

Problem set 1 (September 9 and 16, 2016)

Consider the one-good, two-types, three-dates economy of Diamond and Dybvig. There are infinitely many, *ex ante* identical, individuals, each endowed with one unit of the good at $t = 0$. Consumption takes place either at $t = 1$ or $t = 2$. With probability λ a consumer needs to consume at $t = 1$, and with probability $1 - \lambda$ at $t = 2$. There is an independent draw for each agent. Ex post the consumers can be divided into group 1, impatient consumers, and in group 2, those who will wait until $t = 2$ (patient consumers). An individual's type is private information. The utility function of a consumer is $u(c) = \frac{c^{1-s}}{1-s}$ where c refers to the level of consumption in the period in which the consumer needs to consume, with $s > 1$. There is no discounting.

The economy has two ways of transferring resources between periods: storage (called a short-term project) with gross return equal to 1, and a long-term investment project, with gross return at $t = 2$, equal to $R > 1$, per unit invested at $t = 0$. If necessary, the long-term project can be liquidated or stopped prematurely at $t = 1$, with a return $L \in (0, 1)$.

- (1) Derive the allocation that maximizes social welfare, as given by expected utility. How is initial wealth allocated between the two investment opportunities? Will there be any liquidation?
- (2) Let optimal consumption be C_1^* for a type 1-individual, and C_2^* for a type 2-individual. Who will have the higher consumption? Explain why an uneven distribution can be optimal. How is the optimal consumption profile affected by s ?
- (3) Assume that in the economy there is a competitive banking sector, where individuals can deposit their unit wealth at $t = 0$. The banks have the same investment opportunities as above. Suppose the banks offer the depositors the opportunity to withdraw at $t = 1$ or at $t = 2$. Explain why and under what circumstances the optimal allocation can be realized as an equilibrium.

- (4) When banks offer the deposit contract $\{C_1^*, C_2^*\}$, explain why there are two (Nash) equilibria that are consistent with rational behavior for all individuals; one where only the early consumers withdraw at $t = 1$, and another one where everyone withdraws at $t = 1$. What will the individual consumption level be in the latter equilibrium if you assume $L = 1$?
- (5) Suppose the banking sector offers the contract $\{C_1^*, C_2^*\}$ to depositors at $t = 0$. Imagine that a financial (or a bond) market is opened at $t = 1$. A bond is here a promise to have one unit consumption at $t = 2$. Late consumers are offered to buy bonds at a price $p = \frac{1}{R}$. Will $\{C_1^*, C_2^*\}$ still be a Nash equilibrium? Explain!
- (6) Consider a different setting. Suppose the draw that determines whether a consumer is an early or a late one is perfectly correlated among the individuals: with probability λ all consumers are impatient, while with probability $1 - \lambda$ all consumers are patient. Do banks improve over autarky in this setting?