Jon Vislie; February, 2010

Problem set 1: Seminars – ECON 4335 Economics of Banking
Some tentative answers

Related to the issue discussed on the seminar I will present a simple, but a somewhat more general model illustrating moral hazard – more general than the one with only two admissible non-observable actions that can be taken by the agent after the loan has been granted. The principal (the lender) wants to induce the agent (the borrower) to take an action that is in the principal's interest. This is difficult because only the outcome (like success), not the action itself, can be verified. Under some circumstances, with limited liability and no equity, the agent/borrower can on his own be led to take a too risky action – from the lender's point of view – by implementing a project, with high return if success (and higher than what will be realized from the good or safe project if success), but given the risky choice or lottery, the success probability is lower.

Consider the following problem. Rather than having only two options or lotteries (good/safe or bad/risky project), we let the borrower be able to affect the probability of success by exerting costly effort. The probability of success, if an amount of effort e is exerted, is $q(e) \in \left(0,1\right)$, and the gross return from the project, if success, is denoted G. (If failure, the gross return is assumed to be zero.) We assume that $q(e)$ is strictly increasing (for finite amounts of effort), concave and that $\lim_{e\to\infty} q(e) < 1$, with $q(0) > 0$. Effort is continuous and has a cost as given by the disutility of exerting effort $v(e)$; strictly increasing and convex, with $v(0) = 0$. Because effort is hard to verify, incentives for exerting effort must be provided.

If effort could be verified, the lender would offer a contract so as to induce the borrower to choose effort meeting the lender's objective as well as the agent's participation constraint. This can be regarded as a **first-best problem**, where the repayment (original loan plus interest charge – a debt contract) conditional on success, is denoted R, has to obey the participation constraint, when the outside option of the borrower is set at zero, by assumption. The participation constraint is simply given by $q(e)\left[G - R\right] - v(e) \geq 0$, which is assumed to hold with equality under verifiable effort. We then have $R = G - \dfrac{v(e)}{q(e)} := R(e)$ which shows the repayment that will induce participation for any amount of effort. Using this in the principal's objective function, $qR - (1+r)$, as the difference between the expected repayment and the funding cost $(1+r)$; with r as the risk-free rate of interest, we get:
$q(e)R(e) - (1+r) = q(e)\left[G - \dfrac{v(e)}{q(e)}\right] - (1+r) = q(e)G - (1+r) - v(e)$. We observe that

the function to be maximized is identical to expected social surplus; hence the choice of effort to be induced in a first best should satisfy: $q'(e)G - v'(e) = 0$; the marginal increase in expected gross surplus should be equal to marginal disutility or cost of exerting effort. Let the first-best level of effort be given by e^* and the associated repayment made by the borrower if success; $R^* = R(e^*)$. As long as $q(0)G > 1 + r$ and $q'(0)G - v'(0) > 0$, the first-best optimal effort is positive implying a repayment obeying $R^* \in (0, G)$. The lender can get the borrower to implement the project by offering a contract with a repayment R^* if $e = e^*$, and some punishment (restricted by limited liability) for any effort choice different from e^*.

Let us turn to the **second-best problem** where effort cannot be verified; only success/failure can. The stages of the new game are: First the principal/lender offers a loan or debt contract; given by some repayment if success and nothing if failure. Then the firm either rejects the offer – and the game ends – or he accepts, and then choose effort optimally contingent on R. Thereafter one observes whether the project did succeed or fail, and the contract is executed.

The agent's optimal choice, contingent on some repayment offer R in case of success – and no repayment if failure – is the solution to the following optimization problem: $e(R) = \arg max_e \left\{ q(e)\left[G - R\right] - v(e) \right\}$, where the optimal choice must obey the first-order condition for a maximum of the agent's expected net return, as given by $(G - R)q'(e(R)) - v'(e(R)) = 0$. (Second-order condition for a maximum requires $(G - R)q''(e) - v''(e) < 0$, which is satisfied.) For the borrower to accept the loan contract the constraint $q(e)(G - R) - v(e) \geq 0$ must hold for the optimal choice $e(R)$. For a repayment so that the participation constraint holds, we can find directly how $e(R)$ will vary with R by differentiating the first-order condition, to give:

$$-q'(e) + \left[(G - R)q''(e) - v''(e)\right]\frac{de}{dR} = 0 \Rightarrow \frac{de}{dR} = \frac{q'(e)}{\left[(G - R)q''(e) - v''(e)\right]} < 0$$

The sign of $\frac{de}{dR}$ follows from the second-order condition, and that $q'(e) > 0$. The less of the gross return that is left to the borrower – the higher is R – the less effort will be exerted by the agent. This seems very reasonable as a higher R means that the difference $G - R$ is reduced, and the agent gets less in return from exerting effort if success.

This best response is rationally anticipated by the lender and used at the stage she is to make an offer as to the size of repayment (loan agreement), or how the expected gain is to be shared between the two parties. (Having to take this best response function as a constraint into the optimization programme makes the problem a second best problem.)

When offering a repayment or sharing scheme, the lender will choose *R*, so as to maximize her own net return, from providing a loan of unit size, when her unit cost of funding is $1+r$, when also taking into account the borrower's best response. Her optimization problem can therefore be expressed as: $Max_{R}\,\{q(e(R))R-(1+r)\}$, or maximizing the expected repayment or expected gross return $q(e(R))R$ as long as this maximized value exceeds the lender's funding cost. Hence, the amount to be repaid if success has to obey the following first-order condition:

$$q(e(R))+Rq'(e(R))\frac{de(R)}{dR}=0=q(e(R))+Rq'(e(R))\cdot\frac{q'(e)}{\left[(G-R)q''(e)-v''(e)\right]}$$

Call this second-best optimal payment $\hat{R}$, with an induced effort chosen by the borrower as given by $e(\hat{R})$. If the repayment is written as $1+\rho:=R$; then the optimal rate of interest on lending, from the banks' point of view, has been determined.

How will the second-best debt contract look like compared to the first-best contract? The optimal effort induced under observable effort satisfied the efficiency condition $q'(e^{*})G=v'(e^{*})$ with $R^{*}=G-\dfrac{v(e^{*})}{q(e^{*})}$ from the agent's binding participation constraint. Effort chosen by the agent when effort was unverifiable, contingent on some repayment R, denoted $e(R)$, obeyed $(G-R)q'(e(R))=v'(e(R))$.
Hence, $e(R)<e^{*}$ for any $R>0$; and also for $R=\hat{R}$. When the lender cannot observe the borrower's choice of effort, the loan agreement or debt contract will induce the borrower to exert less effort than what would have been induced in a first best optimum. The reason for this underprovision of effort is that in a first best, the investor, borrower or firm should reap the full marginal benefit of effort, as long as both parties are risk-neutral. Because the debt contract, given no equity and limited liability, specifies a repayment contingent on success, will make the borrower's expected marginal benefit from exerting effort smaller.

A "paradoxical" result is then: To get first best implemented, the borrower should get the loan free of charge; hence only if $R=0$, will get the firm to exert socially optimal effort e^{*}.

Problem 3.
Consider a financial contracting problem with a monopolistic lender – a bank – providing a borrowing firm with a loan. The loan is to be used to buy new equipment (with price set equal to one). Let the size of the loan be *k*. We let the amount being repaid after one period be denoted *t*. The bank's payoff is given by $V=t-(1+r)k$, when financing the loan is done in an international market at the given rate of interest *r*.

The borrower's payoff function is $U = sf(k) - t$, where $f(k)$ is a neoclassical production function; with $f'(k) > 0$, $f''(k) < 0$. The parameter s is private information (observed only by the firm) and can be interpreted as a type-parameter or as a productivity shock, with $s \in \{\underline{s}, \overline{s}\}$, and $\Pr(s = \underline{s}) = p$ for being a low-type, and $\Pr(s = \overline{s}) = 1 - p$ for being a high-type; $\overline{s} > \underline{s} > 0$. (This probability distribution is common knowledge.) Assume that the bank's objective is to maximize expected profits. However, the lender cannot distinguish between the two types. A borrower will not accept a financial contract offering her a negative payoff.

a) What would be the lender's optimal contract under complete and symmetric information; as given by a pair $\{t, k\}$, one for each type of the borrower?
b) When only the borrower knows his true type, as given by the value of s, show that if the contract from a) above should be offered, then a $\overline{s}$-firm would pretend to be a low-type firm.
c) How can the lender, by properly designing the set of contracts, induce the high-type to choose the contract designed for it?

a) If the lender was perfectly informed about the type of the borrower, she would offer a contract so that the borrower's participation constraint is satisfied:
The lender, in the role as principal, can choose $\{t, k\}$ so as to solve the following program:
$Max_{(t,k)} \left\{t - (1+r)k \middle| sf(k) - t = 0\right\} \Leftrightarrow Max_k \left\{sf(k) - (1+r)k\right\}$, with the first-best solution obeying $sf'(k^{FI}) = 1 + r$, with $k^{FI}(\underset{+}{s}, \underset{-}{r})$. The repayment $t^{FI} = sf(k^{FI})$. The size of the loan is higher the more productive (higher s) is the borrower, and also the lower is the bank's funding cost (r).

b) Because the most efficient type $(s = \overline{s})$, now can pretend to be less efficient; accepting the loan $\underline{k}^{FI}$ with a repayment schedule $\underline{t}^{FI} = \underline{s}f(\underline{k}^{FI})$, and by so making an **informational rent** as given by: $\overline{s}f(\underline{k}^{FI}) - \underline{s}f(\underline{k}^{FI}) := \Delta s \cdot f(\underline{k}^{FI}) := U(\overline{s}, \underline{s})$
where $\Delta s := \overline{s} - \underline{s} > 0$. By offering the set of first-best contracts under asymmetric information, we get *involuntary pooling*; **both types will prefer the contract designed for the inefficient type**. There is a social loss, and the credit market does not fulfil its goal to allocate credit (savings) in a socially efficient way. How to cope with this problem?

c) The trick is to get the efficient type to choose the loan intended for this $\overline{s}$-type. To accomplish that, the lender or bank has to offer a somewhat more sophisticated set of contracts; by not only relying on participation constraints, but also on *self-selection constraints* or incentive compatibility constraints.
A second-best optimal financial contract must therefore take into acc that it is never in the inefficient type's interest to pretend to be efficient, but only that the efficient type has an incentive to disguise himself.
The set of financial contracts $\left\{(\underline{t}, \underline{k}), (\overline{t}, \overline{k})\right\}$, one ("price-quantity") pair for each type, has to obey:

A binding participation constraint for the inefficient type:

$\underline{U} := U(\underline{s},\underline{s}) = \underline{s}f(\underline{k}) - \underline{t} \geq 0$ (binding in equilibrium – no point giving a rent to this type)

A binding self-selection constraint for the efficient type:

$\overline{U} := U(\overline{s},\overline{s}) = \overline{s}f(\overline{k}) - \overline{t} \geq U(\overline{s},\underline{s}) = \overline{s}f(\underline{k}) - \underline{t}$ (also binding in equilibrium; when equality, we assume the type to behave as desired; we could offer ε above the right hand side.) This constraint requires that the gain from acting in accordance with one's true (efficient) type should not fall below what the efficient type can gain by pretending to be inefficient.

The principal's program is therefore to solve:

$$Max\left\{p\left[\underline{t} - (1+r)\underline{k}\right] + (1-p)\left[\overline{t} - (1+r)\overline{k}\right]\right\}$$ s.t. the two constraints above.

Inserting for $\underline{t} = \underline{s}f(\underline{k})$ from the participation constraint and $\overline{t} = \overline{s}f(\overline{k}) - \overline{s}f(\underline{k}) + \underline{t} = \overline{s}f(\overline{k}) - \overline{s}f(\underline{k}) + \underline{s}f(\underline{k})$ from the self-selection constraint, we can write the objective function as a function only of the two loans $(\underline{k},\overline{k})$:

$$Max_{(\underline{k},\overline{k})}\left\{W(\underline{k},\overline{k}) := p\left[\underline{s}f(\underline{k}) - (1+r)\underline{k}\right] + (1-p)\left[\overline{s}f(\overline{k}) - \Delta s \cdot f(\underline{k}) - (1+r)\overline{k}\right]\right\}$$

The optimal financial contract must obey:

$\dfrac{\partial W}{\partial \overline{k}} = 0 \Leftrightarrow \overline{s}f'(\overline{k}) = 1 + r \quad \Leftrightarrow \overline{k} = \overline{k}^{FI}$; loan as under symmetric information

$\dfrac{\partial W}{\partial \underline{k}} = 0 \Leftrightarrow \underbrace{p\left[\underline{s}f'(\underline{k}) - (1+r)\right]}_{+}\underbrace{-(1-p)\cdot\Delta s\cdot f'(\underline{k})}_{-} = 0 \quad \Leftrightarrow \overline{k} < \underline{k}^{FI}$; loan below what would have been offered under symmetric information for the inefficient type. We can rewrite the last condition to become: $\left[\underline{s} - \dfrac{1-p}{p}\Delta s\right]f'(\underline{k}) = 1 + r$, when $\underline{s} - \dfrac{1-p}{p}\Delta s > 0$ by assumption. (What would happen if this is not the case?)

The induced distortion from first best is made to get the efficient type not to pretend to be inefficient; accomplished by lowering the loan to the inefficient type, so that the scope over which the efficient type can exploit its comparative advantage, as given by Δs, is reduced. The rent is therefore reduced, but the overall efficiency is increased, as well, as the efficient type now will take the loan designed for efficient investment for the $\overline{s}$ – type.