

All references are taken from the syllabus to this course.

## Part A

1) In this case, "fundamental value" means the discounted present value of payoff from an asset.

The discount rate is the opportunity cost of the investor. For example, consider an asset whose price at time  $t$  is given by

$$V_t = R^{-1}[d_{t+1} + E_t V_{t+1}]$$

Here,  $R^{-1}$  is the risk-free interest rate (the opportunity cost), and  $d_{t+1}$  is the future dividend of the asset. A fundamental solution for the value would be

$$V^* = \sum_{i=1}^{\infty} R^{-i} E_t d_{t+i}$$

Where  $V^*$  is the fundamental value of the asset. In a bubble, the value of the asset would be higher than this.

2) We have a rational bubble when investors know the fundamental value of an asset, and are fully aware that they are buying into a bubble. So the only reason they invest in the bubble, is that they believe it will get even bigger.

There are several conditions that are necessary for a rational bubble to occur. First of all, there has to be infinitely many investors that the bubble can be passed on to. In addition, the expected return must be at least equal to the risk-free interest rate, otherwise, no-one would invest in the bubble. In fact, the expected growth rate of the bubble must be equal to the interest rate. If below, everyone would borrow and buy into the bubble, if below, no-one would want to invest in the bubble. Neither can the investable funds that make up the bubble grow faster than the economy forever. If that were the case, the next generation would eventually not be able to buy into the bubble.

3) It is argued that some rational bubbles can reduce real investment by a crowding-out effect. Since some assets are priced higher than their fundamental value, some portion of savings will be spent on buying into the bubble, instead of real investment. Thus the capital stock is reduced.

4) Because of the bubble, future firm prices are high, which leads to an increase in the amount of credit available to firms. This again leads to more efficient investments, and help offset the crowding out-effect from question 3. More and more small bubbles pop up, and we have an expansionary effect. This is interesting because we might actually get a stationary equilibrium with a bubble. The bubbles can start out small, and then grow for a long period of time. This model with expansionary bubbles could also be used to explain crisis that occur without any particular technological shock. For an historical example of this, the recent financial crisis is probably a good example of an expansionary bubble bursting (Martin and Ventura 2011).

5) Bubbles due to extrapolation would not be distinguishable from the ones due to rational expectations, as in both cases, we would see an increase in investment and credit growth, as investors expect the bubble to keep on growing. It would be more revealing to look at consumption, for instance. If we have an extrapolation of trends, a small one-time positive shock to dividends would lead to an immediate increase in consumption, as the consumers expect the trend to continue. However, as this is not the case, consumption would fall back, below the original level (to make up for the extra consumption). In a rational bubble, consumers will not react in this way.

6) The bubble and expectations theories have quite a different take on deregulation. If we have a rational bubble, deregulation could be a good thing, as the bubble could lead to higher investment, although there might be negative effects if the bubble bursts. With expectation theory however, regulation might be a necessary tool used in order to keep the extrapolation of trends in check. In this case, imposing some constraints on banks might reduce the risk of this happening.

## Part B

1) Since the sum of all probabilities are one, the probability of one mortgage being repaid must be

$$\Pr(\text{one loan repaid}) = 1 - \Pr(\text{both default}) - \Pr(\text{both repaid})$$

If we set the  $\Pr(\text{one loan repaid}) = q$ , we get

$$q = 1 - p^2r - (1-p)^2r$$

So we simply subtract the chances of both or none of the loans being repaid, and we have the probability we seek.

2) For the senior class asset, the expected return is given by:

$$E_s = q*1 + \Pr(\text{both repaid})*1 + 0*\Pr(\text{both default}) = 1 - p^2r - (1-p)^2r + p^2r = 1 - (1-p)^2r$$

And for the junior asset:

$$E_j = p^2r*1 = p^2r$$

We can now calculate the expected returns for the senior and junior asset for  $p$  taking the values 0.8 and 0.9, and  $r$  taking the values 1, 1.1, and 1.2.

For the senior asset:

P	r	Expected return = $1 - (1-p)^2r$
0,8	1	0,96
0,8	1,1	0,956
0,8	1,2	0,952
0,9	1	0,99
0,9	1,1	0,989
0,9	1,2	0,988

And likewise for the junior asset:

P	r	Expected return = $p^2r$
0,8	1	0,64
0,8	1,1	0,704
0,8	1,2	0,768
0,9	1	0,81
0,9	1,1	0,891
0,9	1,2	0,972

Let us start by noting the obvious: That the expected return is higher and varies much less with the parameters for the senior asset. Changes in P seems to have the biggest effect on the expected return. Not too surprisingly, only needing one mortgage repaid reduces the risk significantly.

3) First of all, it seems obvious that a mistake in the estimation of the parameters will have the biggest impact on the junior asset, as the expected return varies the most with changes in the parameters. We see that the effect of an error in estimating p would have a bigger impact than an error in estimating r, as we have a bigger change in expected return when p changes than when r does.

4) Since we are told to ignore foreclosures in the case of default, the effect of falling housing prices will be somewhat mitigated. However, a fall in housing prices might make it more probable that homeowners default, as the reduced value of the house now means that the home owners might suffer losses if trying to sell the house. It might for instance no longer be viable to sell the house and buy something less expensive in order to pay the mortgage. In addition, there is of course the possibility that a fall in housing prices is correlated with a general economic downturn, making default more likely. As noted in 2) and 3) the junior asset is the most likely to suffer from this.

## Part C

The main reason we have a difference between the policy interest rate and the lending rates, is that banks incur costs when acting as intermediaries between savers and borrowers. However, the banks might not have an endless "supply of intermediation" (Woodford 2010), that is, borrowers might want the banks to induce more savers to lend to them, than the banks are able to provide. When the policy interest rate rises, this "supply" might suffer a negative shock, which might again lead to banks lending less, at a higher interest rate, which would negatively affect the economy and enhance the effect of the policy rate increase. A change in the policy rate could also affect the banks more indirectly. For instance, an increased policy rate could change attitudes regarding what degree of leverage it is acceptable for banks to have, letting them increase profits. Another effect could be if the increased rate forces the bank to sell off assets in order to get more capital, and suffers losses because of this. This effect will be further enhanced if, not unlikely, several banks are selling similar assets at the same time.