ECON 4335 Economics of Banking, Fall 2016

Jacopo Bizzotto

Supplement to Holström and Tirole (1997): Market Equilibrium

The model outlined in Holmström and Tirole (1997) illustrates the role of capital among entrepreneurs, uninformed investors and banks, for the undertaking of investment projects under market failure caused by moral hazard.

Entrepreneurs:

- Entrepreneurs need I at $t = 0$ to start a project,
- At $t = 1$, a project yields $y > 0$ (success) or 0 (failure),
- Different entrepreneurs have different endowments at $t = 0$. An entrepreneur's endowment will be denoted A. The values of the endowments are identically and independently distributed according to $G(A)$ on the interval $[0, \hat{A}]$ and the corresponding probability density function will be denoted $g(A)$,
- The entrepreneur can privately choose among 3 versions of the project:

(1) *Good*: probability of success p_H, no private benefit;

(2) *Bad with high private benefit*: probability of success $p_L < p_H$, where $\Delta p \equiv p_H - p_L$, with private benefit B (say, divert resources to buy a car: high private benefit but hard to conceal);

(3) *Bad with low private benefit*: probability of success p_L, with private benefit $b < B$ (say, spend the day at the work desk checking how your stocks are doing: low private benefit but easy to conceal).

Consumers do not observe which type of project the entrepreneur chose. Banks can spend c to observe whether the entrepreneur chose a bad project with high private benefit (but cannot distinguish between a good project and a bad project with low with private benefit).

Consumers:

- Consumers have unlimited endowments at $t = 0$.

We normalize the time discount to 1 for everyone. We assume that everyone is risk-neutral. Let r denote the rate of return required by uninformed investors, and let λ denote the rate of return required by banks. We assume that $\lambda \geq r$. These rates will be determined on the market for credit, but for now they are considered exogenous.

Assumption 1: $p_H y - I\lambda - c > 0 > p_L y - Ir + B$.

Critical values will be derived, from which we can divide the entrepreneurs into three groups: (1) the capital-rich entrepreneurs who solely use direct finance, (2) entrepreneurs with medium-size capital who get indirect finance (from banks acting also on behalf of uninformed investors), and (3) capital-poor entrepreneurs who will not undertake the investment but will make their funds available on the market. The members of this last group have too little capital to put into the project and cannot convince lenders that they will choose the good project. A borrower needs to have enough "stake in the game" in order to have any incentive to choose the good project; this incentive is related to the amount of own capital put into the project.

EXOGENOUS INTEREST RATE

The project is externally financed if and only if investors get an expected return large enough to cover the opportunity cost of investing the amount $I - A$. We will denote the payment from entrepreneurs to consumers in case of success as d^c. The expected return must satisfy the participation constraint of the consumer:

$(\overline{IR}_c)$: $p_H d^c \geq r(I - A)$,

and the entrepreneur must prefer to choose the good project:

$(\overline{IC}_e)$: $p_H(y - d^c) \geq p_L(y - d^c) + B$.

Also the participation constraint of the entrepreneur $(\overline{IR}_e)$ must be satisfied. Assumption 1 ensures that $\overline{IR}_e$ is satisfied whenever $\overline{IR}_c$ and $\overline{IC}_e$ are satisfied.[1] The constraints $\overline{IR}_c$ and $\overline{IC}_e$ are satisfied at the same time if and only if $A \geq \overline{A}(r)$ where

[1] $(\overline{IR}_e)$: $p_H(y - d^c) \geq Ar$. As discussed below, $\overline{IR}_c$ and $\overline{IC}_e$ are satisfied if and only if d^c satisfies $y - \frac{B}{\Delta p} \geq d^c \geq \frac{r}{p_H}(I - A)$. To prove that $\overline{IR}_c$, $\overline{IC}_e$ and $\overline{IR}_e$ can all be satisfied at the same time, it is enough to check whether $\overline{IR}_e$ is satisfied at least for $d^c = \frac{r}{p_H}(I - A)$. Substituting $d^c = \frac{r}{p_H}(I - A)$ in $\overline{IR}_e$, you obtain: $p_H y - rI \geq 0$. This inequality holds by Assumption 1.

$$\overline{A}(r) \equiv I - \frac{p_H}{r}\left(y - \frac{B}{\Delta p}\right).$$

$\overline{A}(r)$ is larger than 0 by Assumption 1, and it is increasing in r, and increasing in the agency cost $\frac{B}{\frac{\Delta p}{p_H}}$ as well. This agency cost is positively correlated with the opportunity cost to the entrepreneur of working hard, B, and negatively correlated with the likelihood ratio $\frac{\Delta p}{p_H}$. The likelihood ratio captures the value to the entrepreneur of choosing the good project.

An entrepreneur with capital below $\overline{A}(r)$ will not be granted credit without bank supervision. There is credit rationing in equilibrium; a market failure.

Banks can monitor and get the entrepreneurs to work with a lower opportunity cost or smaller private benefit from choosing a bad project: b instead of B.

The bank will lend I_b only if the compensation in case of success, d^b, is sufficiently large:

($\underline{IR}_b$): $p_H d^b - c \geq \lambda I_b$,

moreover banks monitor only if they find it convenient to dissuade the entrepreneur from choosing a bad project:

($\underline{IC}_b$): $p_H d^b - c \geq p_L d^b$.

Putting together these constraints, it must be the case that $I_b \geq I_b(\lambda)$, where:

$$I_b(\lambda) \equiv \frac{p_L c}{\lambda \Delta p}.$$

$I_b(\lambda)$ represents the smallest amount an entrepreneur needs to borrow from a bank. Indirect finance is more expensive than direct finance, so the entrepreneur will borrow exactly $I_b(\lambda)$ from the bank, and nothing more. When the entrepreneur borrows $I_b(\lambda)$ from banks, and banks supervise the entrepreneur, the participation constraint of the consumer becomes:

($\underline{IR}_c$): $p_H d^c \geq r(I - A - I_b(\lambda))$,

while the incentive-compatibility constraint of the entrepreneur becomes:

($\underline{IC}_e$): $p_H(y - d^c - d^b) \geq p_L(y - d^c - d^b) + b$.

Putting these constraints together, the entrepreneur endowment must satisfy: $A \geq \underline{A}(r, \lambda)$ where:

$$\underline{A}(r,\lambda) \equiv I - \frac{p_H}{r}\left(y - \frac{\left(b + c - \frac{p_H}{\lambda p_L}cr\right)}{\Delta p}\right).$$

$\underline{A}(r,\lambda)$ is smaller than $\overline{A}(r)$, and therefore we have firms with capital $A \in \left[\underline{A}(r,\lambda), \overline{A}(r)\right]$, in "number", $G(\overline{A}(r)) - G(\underline{A}(r,\lambda))$, each being financed in part by banks with a loan of size $I_b(\lambda)$. The total demand for bank credit is:

$$D_b(r,\lambda) \equiv \left[G(\overline{A}(r)) - G(\underline{A}(r,\lambda))\right] I_b(\lambda).$$

Note that for given r, $\underline{A}(r,\lambda)$ is increasing in λ; hence the number of firms being granted bank credit decreases as λ becomes larger. At the same time, $I_b(\lambda)$ is decreasing in λ. As a result, a higher λ will reduce total demand for bank credit: $\frac{\partial D_b}{\partial \lambda} < 0$. Let K_b denote the exogenous bank capital. The equilibrium condition for bank credit is:

$$K_b = D_b(r,\lambda).$$

This is a market clearing condition. The condition determines a unique equilibrium rate of interest for bank credit (or "informed capital"), denoted $\lambda(r, K_b)$. Note that for a smaller value of K_b, λ must be larger. Why? Suppose the opposite, claiming that λ decreases as K_b becomes smaller. As a result of a decrease in λ, $I_b(\lambda)$ would increase, and $\underline{A}(r,\lambda)$ would decrease. As a result, more firms would obtain loans from the bank and each loan would be larger. This is not compatible with lower bank capital. Hence the rate of interest for bank credit must increase as K_b becomes smaller.

ENDOGENOUS INTEREST RATE

To go a step further, we can look at the full market equilibrium, where we consider the pair of interest rates (r,λ) that will simultaneously clear the market for bank credit as well as the market for uninformed capital. Let consumer savings be given by a function $S(r)$, increasing in r, and let this be part of the supply of uninformed credit, along with supply by entrepreneurs with endowments $A < \underline{A}(r,l)$. The overall supply of capital is

$$S(r) + \int_0^{\underline{A}(r,\lambda)} Ag(A)dA + K_b$$

whereas the overall demand for capital is given by

$$\int_{\underline{A}(r,\lambda)}^{\hat{A}} [I - A]\, g(A)dA.$$

The interest rates r and λ must adjust to ensure that overall demand and supply of capital are equal. What is the impact of changes in exogenous variables?

Consider a shock explained by a "collateral squeeze", that is, the market value of the entrepreneurs' collateral shrinks. The effect is that entrepreneurs that have their project financed even after the decrease in collateral will need to borrow more, and the capital-poor entrepreneurs will have less capital to lend.[2]To restore the equilibrium in the market for capital at least one of the interest rates must increase so as to increase $\underline{A}(r, \lambda)$. The "weakest" entrepreneurs, those entrepreneurs with capital close to $\underline{A}(r, \lambda)$ are hurt. Then, we have the following: lower values of capital among entrepreneurs (as will happen during a recession) will reduce the financing options for undertaking profitable investment projects, and hence amplify the initial recession. This is sometimes called "an accelerator effect".

Suppose instead that supply of capital decreases. The supply of capital can go down either because K_b is reduced, or because savings among consumers are subject to a negative shift. To restore equality, the demand for capital must decrease.

If the reduced supply is the outcome of a reduction in K_b, λ will increase (as we discussed above). Hence, $\underline{A}(r, \lambda)$ will go up: entrepreneurs with capital close to this critical value will now be moved from undertaking the project to become suppliers of capital in the uninformed segment, whereas entrepreneurs close to $\overline{A}(r)$ are not directly affected. However, as the supply of uninformed capital increases, r *might go down*. A reduction in r will reduce the amount of savings provided by consumers.

[2]The is also a countervailing effect: some entrepreneurs that before the shock where demanding capital as they had endowments $A > \underline{A}$ will now have endowments $A < \underline{A}$ and will therefore supply capital. This effect tends to decrease the demand for capital and increase the supply of capital. We assume here that this effect is small and we disregard it.

If instead the supply of capital is reduced because consumers reduce their savings, through a negative shift in the supply function $S(r)$, while K_b is kept fixed, the effect is that r *increases* and $\underline{A}(r, \lambda)$ must increase.