ECON 4335 Economics of Banking, Fall 2016

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Supplement to the lecture on the Diamond-Dybvig model

The model in Diamond and Dybvig (1983) incorporates important features of the real world: (1) consumers have uncertain preferences for expenditure streams, which is turn creates a demand for liquid assets. (2) Real investment projects are often irreversible. (3) Consumers have private information about their need for liquidity. (4) the uncertainty about preferences for consumption streams induces banks to offer contracts that look like demand deposits.

Here is a description of the model in Diamond and Dybvig (1983) (the reference is ch. 12 in Tirole's textbook).

The game has 3 periods: $t = 0, 1, 2$. There is a continuum of agents, each endowed with one unit of wealth that can be consumed or invested. No consumption takes place at $t = 0$. At $t = 0$, no agent knows what type she will be at $t = 1$. Every agent could end up being an early consumer (with probability λ) or a late consumer (with probability $1 - \lambda$). The probability distribution is common knowledge. The types of the agents are independent. Each agent learns her own type at time $t = 1$, but has no information about the types of the other agents (though all agents know the ex-ante distribution of types). Types cannot be verified by a third party, and therefore no insurance company will offer insurance contracts based on such information. The objective of any agent in period $t = 0$ is to maximize her expected utility.

$$U = \begin{cases} u(C_1) & \text{if impatient} \\ u(C_2) & \text{if patient.} \end{cases}$$

where C_t denotes consumption in period t. $u(.)$ is increasing, strictly concave (so agents are risk averse) with $u'(0) = \infty$, and a coefficient of relative risk aversion (or an inter-temporal elasticity of substitution) $\left|\frac{C_t u''(C_t)}{u'(C_t)}\right| > 1$ for all C_t. Being an early consumer is the equivalent of a liquidity shock: the agent is exposed to an early liquidity need or demand for cash.

[1]This note is almost entirely based on the note written by Jon Vislie for the Fall 2014 Banking course.

Ex-ante there are two investment opportunities. The first is a short-term project, which is a pure storage technology: it lasts one period and the gross return is 1 per unit invested. The second is a long-term project with a gross return reaped after two periods; the gross return is $R > 1$ per unit invested. This long-term project can, if necessary, be liquidated at $t = 1$, with a gross return $l < 1$ per unit invested. Let $I \in [0, 1]$ be the fraction of one's wealth that is invested in the long-term project.

SOCIALLY OPTIMAL ALLOCATION

A socially optimal allocation can be characterized as follows. At the ex-ante stage, a social planner solves the following risk-sharing/investment program, while relying on the law of large numbers. As agents are ex-ante identical, the planner solves problem: of a *representative agent*:

$$(1) \qquad \max \{\lambda u(C_1) + (1-\lambda)u(C_2)\} \quad s.t. \quad \begin{cases} \lambda C_1 & = 1 - I \\ (1-\lambda)C_2 & = RI \end{cases} .$$

Combining the two constraints:

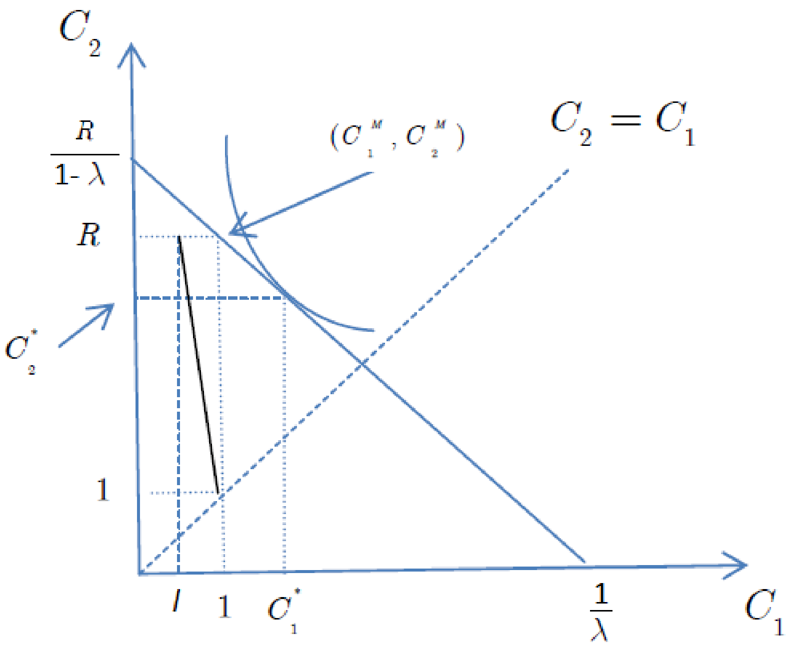
$$(2) \qquad \lambda C_1 = 1 - \frac{1-\lambda}{R} C_2 \leftrightarrow \lambda R C_1 + (1-\lambda)C_2 = R \to \frac{dC_2}{dC_1} = -\frac{\lambda R}{1-\lambda}.$$

This is an efficiency-locus and the frontier of the opportunity set. The first-best solution is found by maximizing expected utility subject to (2).

Finding the first best is a straightforward optimization problem that can easily be illustrated as follows. In the first best, the slope of an indifference curve is equal to the slope of the efficiency locus; i.e., we must have:

$$-\frac{\lambda u'(C_1^*)}{(1-\lambda)u'(C_2^*)} = -\frac{\lambda R}{1-\lambda} \leftrightarrow \frac{u'(C_1^*)}{u'(C_2^*)} = R,$$

where the last equality says that the required rate of return from saving should be equal to the net return from long-term investment. Hence we have: $u'(C_1^*) = Ru'(C_2^*)$.



The assumption $\left|\frac{Cu''(C)}{u'(C)}\right| > 1$ is equivalent to $\frac{d}{dC}(Cu'(C)) < 0$. Therefore $R > 1$, implies $u'(1) > Ru'(R)$.

$(C_1, C_2) = (1, R)$ is a feasible allocation and lies on the efficiency locus – hence $\frac{u'(1)}{u'(R)} > R$ implies that $R > C_2^*$ and $C_1^* > 1$ (hence the socially optimal allocation provides some insurance against a liquidity shock). As $R > 1$, we have $u'(C_1^*) > u'(C_2^*)$. As $u'' < 0$, then $u'(C_1^*) > u'(C_2^*)$ implies $C_1^* < C_2^*$ (hence the socially optimal allocation ensures less than full insurance against a liquidity shock).

AUTARKY

Under **autarky,** or self-insurance, each agent must provide for her own needs by hoarding liquidity to meet a liquidity shock. If the agent becomes an early consumer, liquidity is provided by liquidating the long-term project along with consuming the return from the short-term project. On the other hand, if she faces no liquidity shock, the short-run investment is rolled over another period, while also reaping the return from the long-term project. In both cases, not knowing one's type will be costly ex-post.

If the consumer ends up having an early liquidity need, she will be able to consume: $C_1 = 1 - I + lI \leq 1$. If she is not exposed to a shock, then her consumption capacity at $t = 2$, is $C_2 = 1 - I + RI \geq 1$.

Note that $C_1 = 1$ only if $I = 0$, in which case $C_2 = 1$. On the opposite end, if $I = 1$, then $C_1 = l < 1$ and $C_2 = R$. In the diagram above, the bold straight line between (l, R) and $(1, 1)$ shows all the combinations that an agent can ensure under autarky. The optimal self-insurance combination $\left(C_1^A, C_2^A\right)$ solves:

$$\max_{I \in [0,1]} \{\lambda u(C_1) + (1 - \lambda)u(C_2)\} \text{ s.t. } \begin{cases} C_1 = & 1 - (1 - l)I \\ C_2 = & 1 + (R - 1)I \end{cases}$$

Substituting in the constraints, the function to be maximized becomes:

$v(I) \equiv \lambda u(1 - (1 - l)I) + (1 - \lambda)u(1 + (R - 1)I)$,

therefore the first order condition is

$v'(I) = -\lambda u'(C_1^A)(1 - l) + (1 - \lambda)u'(C_2^A)(R - 1) = 0$.

$v'(0) = -\lambda u'(1)(1 - l) + (1 - \lambda)u'(1)(R - 1) = u'(1) \left[(1 - \lambda)(R - 1) - \lambda(1 - l)\right]$.

If $\frac{R-1}{1-l} \leq \frac{\lambda}{1-\lambda}$, then $v'(0) \leq 0$ and therefore $I^A = 0$. On the other hand, if $v'(1) \geq 0$, then $I^A = 1$.

A solution $I^A \in (0, 1)$ is characterized by: $\frac{\lambda u'(C_1^A)}{(1-\lambda)u'(C_2^A)} = \frac{R-1}{1-l}$.

Note that ex post, the investment decision is always inefficient.

FINANCIAL MARKET

A different institutional arrangement is possible: announce at $t = 0$, that a financial market will open at $t = 1$. The financial market allows exchange between agents of different types, say by trading bonds.

Let a bond market open at $t = 1$. Let 1 unit of a bond entitles a consumer to 1 unit of consumption in period $t = 2$. Suppose an agent has invested some or all her wealth in the long-term investment and then is hit by the liquidity shock, she can go around the direct liquidation of her long-term investment: she can instead **sell bonds** in a number equal to the full return from her long-term investment.While early consumers will choose to convert returns in $t = 2$ into consumption in $t = 1$ by **selling bonds (=borrowing)**, late consumers will instead choose to convert returns in $t = 1$ into consumption in $t = 2$ by **buying bonds (=lending)**.

Let p be the price of a bond, which measures the number of period-1 consumption units per unit of consumption in period 2. Hence, p is the relative price of late consumption (in units of early consumption). An agent being exposed to a liquidity shock will like to borrow, that is, sell RI units of bonds. The amount RI is equal to her future income, that is, the return from her long-term investment that will be reaped at $t = 2$. An early agent will then have the following consumption opportunity. Hence an early consumer will have: $C_1 = 1 - I + pRI$.

A late agent buys bonds and sells her short-term surplus of goods at $t = 1$. Hence, she transforms this surplus into consumption at $t = 2$. In period $t = 2$, a late agent can therefore consume: $C_2 = RI + \frac{1}{P}[1 - I]$.

Observe that $C_1 = pC_2$. Suppose that $p > \frac{1}{R}$. Then, the rate of return from investing in the short-term project (either $C_1 = 1$ or $C_2 = 1\frac{1}{p} < R$) is smaller than the rate of return from investing in the illiquid long-term project (either $C_1 = Rp > R\frac{1}{R} = 1$ or $C_2 = R$). Hence, at $t = 0$, every agent will choose $I = 1$. That means that at $t = 1$, there will be an excess demand for goods (or an excess supply of bonds, which is the same thing). This cannot constitute a market equilibrium. If on the other hand, $p < \frac{1}{R}$, the rate of return from investing in the long-term project is smaller than the rate of return from investing in the short-term project. In $t = 0$, every agent will choose $I = 0$, creating an excess supply of goods or (equivalently) an excess demand for bonds at $t = 1$. Not an equilibrium either.

If $p = \frac{1}{R}$, then every agent will be indifferent between investing in the short-term project or in the long-term one. Hence any choice $I \in [0, 1]$ is as good as any other. Suppose, for simplicity, that every agent chooses the same I. Then, the market-clearing condition in the bond market is: $\lambda RI = (1 - \lambda)\frac{1-I}{p}$. Hence in equilibrium $p = \frac{1}{R}$ and $\lambda RI = (1 - \lambda)\frac{1-I}{p}$.

In equilibrium, $I = I^M \equiv 1 - \lambda$ and $C_1 = C_1^M = 1$ while $C_2 = C_2^M = R$.

It should not come as a surprise that the market equilibrium Pareto-dominates the autarky equilibrium:

$C_1^M = 1 \geq C_1^A$ and $C_2^M = R \geq C_2^A$.

Note however that the market equilibrium ensures an allocation that is (strictly) worse than the socially optimal allocation.

The market equilibrium is not ex ante Pareto-efficient because we have not a complete set of risk or insurance markets ex ante. As a result, liquidity risk cannot be properly allocated. The market equilibrium shows higher volatility in consumption than the socially optimal allocation. At the same time, because risk is not properly shared, people will take more cautious actions by investing more in the long-term project than what is optimal.

BANKS

The last institutional arrangement we consider are banks. Banks let agents deposit their initial endowments. The bank will undertake the required asset transformation by investing in the long-term project, while offering demand deposits to all agents. For banks to implement the socially-efficient allocation, it must be the case that banks allow each agent to withdraw C_1^* units in period $t = 1$, or C_2^* in period $t = 2$. We saw above that $C_1^* < C_2^*$. This property makes the socially-efficient allocation *incentive efficient:* no late consumer will be motivated to act as an early consumer. A late consumer can always pretend to be impatient, "take out" C_1^* at $t = 1$, and invest this amount in a short-term project with a gross return C_1^* to be consumed at $t = 2$. However, this behavior will incur a utility loss as $C_1^* < C_2^*$.

In a **competitive banking equilibrium**, banks compete for deposits.

A free-entry banking equilibrium is characterized by banks offering a contract (which corresponds to a consumption profile (C_1, C_2)) to agents so that:

$\max_{(C_1,C_2)} \{\lambda u(C_1) + (1-\lambda)u(C_2)\}$

s.t. $(1-\lambda)C_2 = (1-\lambda C_1)R$.

With free entry in the banking industry, bank profits will be zero and equilibrium is characterized by each bank offering the efficient contract. If a bank offers a profile different from the one that maximizes the agents' expected utility, another bank will gain by offering the one that is preferred by the agents. The zero-profit condition says that what is left for late consumers is equal to the long-term return from investment (lending), after having put aside, as liquid reserves, an amount equal to what early consumers are expected to withdraw. Hence an amount $1 - \lambda C_1$ is being lent out at $t = 0$, with a gross return equal to R. Note that the constrained maximization problem of banks corresponds to (1). Hence, the competitive equilibrium deposit contract will coincide with the efficient consumption profile. Banks implement the efficient allocation: they can eliminate the mismatch between the maturity structure and the population's need for liquidity.

When agents are exposed to idiosyncratic (diversifiable) liquidity shocks, the allocation of the financial market equilibrium can be improved upon by having banks offering deposits contracts. Banks or financial intermediaries act like insurance companies and eliminate the cost of maturity mismatch by undertaking the optimal long-term investment.

BANK RUNS

The desirable properties of the bank solution are highly dependent on the fact that only a fraction λ of depositors withdraw early. The efficient deposit contract constitutes a Nash equilibrium. [2] On the other hand, as shown by Tirole, there is also another Nash equilibrium where all depositors withdraw early under a system "first-come-first-serve".

To see how this work let's assume every consumer deposited her money in a bank and banks offer C_1^* and C_2^* in the two periods, respectively. Every early consumer finds it optimal to withdraw in period $t = 1$. Hence, it must be the case that the fraction of consumers withdrawing early is some $f \geq \lambda$. In the equilibrium we considered above, $f = \lambda$. In this case, every late consumer waits. Hence in period $t = 2$ a fraction $1 - \lambda_2$ of consumers all demand C_2^*.

As the bank invested $1 - \lambda C_1^*$ in the long-term project, you can check that the bank will have just enough funds to satisfy the request of every late consumer. If instead, for some reason, some late consumers withdraw in period f, then the bank will not have enough resources to honor the contract with every consumer that withdraws in period 2 (in this case the bank will share what is left among all late withdrawers).

[2] You can think of the Nash equilibrium as a self-fulfilling prophecy: a situation in which everyone has correct beliefs about how everyone else will behave, and everyone chooses the most profitable option, given those beliefs.

If $\left(1-\lambda-\frac{f-\lambda}{l}\right)\frac{R}{1-f}\geq 1$ (or, equivalently $f\leq\frac{((1-\lambda)l+\lambda)R-\lambda}{R-\lambda}$), then every late consumer is better off waiting until $t=2$.

If instead $\left(1-\lambda-\frac{f-\lambda}{l}\right)\frac{R}{1-f}<1$ (or, equivalently $f>\frac{((1-\lambda)l+\lambda)R-\lambda}{R-\lambda}$), then every late consumer is better off withdrawing in $t=1$.

Hence if, for some reason, everyone believes $f>\lambda+(1-\lambda)l$ then everyone withdraws in period $t=1$: the realized f is $f=1$. Note that as $1>\lambda+(1-\lambda)l$, then there is a Nash equilibrium in which everyone withdraws in period $t=1$.

Note also that there are only 3 Nash equilibria:

1) $f=\lambda$: every late consumer waits;

2) $f=\lambda+(1-\lambda)l$: $f-\lambda$ late consumers withdraw in $t=1$ and $1-f$ late consumers withdraw in $t=2$;

3) $f=1$: every late consumer withdraws in $t=1$.

Note that if there was a third player in the game, namely a public authority that ensures deposits in the bank, then the only equilibrium would be the one in which every late consumer waits.

ROLLING OVER DEPOSITS

The model considered so far overestimates the need of banks to invest in short-term projects. This is the case as the model considers only one generation of agents. All agents deposit at the same time, and all withdraws take place in one of two periods. In reality, while some investors withdraw their deposits there are new deposits being made. This feature can be captured in an overlapping-generation model.

Consider the following variation of the model:

- a new generation t invests its endowment (1 unit per consumer) at time t and lives up to period $t+2$;
- each member of generation t discovers at time $t+1$ whether she is an early or a late consumers;
- the total population is constant;
- the technology is the same as the one described above.

Consider a situation of steady state (= enough periods have passed from the beginning of the game). Suppose there is only one active bank and the bank offers a consumption profile that maximizes the depositors' utility (in order to deter entry from other banks):

$\max_{(C_1,C_2)} \{\lambda u(C_1) + (1-\lambda)u(C_2)\}$.

The bank can invest all deposits in the long-term project: in period $t+2$ the bank uses the total return R on the deposits of generation t to honor the withdraws of early consumers of generation $t+1$ and late consumers of generation t. Therefore the bank makes NO investment at all in the short-term project.

The budget constraint becomes:

$\lambda c_1 + (1-\lambda)c_2 = R$.

As u is concave, the bank could maximize the depositors' expected utility by offering full insurance: $C_1 = C_2 = R$. Nevertheless, in this case late consumers of generation t would withdraw in period $t+1$ and then deposit again and withdraw in $t+2$. This way late consumers would get $R^2 > R$. Hence full insurance is not robust to arbitrage. The no-arbitrage condition is: $(C_1)^2 \leq C_2$. The utility-maximizing contract is then $(C_1)^2 = C_2$.