 = 50\%$ α

Share $E_2 = 50\%$ $1-\alpha$

Expected gross return (R_i) $= \mu > 1$ for both
Investment expenditure $= I = 1$ for both

Project 1: $\mu = p \cdot G + (1-p) \cdot 0 = pG = R_1$

Project 2: $\mu = q \cdot B + (1-q) \cdot 0 = qB = R_2$

Hence $pG = qB = \mu$

Average $Pr(S)$ in economy:
$m = 0{,}5 \cdot p + 0{,}5\, q = \frac{1}{2}(p+q)$ ☆

Given $Pr(\text{success})$ (S): $p > q \Rightarrow B > G$, ie. gross return is greater for project 2 if successful: $R_2|S > R_1|S$

Entrepreneurs for both types of projects need a loan from a RN-bank, but have to put up collateral $K < I$

If failure (F) the bank will catch the collateral, and hence loose maximum: $I - K > 0$
while entrepreneur will loose: K

$\rightarrow \sum loss|F = I$

☆ $q < m < p$

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i) Variance of the two projects:

$$\sigma_{R_1}^2 = \sum_{i=1}^{2} p_i (R_i - \mu)^2 = \sum (p_i X_i^2) - \mu^2$$

$$\sigma_{R_1}^2 = pG^2 + (1-p)0^2 - \mu^2 = pG \cdot G - \mu^2 = \mu G - \mu^2 = \mu(G-\mu)$$

$$\sigma_{R_2}^2 = qB^2 + (1-q)0^2 - \mu^2 = qB \cdot B - \mu^2 = \mu B - \mu^2 = \mu(B-\mu)$$

Since $B > G \rightarrow$ Project 2 has the greater variance.

- Define risk-adjusted return: Sharpe ratio $= S_i = \frac{E(R_i)}{\sigma_{R_i}}$

Project 1: $S_1 = \frac{pG}{\sqrt{\mu(G-\mu)}} = \frac{\mu}{\sqrt{\mu(G-\mu)}} = \sqrt{\frac{\mu}{G-\mu}}$

Project 2: $S_2 = \frac{qB}{\sqrt{\mu(B-\mu)}} = \frac{\mu}{\sqrt{\mu(B-\mu)}} = \sqrt{\frac{\mu}{B-\mu}}$

$S_1 > S_2 \Rightarrow$ project 1 has greater X-ante risk-adjusted return

Since all entrepreneurs + lender are risk-neutral (RN), this is not relevant for this model as all agent only care about returns...

ii) Entrepreneur's Expected profits, assuming limited liability (LL) for both types:

$$E(\Pi_1) = p[\overbrace{G-(1+R)I}^{+}] + (1-p)[0-K]$$

$$= pG - pI(1+R) - (1-p)K$$

$$E(\Pi_1) = \underbrace{\mu - pI(1+R)}_{\text{Net profit if success}} \underbrace{-(1-p)K}_{\text{Loss of collateral if failure}} = \mu - p(1+R) + K(p-1)$$

$$E(\Pi_2) = q[\overbrace{B-(1+R)I}^{+}] + (1-q)[0-K]$$

$$= qB - qI(1+R) - (1-q)K$$

$$E(\Pi_2) = \mu - qI(1+R) - (1-q)K = \mu - q(1+R) + K(q-1)$$

$$E(\Pi_1) \gtrless E(\Pi_2) \; ?$$

$$\cancel{\mu} - pI(1+R) - (1-p)K \gtrless \cancel{\mu} - qI(1+R) - (1-q)K \quad \Big| \times -1$$

$$(p-q)I(1+R) \lesseqgtr K(1-q-(1-p))$$

$$(p-q)I(1+R) \lesseqgtr K(p-q)$$

$$I(1+R) \lesseqgtr K \Rightarrow K < I(1+R)$$

Collateral $K < I \Rightarrow \forall R > 0$; project 2 has the highest expected profits. The expected gross return is μ for both, while the expected negative cash-flow (= repayment + collateral loss) is lower when Pr(success) is smaller.

iii) Critical rate of interest R_C making entrepreneurs indifferent btw. investing or not.

This economy consist of Project 1 = "safe" = P_S,
and Project 2 = "risky" = P_R

Given two groups of entrepreneurs of equal size, I assume that both kind of investments are necessary to keep the economy going, i.e there are no "good" or "bad" projects/investors; both have the same expected return = μ, but P_R yields higher returns given success, but with lower $Pr(S) = q < p$.
Hence there is <u>no</u> moral-hazard problem, and the decision on investing or not reduces to the simple participation constraint for both types of investors/projects:

$$PC_1: \quad pG - p(1+R) \geq 0$$
$$pG - pr \geq 0 \Rightarrow r \leq G$$

$$PC_2: \quad qB - q(1+R) \geq 0$$
$$qB - qr \geq 0 \Rightarrow r \leq B$$