Jon Vislie; February, 2010

Supplements (and some necessary corrections) to
Lectures in ECON 4335 – The Economics of Banking

In this note I will make some remarks as to some of the issues presented during my lectures in banking. These remarks are related to:

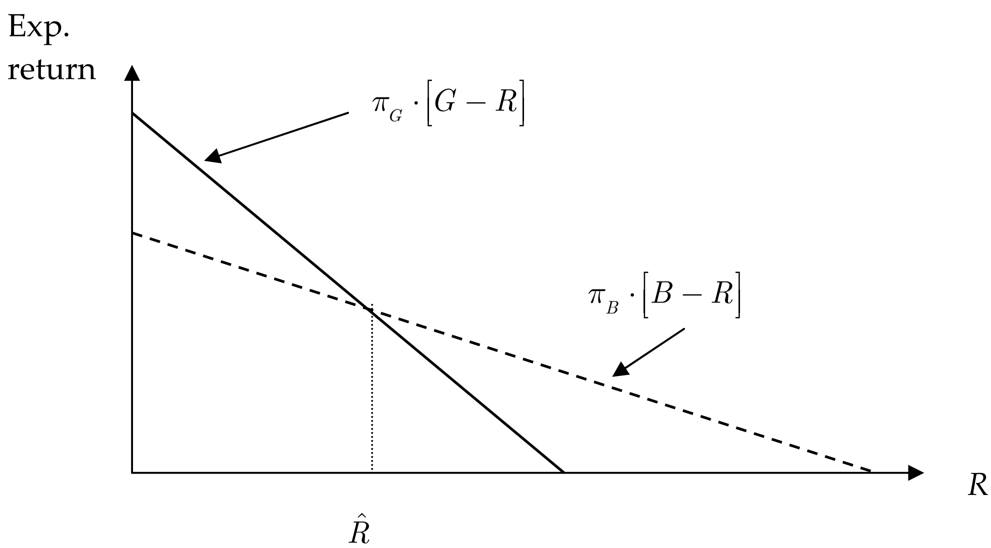
- Some features of a model that were ignored in the lecture #3 – see section 1
- Correct some confusion produced by me; from lecture #4 – see section 2
- Some support for the reading of 3.5.3 – especially page 96; see section 3 [1]

1. In the model discussed in section 2.5.1 – "A simple model of the Credit Market with Moral Hazard", cf. Lecture #3, it is said that *if moral hazard is not too important, we will have credit market equilibrium with no monitoring only if the good project is chosen by the firm*.[2] In this model moral hazard is related to an informational problem where the informed agent – the borrower/firm that is implementing a project – can take an unverifiable action after a loan or a financial agreement has been settled, and by so affect the probability of fulfilling the contractual obligations. One action the lender/the uninformed party/the principal wants to avoid is that the firm chooses a too risky project, with no repayment if failure, but will be able to repay only if success. However, the probability of repayment by the borrower will be lower the more risky project is chosen. The lender can then, by offering a repayment schedule or a loan agreement, induce the borrower to take the action which is more in accordance with the lender's goal or objective. Hence, for any repayment the borrower has a best response. If symmetric information, as would be the case if the action taken by the borrower could be verified, then the borrower could be forced to make a choice in accord with the interest of the lender.

A brief summary of the first part of the model is: A firm wants to finance a project of size one. With no equity the entire project is financed by external funds, demanding a repayment R if the project succeeds; and nothing if failure. (The risk-free rate of interest is equal to zero, by assumption.)
After the loan is granted, the firm could choose to undertake a project with a specific quality. The firm has two options. The first one yields a good lottery $\{(\pi_G, G);(1-\pi_G, 0)\}$; the second option yields a bad lottery $\{(\pi_B, B);(1-\pi_B, 0)\}$. The first one produces a return G with probability π_G and zero with probability $1-\pi_G$, whereas the second one produces a return B with probability π_B and zero with probability $1-\pi_B$. It is assumed that $\pi_G G > 1 > \pi_B B$, with $B > G$; hence we must

[1] I did not have time to present the material about *competition and screening* (section 3.5.3 in F&R) in lecture #4.
[2] I present some supplementary remarks to the model used in section 2.5.1 in F&R.

have $\pi_G > \pi_B$. The risky project will have a higher gross return than the good project, but will succeed less often. The condition $\pi_G G > 1 > \pi_B B$, says that only good projects, with a positive expected net present value, should be implemented. It is assumed that only the **success** of the project (a positive return, but not the size of the return, and hence not the choice of project) can be verified by a third party. For some promised amount being repaid if success, R, expected net return to the firm from undertaking a good project is $\pi_G \cdot \left[G - R\right]$ and the one for the bad project is $\pi_B \cdot \left[B - R\right]$. There is no repayment if failure. We can then draw these project-specific firm-payoffs as function of *R*:



As seen from the diagram, there exists a critical repayment value, $\hat{R}$, so that for any payment below this value, the firm will choose the G-project, which has the highest expected return to the borrower. This net return is more heavily affected by an increase in *R*; larger slope. Above the critical level, the firm will choose the B-project. This critical value obeys:

$$\pi_G \cdot \left[G - \hat{R}\right] = \pi_B \cdot \left[B - \hat{R}\right] \Leftrightarrow \hat{R} = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B}$$

The lender's gross return will then be: If the good project is undertaken the expected gross return is, $\pi_G R$, and $\pi_B R$ if the bad project is chosen. For $R < \hat{R}$, the firm will choose the good project; hence for $R < \hat{R}$, the expected net return for the lender is $\pi_G R - 1$. For a higher amount to be repaid, if success, the firm will choose the B-option, with a net return to the lender as given by $\pi_B R - 1$. Because the lender will correctly anticipate that the firm will make an optimal response to any value of R, the *G*-project will be chosen (preferred to project B) if $R < \hat{R} := \dfrac{\pi_G G - \pi_B B}{\pi_G - \pi_B}$; which can be interpreted as *G* being the best response by the investor if *R* is below the critical value $\hat{R}$. For any value of *R* above this critical value, the best response is the *B*-project. Hence, the lender can then correctly anticipate the best-response function for the borrower, as given by $\pi(R)$, which is here the implied probability for success, contingent on the size of the required repayment:

$$\pi(R) = \begin{cases} \pi_G \ if \ R \leq \hat{R} \\ \pi_B \ if \ R > \hat{R} \end{cases}$$

A competitive credit market equilibrium with **no** monitoring (no banks operating) has to obey the condition: $\pi(R)R = 1$; i.e. the expected net return to lenders from a repayment scheme *R* and the derived best response, is set equal to the required cost of undertaking the project. If the condition is not satisfied, we cannot have equilibrium. (If $\pi(R)R > 1$, net expected return to a lender is positive, and entry of new lenders will take place. On the other hand, if $\pi(R)R < 1$, lenders will exit. The only situation characterized by no entry/exit is the one with $\pi(R)R = 1$.)

Because we have $1 > \pi_B B \geq \pi_B R$ for any $R \leq B$; a credit market equilibrium will be possible only if the G-project is undertaken. Hence; for the borrower to choose G, we must have $R < \hat{R}$.
Now, for trade to take place in equilibrium, both parties have to benefit from trade: The lenders must not lose; hence their net expected return must be non-negative; i.e., $\pi_G R - 1 \geq 0$, and also, the borrowers must benefit; hence $\pi_G (G - R) \geq 0$. In equilibrium, with $\pi_B B - 1 < 0$, we therefore have to have $R < \hat{R}$ for the *G*-project to be chosen. Hence, $\pi_G \hat{R} - 1 > \pi_G R - 1 \geq 0$ must be satisfied.
We then have to check under what circumstances $\pi_G \hat{R} \geq 1$ will hold. The claim is that this will hold when "*moral hazard is not too important*"; cf. F&R, p. 35.
Observe that $\frac{\partial \hat{R}}{\partial \pi_B} = \frac{\pi_G (G - B)}{(\pi_G - \pi_B)^2} < 0$ as $G < B$ and $\pi_G > \pi_B$. When $\pi_B \downarrow 0$, then $\hat{R}$ will increase and approach *G*; $\hat{R} \uparrow G$; hence we must have $\pi_G \hat{R} \uparrow \pi_G G > 1$, by assumption. As the probability for success when choosing the *B*-lottery almost vanishes, moral hazard is not an issue. In that case, we can have market equilibrium with direct finance or external (public) debt!

If on the other hand, $\pi_G \hat{R} - 1 < 0$, which is the case if moral hazard matters, no trade will take place in equilibrium. The credit market with direct or public debt collapses, as good projects cannot be financed because no lender will grant a loan due to the fact that if moral hazard matters a lot, we have: $0 > \pi_G \hat{R} - 1 > \pi_G R - 1$ for $R < \hat{R}$, and bad projects are unprofitable, by assumption.

What is the role for a banking industry in this economy? Introduce a specialized banking sector with access to a costly monitoring technology such that by incurring a cost *C*, a bank is able to prevent the borrower to undertake the risky or bad project. It is a prevention cost – "sitting on the borrower" will prevent or deter him from taking a bad choice (from the lender's point of view) or to choose a too risky project. By spending resources or by paying *C*, the bank can get the borrower to choose the good project, *if* that is profitable.
In a competitive equilibrium, with monitoring, expected profits have to be zero, or that $\pi_G R^* - 1 - C = 0$ in this case, where R^* is the value of the bank loan or repayment in equilibrium.

For bank finance or bank lending to take place in equilibrium, two conditions have to be satisfied, given that $R^* = \dfrac{1+C}{\pi_G}$:

- Borrowers must have an expected gain; $\pi_G(G - R^*) > 0 \Leftrightarrow \pi_G G > \pi_G R^* = 1 + C$ or $\pi_G G - 1 > C$. (Monitoring cost cannot exceed the net present value of a good project.)
- Direct lending cannot be profitable; so we must have $\pi_G \hat{R} < 1$; hence, moral hazard has to play a role!

We can then conclude: We will have bank lending in a credit market equilibrium if $\pi_G \in \left[\dfrac{1+C}{G}, \dfrac{1}{\hat{R}}\right]$, or when moral hazard plays some role and when $\dfrac{1+C}{G} < \dfrac{1}{\hat{R}}$. If that is the case, we can identify three equilibrium regimes:

a) If $\pi_G > \dfrac{1}{\hat{R}}$, we have direct lending because moral hazard now is not very important; the firm issues direct debt with repayment in equilibrium as given by R_1 so that $\pi_G R_1 = 1$ or $R_1 = \dfrac{1}{\pi_G}$. *G*-projects are undertaken.

b) If $\pi_G \in \left[\dfrac{1+C}{G}, \dfrac{1}{\hat{R}}\right]$, bank lending will take place, as firms borrow from banks at a repayment rate R_2 obeying the equilibrium condition: $\pi_G R_2 = 1 + C$; hence $R_2 = \dfrac{1+C}{\pi_G} > R_1$. Bank debt is more expensive than direct lending.

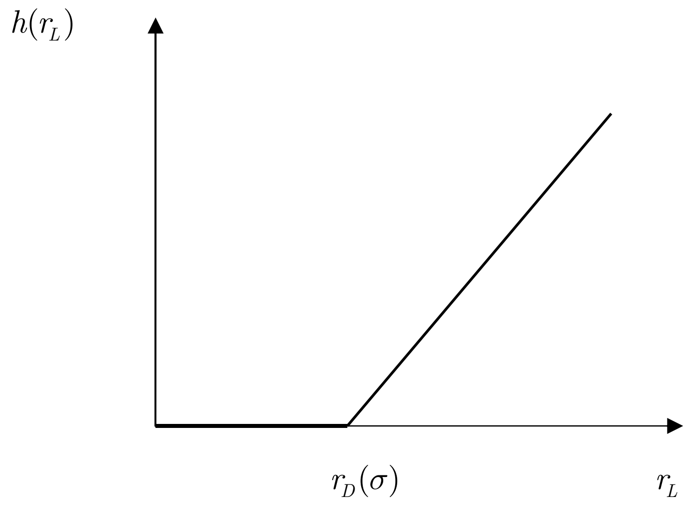
c) If the probability for success from undertaking a good project is low; i.e., $\pi_G < \dfrac{1+C}{G}$; then $\pi_G G < 1$. In equilibrium there is no trade and the credit market collapses. Projects are not implemented at all because no lenders will grant loans.

2. I find it necessary to correct some confusion (produced by me) regarding the presentation of the model in section 3.5.1 in F&R. The issue that was under consideration was:

- **Competition and risk-taking**. Will more competition in the banking industry induce banks to take riskier actions or to increase the riskiness on their loans? If that is the case, we should, perhaps, impose regulations so as to combat or fight fierce competition in the banking sector?

This problem is analyzed within a model where depositors are not insured, there is no bank capital, and banks can choose the riskiness on their loan portfolio, when riskiness is measured by the parameter σ. The bank's return on loans granted to firms will then depend on the ability of the borrowers to repay. Because a borrower will default with some probability, the gross return to the bank is a random variable. All actors involved are aware of the fact that the bank will default or fail if the realized rate of return on its loans, r_L falls below the required rate of return on deposits, denoted $r_D(\sigma)$. Any realized rate of return below $r_D(\sigma)$ will accrue to depositors because of their superiority; hence the bank's return in that case will be zero. (It is of course a difference between a situation where depositors can observe the true value of σ (transparency) or when they have to make some rational conjecture about the riskiness chosen by the bank.) The balance sheet is simply $L = D$, where L is the amount of loans, and D is the amount of deposits. It is assumed that the required rate of return on deposits depends on the riskiness of the bank's loan portfolio. (We may find it reasonable that depositors will require a higher rate of return on their deposits if they knew that the bank had chosen a very risky loan portfolio.) Because gross return is random, the probability for the bank to default is $\Pr(r_L < r_D(\sigma)) := 1 - p(\sigma)$, with $p(\sigma) = \Pr(r_L > r_D(\sigma))$ as the success probability, expected to be declining in σ. The value that is lost if default is the long-run value of the bank, called the charter value; denoted V. In the model the degree of competitive pressure in the banking industry is inversely related to the size of V. At one extreme – fierce competition – it is assumed that $V \approx 0$, and V is higher for an imperfectly competitive industry.

Define the function $h(r_L) = \max\left[0, r_L - r_D(\sigma) | \sigma\right]$, showing the random net return to the bank, for some given σ. This function is convex as shown by the bold curve below, with r_L measured along the horizontal axis (getting rid of some confusion, I hope), and $h(r_L)$ along the vertical axis, defined for some given value of σ:



(Even though the bank is assumed to be risk neutral, limited liability will, due to convexity of $h(r_L)$, induce the bank to act almost as a risk-lover.)

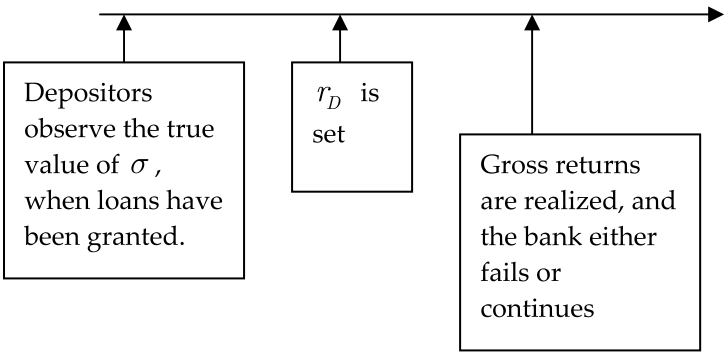
One way of formalizing that "a random variable becomes more risky than another random variable", is: We translate this notion into the concept of greater risk in the sense of a *Mean-Preserving Spread*. This is done by having a higher value of σ while keeping the expected gross return $E[r_L | \sigma]$ constant. (Restricting attention to this kind of increase in risk may be justified by imagining that even though the bank can choose the riskiness on its loan portfolio, the expected gross return from investment projects in the economy is given and is not affected by the bank's choice of loan portfolio.)
If we change the probability distribution for r_L by displacing or taking away weight from the centre of the distribution to the tails of the distribution so that the dispersion in the distribution increases, while keeping the expected return constant; $E[r_L | \sigma] \equiv \mu$ for any value of σ, then we say that *the increase in σ is a mean-preserving increase in risk*.[3]
The important property to be used is that if the increase in σ is a mean-preserving increase in risk (so that $E[r_L | \sigma] \equiv \mu$ is kept constant) then, because $h(r_L)$ is convex, as illustrated above, $E\{h(r_L) | \sigma\}$ will be increasing in σ. Let this expected value be the expected current (or short-run) profit per unit of *D*, written as $\frac{\pi(\sigma)}{D}$. The expected long-term gain from not defaulting, is $p(\sigma)V$, which is declining in σ. In general there is an intertemporal trade-off as to the riskiness on the loan portfolio, which is settled by the bank choosing σ so as to maximize its long-run objective $\{\pi(\sigma) + p(\sigma)V\}$ subject to a participation constraint among the depositors.

[3] See Stiglitz & Weiss (1981); p. 395 for more details.

We will consider the solution to this problem in two informational settings:

- Perfect information: The depositors have observed the choice of σ when they put forward their requirement on the rate of interest on deposits. This is the transparent banking regime.
- Imperfect information: The depositors have to make a rational conjecture as to the bank's choice of riskiness.

Consider first perfect information. In that case we have the timing of actions as illustrated by the time line below:



When the depositors require a rate of interest on their deposits, they know in the present case the true riskiness on the bank's loan portfolio, and will require a rate of interest so that their expected rate of return is equal to (or above) zero (risk-free rate of interest is set equal to zero), knowing that the bank will default or fail if the realized return on its loans falls below the required deposit rate; i.e. if $r_L < r_D(\sigma)$. Because deposits are superior, they will get whatever gross return accrues to the bank when this falls below $r_D(\sigma)$, but will receive their agreed-upon return when the bank does not fail. When having observed the true value of σ, the depositors will require that the following constraint holds: $E\{\min(r_L, r_D(\sigma)) | \sigma\} \geq 1$, when remembering that the risk-free interest rate is set at zero. In the current period, the bank's expected profit for some σ can be written as $\pi(\sigma) =_{(D=L)} D \cdot E\{Max(0, r_L - r_D(\sigma)) | \sigma\}$, with the long-run objective as given by $Max_\sigma \ \{W(\sigma) = \pi(\sigma) + p(\sigma)V\}$ subject to a participation constraint among depositors which holds as an equality in equilibrium; $E\{\min(r_L, r_D(\sigma)) | \sigma\} - 1 = 0$. Now we use that $\max[(0, r_L - r_D(\sigma)) | \sigma] + \min(r_L, r_D(\sigma)) | \sigma) = (r_L | \sigma)$. On taking expectations, we get that the first term becomes $\frac{\pi(\sigma)}{D}$ by definition, the second being equal to one in equilibrium, and the RHS is, by definition of the mean preserving

spread, equal to μ for any value of σ; hence we have $\frac{\pi(\sigma)}{D}+1\equiv\mu$ or $\pi(\sigma)\equiv(\mu-1)D$; independent of the value of σ.
The bank's long-run objective is then: $Max_{\sigma}\{(\mu-1)D+p(\sigma)V\}$.

If the success probability is declining in σ, the bank will, for any $V>0$, choose its riskiness as low as possible without losing its future value. The future value of doing business will discipline the bank not to choose a too risky loan portfolio. What is the intuition? One explanation might be that when the bank knows that the depositors are perfectly informed about the riskiness chosen at the stage when the depositors are requiring a rate of return, the bank's current (or short-run) profit will be a constant, independent of σ. The riskiness chosen for the loan portfolio will only affect the probability for success, and hence the long-run value. Perfect information along with a binding participation constraint for the depositors, will determine a unique value of $r_D(\sigma)$, leaving no short-run discretion for the bank. (In this case the bank will be "forced" or disciplined to choose the minimal level of risk.)

But what if depositors cannot observe the riskiness on the bank's loans? This is more interesting as the bank management then might be tempted to take too risky actions, causing perhaps financial instability or stress.

Suppose that the depositors' claim on their return r_D now must be based on a conjecture without knowing exactly the bank's true choice of σ. However, depositors are rationally anticipating that the bank will maximize long-term profits. If the depositors anticipate a risk level $\hat{\sigma}$, they require a deposit rate of interest as given by $E[\min(r_L,r_D(\hat{\sigma}))|\sigma]=1$.
The bank will choose σ to solve: $\sigma^*=\arg\max_{\sigma}\{\pi(\sigma)+p(\sigma)V\}$, taking into account the "best rational conjecture" of the depositors and given their binding participation constraint. In a rational expectations equilibrium, we must have $\sigma^*=\hat{\sigma}$. However, the important difference from previous case, is that $\pi(\sigma)=D\cdot E[\max(0,r_L-r_D(\hat{\sigma}))|\sigma]$ is a convex function, and will therefore be increasing in σ. Hence we get: If $V=0$, as with fierce competition, σ is set as high as possible; at its maximal level! In equilibrium this will be anticipated by depositors. If the bank has some market power, with $V>0$, there is a weaker incentive to choose too risky loans. In the present case, with none-transparency, there is an intertemporal trade-off; as $\pi(\sigma)$ is increasing and $p(\sigma)V$ declining in riskiness. This trade-off is highly influenced by the value of *V*, related to, among other things, the competitive pressure in the banking industry.

(One might wonder about the following policy issue: How is the bank's incentive to choose a risky loan portfolio affected by an ex ante commitment by the government to support or save banks that fail?)

3. One question that we did not have time to go into during my last lecture was: How is a bank's incentive to screen its loan applicants affected by competition? Will the banks let more risky or bad projects be financed if there is fierce competition?

One claim is that when firms can apply for loans in sequence from different banks, a rejection from one bank will produce an externality on other banks, as "subsequent" banks are left with "bad" loan applicants, with lower credit-worthiness. Banks offering low repayment rates will be able to attract the best projects, while leaving bad borrowers to the competitors; a typical adverse selection problem.
To get some idea about this issue consider a model where all actors are risk-neutral, interest rates are at zero, and there is a continuum of loan applicants, each applying for a loan to undertake some investment project of size one. The project, will produce a gross return y if success, and nothing if failure.[4]
In the economy we have good firms and bad firms, with a known fraction of good firms in the population as given by ψ, and a fraction of bad firms $(1-\psi)$. For a good firm, the probability for success is $p_G = \Pr(success|good) > p_B = \Pr(success|bad)$, with p_B being the probability of success for a bad firm. Assume that $p_G y > 1 > p_B y$, so it is never profitable to grant loan to a bad firm.

Suppose we have two banks in the economy; each bank will screen or test any applicant, and the decision to grant loan is based on the updated beliefs for success, using Bayes's rule. Because of imperfect screening, a bad firm that has passed the test may be offered a loan. However, the offer will normally depend on the firm's test record. We want to update the belief that the applicant is a good type conditional on passing the test, as given by the posterior belief $\psi(pass) := \Pr(good|test\ passed)$ which is the likelihood that the firm is good conditional on having passed the test. This updating of prior beliefs is done by using two sources of information: The quality of the test as given by the likelihood of passing the test given that the applicant is good, and the likelihood of passing the test, given that the applicant is bad. Then the marginal likelihood for passing the test can be calculated as:
$\Pr(pass) = \Pr(pass|good)\cdot\Pr(good) + \Pr(pass|bad)\cdot\Pr(bad)$. To calculate $\psi(pass)$ we use the relationship that the likelihood for *both passing and being good* is given by $\Pr(pass \cap good) = \Pr(pass|good)\cdot\Pr(good) = \Pr(good|pass)\cdot\Pr(pass)$.
From the last equality, we get the likelihood we want to derive, when using the marginal likelihood in the denominator:

[4] The argument is based on an article by T. Broecker in Econometrica (1990) – see F&R p. 122 for details.

$$\psi(pass) := \Pr(good|pass) = \frac{\Pr(pass|good)\cdot\Pr(good)}{\Pr(pass)}$$
$$= \frac{\Pr(pass|good)\cdot\Pr(good)}{\Pr(pass|good)\cdot\Pr(good) + \Pr(pass|bad)\cdot\Pr(bad)}$$
$$= \frac{\Pr(pass|good)\cdot\psi}{\Pr(pass|good)\cdot\psi + \Pr(pass|bad)\cdot(1-\psi)}$$

We also have:

$$1-\psi(pass) = \Pr(bad\,|\,test\ passed) = \frac{\Pr(pass|bad)\cdot(1-\psi)}{\Pr(pass|good)\cdot\psi + \Pr(pass|bad)\cdot(1-\psi)}$$

The quality of the test is reflected in a high value of $\Pr(pass|good)$ and a low value of $\Pr(pass|bad)$. If we have good tests, the likelihood of granting loans to bad firms will be low.

The population of loan applicants a bank is facing depends on the repayment offered by both banks. The bank offering the lower repayment rate R_L will face the entire population of borrowers. The average probability for repayment to this bank is then given by $p(1) = \psi(pass)\cdot p_G + (1-\psi(pass))\cdot p_B$

Suppose the other bank quotes $R_H > R_L$, and only applicants rejected by the first bank will apply for loan in the second bank. Consequently, the second bank will face a population of loan applicants with lower credit worthiness and lower average probability for repaying; this is the adverse selection effect. The bank offering the high repayment rate can now calculate:

$$\psi(pass|fail) = \Pr(good\,|\,first\ test\ failed \cap \mathrm{sec}\,ond\ passed)$$

in a manner as above, and then calculate the average probability for repayment to the second bank as given by $p(2) = \psi(pass|fail)p_G + (1-\psi(pass|fail))p_B$

As the second bank gets the worst applicants, we have

$$\psi(pass|fail) < \psi(pass) \Rightarrow p(2) < p(1)$$

This set-up can be used, as shown in the book, in section 3.5.3, in various types of games a one-stage game with simultaneous moves by the banks, and then a two-stage game without commitment, meaning that initial repayment first-stage offers can be withdrawn in the second stage.
What is perhaps more interesting for us, especially in the two-stage game, is how the number of banks will affect the probability of not repaying. Whereas traditional

theory says that more competition normally will be beneficial for consumers and for welfare, more banks or a more competitive banking industry will increase the likelihood for a bad firm to be granted a loan, at less favorable conditions, and so reduce the overall efficiency in the economy. If the test is good in screening bad firms, so that $\Pr(fail|bad)$ is high (close to one) and $\Pr(fail|good)$ is low, then we may expect that the number of good firms being granted loan will increase with the number of banks – shown in the book – hence, overall efficiency is improved.
If that is not the case, then more competition as given by an increasing number of banks may not be a good thing.