Answers to some questions in Problem Set 6 – ECON 4335 Economics of Banking

Question 1.

Given the assumptions in **3**, we have: $\Pi_A = p_A(R_A - (1-k)R_D) - kR_E$ and $\Pi_B = p_B(R_B - (1-k)R_D) - kR_E$, when, combined with $p_A R_A > R_E \geq R_D$, then project A is always profitable. We have:

$$\Pi_A = p_A(R_A - (1-k)R_D) - kR_E = \underbrace{p_A R_A}_{>R_E} + \underbrace{(1-p_A)(1-k)R_D}_{>0} - \underbrace{\left[kR_E + (1-k)R_D\right]}_{\leq R_E}$$

where we have added and subtracted $(1-k)R_D$. Given our assumptions, $\Pi_A > 0$.

In **4** we have that project B is more profitable than project A if:

$$\Pi_B = p_B(R_B - (1-k)R_D) - kR_E > \Pi_A = p_A(R_A - (1-k)R_D) - kR_E \Leftrightarrow$$
$$p_B(R_B - (1-k)R_D) > p_A(R_A - (1-k)R_D) \Leftrightarrow (1-k)(p_A - p_B)R_D > p_A R_A - p_B R_B$$
$$\Leftrightarrow (1-k)R_D > \frac{p_A R_A - p_B R_B}{p_A - p_B} \Leftrightarrow (1-k) > \frac{p_A R_A - p_B R_B}{(p_A - p_B)R_D} \Leftrightarrow k < 1 - \frac{p_A R_A - p_B R_B}{(p_A - p_B)R_D} := \underline{k}$$

This could happen because the gross return from a successful outcome when project B is implemented is higher than for project A, while both projects have the same return if failure (non-success), if capital requirment is not too strict and competition implies a high R_D. Here it is possible to have $\Pi_A \leq 0$, and at the same time $\Pi_B > 0$.

For $\Pi_A \leq 0$, the capital requirment has to obey $k \geq \dfrac{p_A(R_A - R_D)}{R_E - p_A R_D} := \overline{k}$, whereas

$\Pi_B \geq 0$ for $k \leq \dfrac{p_B(R_B - R_D)}{R_E - p_B R_D} := k^*$. If $\dfrac{p_B(R_B - R_D)}{R_E - p_B R_D} > \dfrac{p_A(R_A - R_D)}{R_E - p_A R_D}$, then we have a non-empty interval, such that for any capital requirement $k \in \left[\overline{k}, k^*\right]$, where $p_A > p_B$ and $R_B > R_A$, cf. 2, we have $\Pi_B > 0 > \Pi_A$. (One should perhaps derive conditions for this to happen. We have, cf. 2, that $p_A > p_B$. Therefore: If

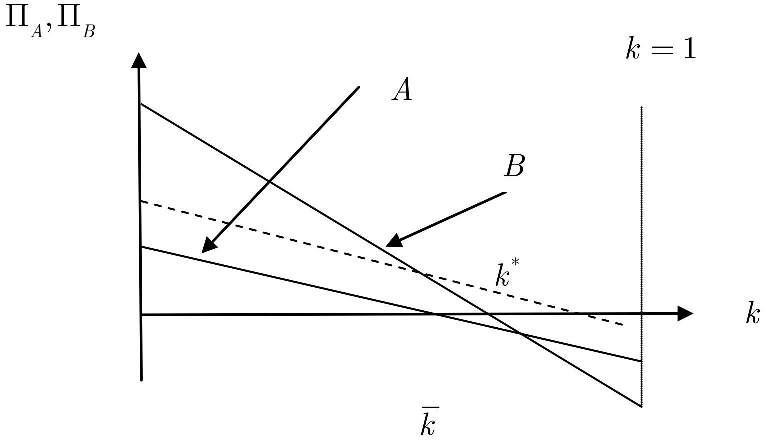
$$R_E - p_A R_D > 0, \textit{then } R_E - p_B R_D > R_E - p_A R_D > 0 \text{ and } \underbrace{\frac{R_E - p_A R_D}{R_E - p_B R_D}}_{<1} > \frac{p_A(R_A - R_D)}{p_B(R_B - R_D)}.$$

A necessary, but not a sufficient, condition for this to take place is $p_A(R_A - R_D) < p_B(R_B - R_D) \Leftrightarrow \underbrace{p_A R_A - p_B R_B}_{>0} < \underbrace{R_D(p_A - p_B)}_{>0}$. If the expected values of the project do not differ much, and/or the deposit rate is high, then there might exist a non-empty interval $\left[\bar{k}, k^*\right]$ so that for any k in this interval, we have $\Pi_B \geq 0 \geq \Pi_A$, with strict inequalities in the interior.)

Under **5** it is assumed that $R_E > p_A R_A > R_D$; hence $R_B > R_A > R_D > p_A R_D > p_B R_D$, and that project A is socially desirable, but project B is not. Hence, the regulator wants to set the capital requirement so as to get project A chosen by the bank with non-negative expected profits from implementing A. The profit from each project is now declining in k; as we have $\Pi_j(k) = p_j(R_j - R_D) - (R_E - p_j R_D) \cdot k$ for $j = A, B$, where $R_E - p_A R_D < R_E - p_B R_D$. (The B-profile has a steeper slope.) We note that

$\Pi_A(0) = p_A(R_A - R_D) > 0$ and $\Pi_A(1) = p_A R_A - R_E < 0$, and $\Pi_B(0) = p_B(R_B - R_D) > 0$ and $\Pi_B(1) = p_B R_B - R_E < 0$ as $R_E > p_A R_A > p_B R_B$.

Because A is socially desirable, this can be interpreted as imposing some k for which $\Pi_A > 0$, and $\Pi_A > \Pi_B$, if such a value of k exists. If the situation is as the one depicted below, with the full-line for project A, no capital requirement regulation will get the bank to choose project A.



In order to get the bank to choose project A, a situation like the one as illustrated by the dashed line for Project A in the figure above has to occur. In that case there will exist a non-empty interval for k over which $\Pi_A \geq \Pi_B$ and $\Pi_A \geq 0$. The upper limit, defined by having $\Pi_A(\bar{k}) = 0$, whereas the lower limit is given by the value of k so that $(1-k) < \dfrac{p_A R_A - p_B R_B}{(p_A - p_B) R_D}$; which is equivalent to have $k > \underline{k}$, implied by the lowest value of k so that $\Pi_A = \Pi_B$.

First, for project A to be profitable, $k < \bar{k}$. For any $k \in \left[0, \bar{k}\right)$, we have $\Pi_A > 0$. However, if there exists a lower value of k, defined so that $\Pi_A(\underline{k}) = \Pi_B(\underline{k}) > 0$, then we have that for any capital requirement obeying $k \in \left(\underline{k}, \bar{k}\right)$, the socially optimal project is being implemented by the bank. (Note that the regulator needs a lot of information; about all parameters describing the projects, as well as the return from both equity and deposits, to ascertain whether such an intreval in fact exists.)

Question 2.

(Some answers and comments.)

In the competitive banking industry, with a zero-profit condition for the bank, $w(D,r) = (1+r_0)\cdot D$, we find the the equilibrium rate of interest under **b** as:

$$r = \frac{1+r_0}{1-p} - \frac{p}{1-p}\frac{y_0 - B}{D} - 1$$

Here we have that $\dfrac{dr}{dD} > 0$; the rate of interest must be increasing in the amount borrowed. Furthermore, with a smaller amount of equity; i.e. the higher is D, the higher is the equilibrium rate of interest.

The entrepreneur's investment criterion under **c** is: Invest if and only if $v(D,r) \geq (1+r_0)\cdot L$. This can be rewritten to become

$$\begin{aligned}
&\Phi(K,r,L) := v(K-L,r) - (1+r_0)L \\
&= py_0 + (1-p)y_1 - py_0 - (1-p)(1+r)D - (1+r_0)L \\
&= \mu - (1+r_0)K - pB \geq 0
\end{aligned}$$

Here the first term $\mu - (1+r_0)K$ shows the net value of the project in a perfect financial world, whereas the last term, the expected bankruptcy cost, can be regarded as the cost due to an imperfect financial system.

In **d** we assume that cash flow can be observed only by the entrepreneur/borrower, not by the bank/lender; hence the magnitude of cash flows cannot be verified by a third party. Following the argument by Bolton & Scharfstein (see Freixas & Rochet, pp. 135 – 137), the borrower will in a one-period setting never report a cash flow other than the lower one, irrespective what the true cash flow is; i.e. y_0 will be reported whatsoever, where $y_0 < D < y_1$. Because the lender has rational expectations, the bank will never grant a loan, due to the loss, because $y_0 < D < (1+r)D$. We have a social loss because the lender never will get a truthful report; hence a project with high cash flow will not get external finance, and therefore not be implemented. (Because no loan is granted, the expected social loss is μ.)

In **e** it is supposed that the project is repeated over two periods (we ignore discounting), with cash flows being independently and identically distributed in the two periods; . If the entrepreneur cannot repay after period 1, the loan will not be continued. If a repayment $(1+r)D$ is made after period 1, the bank has made commitment to grant a similar loan as the one granted in the first period, at the beginning of period 2 so that the entrepreneur can implement a similar project as above. (Because the bankruptcy cost creates some problems, let us ignore this contingency, by simply putting $B=0$.)

At the end of period 2 the borrower has no incentive to report a high cash flow if that should be the outcome; in that case, the low cash flow accrues to the bank, as in the one-period case.

One interpretation of the problem raised here is whether a competitive banking industry is less able to support long-term relationships, as compared to an imperfect system where the lender offers financial contracts. (This question is not answered here, but raised as an issue.)

The bank's expected net profits from the proposed contractual setting will now be:

$$\Pi = -(1+r_0)D + py_0 + (1-p)\Big[(1+r)D + y_0 - (1+r_0)D\Big]$$

where $\Pi = 0$ in equilibrium. The first term is the first-period funding cost, the second term is the first-period expected profit if default, whereas the last term is the expected profit from continuing the relationship. This is a condition determining the equilibrium rate of interest. Here we find:

$$
\begin{aligned}
&(1-p)\left[(1+r)D + y_0 - (1+r_0)D\right] = (1+r_0)D - py_0 \\
&\Rightarrow (1+r)D = \frac{1+r_0}{1-p}D - \frac{p}{1-p}y_0 - y_0 + (1+r_0)D \\
&\Rightarrow 1+r = \frac{(1+r_0)(2-p)}{1-p} - \frac{y_0}{D}\frac{1}{1-p} \\
&\Rightarrow r = \frac{(1+r_0)(2-p)}{1-p} - \frac{y_0}{D}\frac{1}{1-p} - 1 \qquad (*)
\end{aligned}
$$

(Compare the interest rate in (*) with the one you derived in b, for $B = 0$.) The borrower must, given the rate of interest in (*), be induced to repay/continue the relationship by truthfully reporting y_1 after period 1, if y_1 is realized. Hence, the question is whether the equilibrium rate of interest can support truth-telling by the borrower. If that should be the case, the following incentive constraint has to be met for the proposed debt contract, providing incentives to repay the first-period loan when cash flow is y_1:

If the realized profit in period 1 is y_1, the expected profit to the borrower from reporting truthfully is:

$$
\underbrace{-(1+r)D + y_1}_{1.\textit{period profit when repaying}} \quad + \quad \underbrace{\left[-y_0 + (1-p)y_1 + py_0\right]}_{\text{exp.}2.\textit{period profit by paying the lower profit}}
$$

which should exceed what the entrepreneur expects to obtain by repaying y_0 at the end of period 1 when realized profit is y_1; i.e., the following inequality must hold:

$$
-(1+r)D + y_1 + \left[py_0 + (1-p)y_1 - y_o\right] \geq y_1 - y_0 \Leftrightarrow
$$

$$
-(1+r)D + (1-p)(y_1 - y_0) \geq -y_0 \;\text{ or }\; (1-p)(y_1 - y_0) + y_0 = \mu \geq (1+r)D
$$

We have to check whether the equilbrium interest rate in (*) can hold along with this condition (not done here).

The condition to be verified is: $\dfrac{\mu}{D} \geq 1 + r = \dfrac{(1+r_0)(2-p)}{1-p} - \dfrac{y_0}{D}\dfrac{1}{1-p}$.