Alle formeler skal stå I matte og ikke tekst språk!

Seminar week 11

$U = sf(k) - t = 0$ $\qquad$ $t = sf(k)$

$V = t - (1 + r)k$ $\qquad$ $(1 + r)$ Interest rate is exogenous

Distinguish between Low-type $\underline{s}$ and High-type $\bar{s}$. Borrowers' payoff is driven to 0.

Probability for being Low-type $Pr(s = \underline{s}) = p$

Probability for being High- type $Pr(s = \bar{s}) = 1 - p$

1)

Bank problem

$U = sf(k) - t$

Profitt max bank extract all profit from firm, down to the firms reservation payoff. The firms problem is now.

$U = sf(k) - t = 0 \Rightarrow t = sf(k) - U$

Firm problem

$V = t - (1 + r)k$

The bank setts t and k to max expected payoff

$t = V + (1 + r)k$

$\bar{s} > \underline{s} > 0$

$\max_k t = sf(k)$ s.t $= v + (1 - r)k$

$sf'(k) = (1 + r) \rightarrow$ Tangent point = Optimal rent

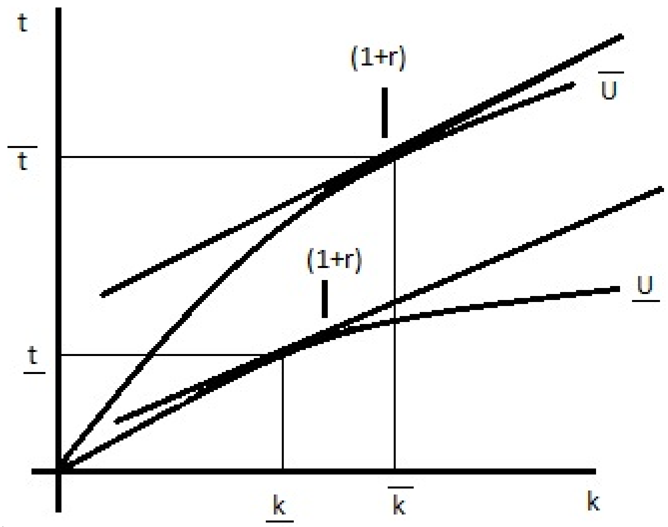
Good Firm

$\bar{s}f'(\bar{k}) = (1 + r)$

Bad Firm

$\underline{s}f'(\underline{k}) = (1 + r)$

With the rate given exogenous, the tangent has the same slope for the different firms



$\bar{s}f'(\bar{k}) = (1+r)$

$\underline{s}f'(\underline{k}) = (1+r)$

$\bar{s}f'(\bar{k}) = (1+r) = \underline{s}f'(\underline{k})$ ➔ $\bar{s}f'(\bar{k}) = \underline{s}f'(\underline{k})$

Because $\bar{s} > \underline{s} > 0$ ➔ $\bar{s} > \underline{s}$ ➔ $f'(\bar{k}) < f'(\underline{k})$ ➔ $\bar{k} > \underline{k}$

$\bar{V} = \bar{t} - (1+r)\bar{k}$ Payoff from lending to $\bar{s}$-firm

$\underline{V} = \underline{t} - (1+r)\underline{k}$ Payoff from lending to $\underline{s}$-firm

This gives

$\bar{k} > \underline{k}$

$\bar{t} > \underline{t}$

The firm in $\bar{s}$ Borrows more then the firm in $\underline{s}$, the bank extracts all profit from firms by setting maximum possible t, the $\bar{t}$ is set higher for firms in s(over) then $\underline{t}$ (under)

2)

The bank offers two contracts

$\bar{U} = \bar{s}f(\bar{k}) - \bar{t} \rightarrow [\bar{t}, \bar{k}]$

$\underline{U} = \underline{s}f(\underline{k}) - \underline{t} \rightarrow [\underline{t}, \underline{k}]$

The state, s, is now private, and the firms can now choose between $[\underline{t}, \underline{k}]$ and $[\bar{t}, \bar{k}]$

If $\bar{s}$-firm choose contract $[\bar{t}, \bar{k}]$, Firm payoff is now $\bar{U} = \bar{s}f(\bar{k}) - \bar{t}$ Banks optimization gives

$\bar{U} = 0$

If $\bar{s}$-firm choose contract $[\underline{t}, \underline{k}]$ Firm payoff is now $\hat{U} = \underline{t} - \bar{s}f(\underline{k})$.

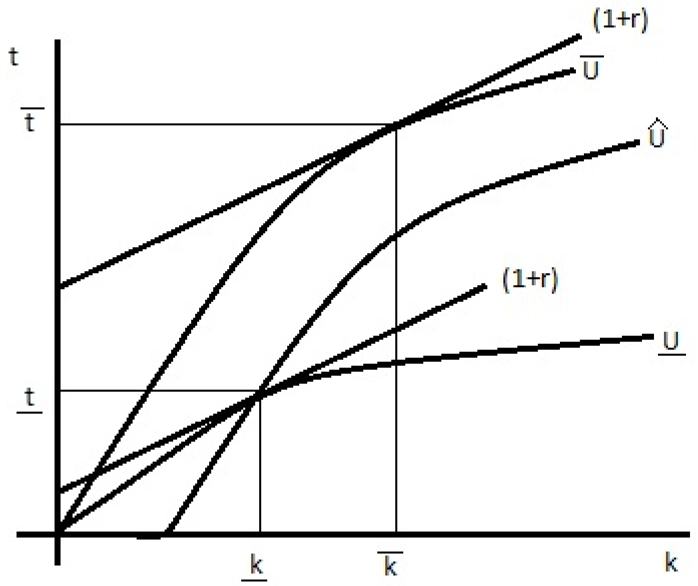
Since $\bar{s} > \underline{s}$ ➔ $\bar{s}f(\underline{k}) > \underline{s}f(\underline{k})$

The $\underline{s}$ gains higher income given the same production level.

This implies

$\underline{t} < \bar{s}\mathrm{f}(\underline{k})$ → $\hat{U} > 0$

$\bar{s}$ Firm gains positive payoff by taking contract $[\underline{t}, \underline{k}]$



3)

The banks problem under full information

$\max_{\bar{k},\underline{k}} p[\underline{t} - (1+r)\underline{k}] + (1-p)[\bar{t} + (1+r)\bar{k}]$

s.t $\bar{s}\mathrm{f}(\bar{k}) - \bar{t} \geq 0$ →$\bar{s}\mathrm{f}(\bar{k}) \geq \bar{t}$

s.t $\underline{s}\mathrm{f}(\underline{k}) - \underline{t} \geq 0$ → $\underline{s}\mathrm{f}(\underline{k}) \geq \underline{t}$

$\frac{\partial}{\partial \underline{k}} = 0$ → $\underline{s}\mathrm{f}'(\underline{k}) = (1+r)$

$\frac{\partial}{\partial \bar{k}} = 0$ → $\bar{s}\mathrm{f}(\bar{k}) = (1+\mathrm{r})$

The bank offers two contracts, but the $\bar{s}$-firm does not choose the $[\bar{t}, \bar{k}]$ contract. The bank wants to offer two contracts, one contract for the $\underline{s}$ firm and another for the $\bar{s}$ firm. The bank does this by incentivizing the $\bar{s}$-firm trough a incentive contract. The firm gets higher payoff by choosing $[\bar{t}, \bar{k}]$ and not $[\underline{t}, \underline{k}]$, For $\bar{s}$-firm this impies

$\bar{U} > \hat{U}$

$\bar{s}\mathrm{f}(\bar{k}) - \bar{t} \geq \bar{s}\mathrm{f}(\underline{k}) - \bar{t}$

From previous deloppgave $\bar{U} = \bar{s}\mathrm{f}(\bar{k}) - \bar{t}$, $\hat{U} = \bar{s}\mathrm{f}(\underline{k}) - \bar{t}$

$\bar{s}\text{f}(\bar{k}) - \bar{t} \geq \bar{s}\text{f}(\underline{k}) - \bar{t}$

$\max_{\bar{k},\underline{k}} p\big[\underline{t} - (1+r)\underline{k}\big) + (1-p)[\bar{t} - (1+r)\bar{k}]$

PC s.t $\underline{s}\text{f}(\underline{k}) - \underline{t} \geq 0$

IC s.t $\bar{s}\text{f}(\bar{k}) - \bar{t} \geq \bar{s}\text{f}(\underline{k}) - \underline{t}$

$p[\underline{t} - (1+r)\underline{k} + (1-p)[\bar{t} - (1+r)\bar{k}]$

s.t $\underline{s}\text{f}(\underline{k}) - \underline{t} \geq 0$ PC

s.t $\bar{s}f(\bar{k}) - \bar{t} \geq \bar{s}f(\underline{k}) - \underline{t}$ IC

$\frac{\partial}{\partial \bar{k}} = 0 \rightarrow \bar{s}f'(\bar{k}) = (1+r)$

$\bar{s}$-contract is unchanged.

$$\frac{\partial}{\partial \underline{k}} = 0$$

$\rightarrow\ p\big[\underline{s}f'(\underline{k}) - (1+r)\big] + (1-p)\big[-\bar{s}f'(\underline{k}) - \underline{s}f'(\underline{k})\big]$

$p\underline{s}f'(\underline{k}) = p(1+r) + (1-p)(\bar{s} - \underline{s})f'(\underline{k})$

$$\underline{s}f'(\underline{k}) = (1+r)\frac{(1-p)}{p}(\bar{s} - \underline{s})f'(\underline{k})$$

$\underline{s}f'(\underline{k}) = (1+r) + \frac{(1-p)}{p}(\bar{s} - \underline{s})f'(\underline{k}) = (1+r) + j$ $\rightarrow$Optimal to offer $[\underline{t}, \underline{k}]$ contract with a stepper slope. Bank offers $\underline{s}$-firm a contract with higher interest rate than to frighten $\bar{s}$-firm from wanting the $\underline{s}$-contract.

Why do we have banks

Nail Ferguson writes in his book “The Ascent of Money” the historical reasons for establishing the banking institution. In general they existed because they were the safest place to store wealth in the form of gold. In a modern economy banks have gained different functions, and thus have different reasons to exist.

Consumption smoothing and cross consumer insurance

In a realistic world, there may exist times when the consumer has a mismatch between the income/endowment and the desired intertemporal consumption path. This time uncertainty problem can be solved by pooling the wealth of many consumers with different desired consumption paths in one bank, and withdrawing the allotted share when needed. The Bank is then delegated the responsibility of ensuring that the sum of the withdrawals is not bigger than the sum of the deposits. The bank functions as an intermediary to guarantee that the different consumers function as insurers to each other.

The function of a cross consumer insurer relies heavily on the consumers trusting the intermediaries with their wealth. In the case the depositors fear that the bank is unable to guarantee the expected withdrawals (the fear being justified or not), the bank runs the risk of a bank run. With the consumers loosing trust in the intermediaries, the consumers also lose the ability of consumption smoothing the banks offered.

Risk adjusted investment diversification

The main result extracted from the capital asset pricing model (CAPM) is that the optimal allocation of investment is gained when the portfolio is diversified, with the amount invested being weighted according to the risk of the asset. The time and cost associated with gaining the information to make an informed decision might lead to the consumer being discouraged to invest in different assets, or even taking high risk. By not participating in an investment, the consumer does not gain the highest consumption level possible.

With the depositors delegating investment decision to the bank, the bank can act as an agent to the depositors. With increasing numbers of n-depositors the cost of an investment c, is diluted to $\frac{c}{n}$, this can lead to higher expected payoff per depositors. The depositors can in theory gain lower individual investment cost and better informed decision (lower risk =more certain consumption). It’s important to note that “informed decision” might be a glorified definition of what some banks might be doing. This also opens up the agent-principle problem if there is a mismatch between the incentives of the depositors and the intermediaries.

Monitoring investment

Lending with the presence of limited liability raises the problem of the borrower reporting the payoff of an investment to be lower than the initial capital that was lent. The low reported payoff can arise, either from misreporting the investment payoff so that the borrower gains private profit, or by the fact that the borrower does not actually do the job the borrower has lent money to do. Either way, the investors expected return is reduced and so is the consumption level.

By monitoring the investment, the lender can insure that the lender is not shirking on the project. But this monitoring comes at a cost. In the presence of multiple lenders there might be a free-rider problem arising from uncoordinated monitoring, or the replication of the monitoring leading to reduced payoff for all the investors. The project might also be a complex one, where technical information is needed to monitor. Again the problem of cost pooling arises. By coordinating monitoring, through the intermediary, the investors can avoid free-riders and reduce the total cost for monitoring. Again the delegation of responsibility can lead to the agent-principle problem as stated above.

Do we need banks?

The bank in itself does not produce wealth; it functions as a middleman that reallocates resources to improve efficiency in the economy. The intermediaries function today is more than keeping the gold secure in a safe. The increased functions of a bank come from different solution the banks offer, as stated above. It is because the multiple functions we have banks. But it is also the same reasons that a bankruptcy (or even the possibility) of a bank?, has a more severe effect on the economy then a non- financial firm, that it needs to be well regulated.