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Chapter 11

A CLASSICAL MODEL OF ECONOMIC GROWTH

I. INTRODUCTION

The purpose of the present paper is to analyse some of the fundamental questions raised by Adam Smith, David Ricardo and other classical economists within the framework of a model which I think is a fair, though simplified, representation of their thinking. I do not claim to have obtained any original or surprising conclusions; the analysis corroborates the logic of classical analysis. The advantage of the present approach is that conclusions which were stated in qualitative terms by classical economists can be given in terms of the parameters of the model.

Since I shall use only well known and often quoted chapters and paragraphs, mainly from Smith and Ricardo, I do not find it necessary to give many references to the literature.

The main questions on which we shall concentrate are the following two:

A. Is growth in the long run possible or is stagnation inevitable?
B. Will the wage rate remain above subsistence level in the long run?

These were central questions in classical thinking on economic development. We shall be less concerned with other questions which have a more special flavour of earlier centuries.

The basic assumptions in classical growth theory which we shall retain throughout are the following:

I. Employment is determined by the amount of capital.
II. Population growth depends upon the wage rate.
III. Current wage rate is high when labour is scarce, and low when labour is abundant.
IV. Savings and investments are determined by profits.

I think these assumptions—which will be given more precise forms below—are obligatory in an analysis of economic growth on classical premises.

With respect to returns to scale and technical progress, which are also important for questions A and B, we shall try some different assumptions.

2. THE BASIC MODEL

We shall proceed on the basis of a one-commodity model. I think this is permissible for analysing the main questions mentioned in the introduction, but of course it prevents us from analysing some other questions which are also prominent in classical writings on growth or development.

We shall use the following notations:

- X = total production per period of time;
- K = capital stock;
- N = total population;
- L = employed labour;
- w = wage rate in terms of goods; it corresponds with Ricardo's 'market price of labour';
- w^* = natural wage rate, corresponding to Ricardo's 'natural price of labour'; its meaning will be clarified below;
- π = total profits.

The natural wage rate w^* is assumed to be a given constant. The other symbols represent variables that are functions of time. When necessary we indicate this by $X(t)$, $K(t)$ etc., t being 'continuous time'.

We assume production and employment to be determined by capital by the following two equations:

$$X = f(K) \tag{1}$$

$$L = bK, \tag{2}$$

where f is a function and b is the (constant) labour/capital ratio. With constant returns to scale (1) is simply

$$X = aK, \tag{1'}$$

where a is a constant.

Equations (1) and (2) seem to imply capital having infinite life. However, we may let X be output net of depreciation, and the labour necessary for repair work to offset depreciation may be accounted for through b.

Equation (2) means that demand for labour is determined by capital stock. The momentary supply function for labour is written as

$$L = Ng(w-w^*), \tag{3}$$

where function g is the *per capita* supply function for labour. We assume the supply of labour to increase with the wage rate, i.e. $g' > 0$. (3) can be inverted so as to be written

$$w = w^* + G(L/N), \tag{4}$$

where we have introduced G for the inverse of the function g.

Here L/N measures the momentary scarcity of labour, and (4) gives the 'supply price' of labour. It represents Ricardo's expression: 'Labour is dear when it is scarce and cheap when it is plentiful.' (*Principles*, Chapter V.)

In the longer run labour supply is influenced by population growth, which is a function of the wage rate:

$$dN/dt = Nh(w-w^*). \tag{5}$$

We shall put

$$h(0) = 0, \quad \text{i.e. } dN/dt = 0 \quad \text{when } w = w^*, \tag{6}$$

which may be interpreted as an implicit definition of w^*; compare Ricardo's definition: 'The natural price of labour is that price which is necessary to enable the labourers, one with another, to subsist and to perpetuate their race, without either increase or diminution.' (*Principles*, Chapter V.)

We shall assume h to increase with $w-w^*$. The function $h(w-w^*)$ is obviously non-linear, but in some connections we shall employ the linear approximation

$$dN/dt = N\lambda(w-w^*) \quad (\lambda = h'(0)), \tag{7}$$

which will be valid when w is not too far away from w^*.

Profits are

$$\pi = X - wL. \tag{8}$$

In most cases we shall for simplicity assume that all profits are invested:

$$dK/dt = \pi. \tag{9}$$

However, we shall in some connections consider the effects of replacing (9) by

$$dK/dt = \alpha(\pi/K)\pi \tag{10}$$

where α, the ratio of profits invested, is assumed to be a function of the profit rate. It may be assumed that $\alpha(\pi/K) = 0$ for some critical value of π/K, and increases with π/K, approaching unity for large values of the profit rate.

Counting unknowns and equations, we now have six of both. A given initial capital stock $K(0)$ and population $N(0)$ is necessary to complete the system and to start it moving. Given $K(0)$, we see from (1) and (2) that $X(0)$ and $L(0)$ are determined. Then from (3) and the given $N(0)$, $w(0)$ is determined. Profits are determined by (8). The transition to 'the next moment of time' then follows from (5) and (9) or (10), and so the system moves.

3. THE CASE OF CONSTANT RETURNS TO SCALE

Let us first consider the case of constant returns to scale, i.e. the special case (1′) of (1). This may be approximately valid for a newly opened country, rich in natural resources, as exemplified by the classical economists by North America.

We define *a steady growth path* as one along which the wage rate w and the proportions between X, K, N, L and π remain constant, these latter variables growing at a common rate per period of time. Let $\bar{w}$ be the wage rate on such a path, and let the common growth rate for the other variables be r, i.e.

$$X(t) = X(0)\,e^{rt}, \quad K(t) = K(0)\,e^{rt}, \text{ etc.}$$

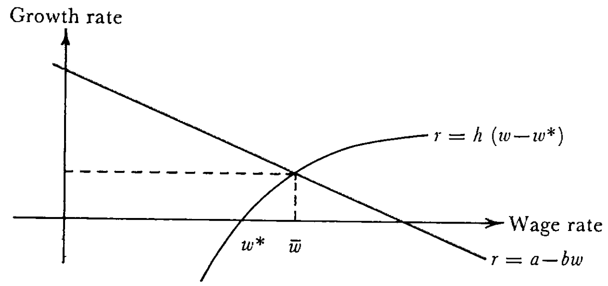

Fig. 1.

Now insert these assumptions in (1′), (2), (3), (5), (8) and (9). It is then easy to see that with appropriate proportions between the variables initially, a steady growth path will exist with a growth rate r and a wage rate $\bar{w}$ determined by the following two equations:

$$r = a - b\bar{w}, \tag{11}$$

$$r = h(\bar{w} - w^*). \tag{12}$$

The solution is shown in fig. 1.

It follows immediately that we have positive growth if and only if

$$a - bw^* > 0, \tag{13}$$

and furthermore, this condition also ensures a wage rate on the steady growth path permanently above the natural wage rate.

Condition (13) is easily interpreted: it simply says that production should yield a profit if labour were paid the natural wage rate. The difference $a - bw^*$ is simply the surplus, in terms of product, per unit of labour, above the natural wage rate.

With the linearized function (7) for the growth of population the solution of (11–12) is:

$$\bar{w} = \frac{a + \lambda w^*}{\lambda + b} = w^* + \frac{a - bw^*}{\lambda + b}, \tag{14}$$

$$r = \frac{a - bw^*}{1 + \dfrac{b}{\lambda}}. \tag{15}$$

Let us consider some classical conclusions in the light of these formulae. First observe that (14) and (15) imply

$$\bar{w} = w^* + \frac{r}{\lambda}, \tag{16}$$

i.e. for given w^* and λ the wage rate on the steady growth path is higher the higher is the growth rate. Compare this with Chapter VIII, Book I of *Wealth of Nations*: 'It is not the actual greatness of national wealth, but its continual increase, which occasions a rise in the wages of labour. It is not, accordingly, in the richest countries, but in the most thriving, or in those which are growing rich the fastest, that the wages of labour are highest.'† Furthermore, see Chapter V of Ricardo's *Principles*: 'Notwithstanding the tendency of wages to conform to their natural rate, their market rate may, in an improving society, for an indefinite period, be constantly above it; for no sooner may the impulse which an increased capital gives to a new demand for labour be obeyed, than another increase of capital may produce the same effect; and thus, if the increase of capital be gradual and constant, the demand for labour may give a continued stimulus to an increase of people.'

Next consider the role played by λ, which measures the effect of higher wages on population growth. A lower λ means a higher wage rate on the steady growth path, and a slower growth. The effect is of course that labour by making labour supply react more slowly to higher wages extracts a greater share of the surplus $(a - bw^*)$, and thereby also decreases accumulation.

It is also interesting to observe the effects of the natural wage rate w^*. This can be interpreted as a strictly defined subsistence wage rate, and then alternative levels are perhaps not interesting. But at least in Ricardo's work one finds the suggestion that w^* is not a subsistence wage rate in this sense, but rather a magnitude determined by 'habits and customs', and by sociological factors. Then one may ask: what is the effect upon

† It is usually assumed that the expression 'a rise in the wages' in the first period refers to a comparison between levels of wages rather than the rate of change through time. This would conform with (16).

the actual wage on a steady growth path of a higher natural wage rate? From (14) we see that

$$\frac{d\bar{w}}{dw^*} = \frac{\lambda}{\lambda+b}, \tag{17}$$

that means $\bar{w}$ increases with an increase in the natural wage rate, though only by a fraction of the increase in w^*. It is interesting, particularly in the light of later developments, to compare this with Ricardo's wishes expressed in Chapter V of *Principles*: 'The friends of humanity cannot but wish that in all countries the labouring classes should have a taste for comforts and enjoyments, and that they should be stimulated by all legal means in their exertions to procure them. There cannot be a better security against a super-abundant population.'

All conclusions drawn above could be drawn also for the case (11–12) without linearizing the function for the increase in population. A proportional shift in the function $h(\bar{w}-w^*)$ would correspond to a change in λ.

Replacing (9) by (10) we would still have a steady growth path in the model, with (13) as a condition for positive growth. Instead of (11) we would now get

$$r = \alpha(\pi/K)(a-b\bar{w}), \tag{18}$$

where the profit rate π/K is constant on the steady growth path. If α is independent of π/K, it is obvious from fig. 1 that a value of α less than unity would result in both a lower growth rate and a lower wage rate on the steady growth path. If α does in fact depend on π/K, (18) and (12) cannot be solved for r and $\bar{w}$ independently of the rest of the system, but it is nevertheless clear that the same conclusion holds if only $\alpha(\pi/K) < 1$.

All conclusions above concerning r and $\bar{w}$ are independent of the form $g(\ )$ of the momentary labour supply function. In fact, on the steady growth path this function only serves to determine the ratio L/N. If we study the stability of the steady growth path, this function is, however, important.

Let us imagine a starting point at $t = 0$ with arbitrary values of $N(0)$ and $K(0)$ which do not correspond to the proportions on the steady growth path. Let us assume that $N(0)$ is 'too big' in proportion to $K(0)$. Total employment would be $L(0) = bK(0)$, which inserted in (3) or (4) would yield a wage rate $w(0) < \bar{w}$. This would on the one hand make population growth slower than on the steady growth path, and on the other hand through higher profits give a faster accumulation. We would by this get an increasing value of L/N, and by (4) an increasing w. The same type of argument could be applied for an initial situation with 'too small' population in proportion to capital. It appears that there is a convergence to the steady growth path. Furthermore, it is clear that the momentary labour supply function, which does not influence $\bar{w}$ and r on

the steady growth path, does influence the wage rate and the growth rate when the economy is not on the steady growth path, and thereby also the speed with which it approaches this path.

4. DECREASING RETURNS TO SCALE

Clearly the classical economists believed that constant returns to scale would be prevalent only up to a certain level of production, after which one would encounter decreasing returns. Let us now analyse the working of the model for the general case (1) of the production function, where, at least for K above a certain level, average productivity $f(K)/K$ decreases with increasing K.

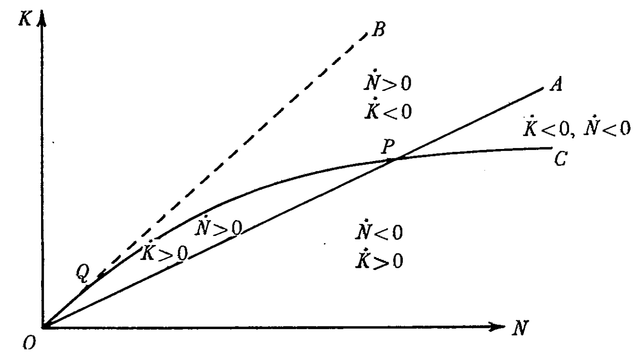

Fig. 2.

By insertion the model can now (using (9), not (10)), be reduced to the following two differential equations in N and K:

$$\frac{1}{K}\frac{dK}{dt} = \frac{f(K)}{K} - b\left[w^* + G\left(\frac{bK}{N}\right)\right], \tag{19}$$

$$\frac{1}{N}\frac{dN}{dt} = h\left(G\left(\frac{bK}{N}\right)\right). \tag{20}$$

We can draw some conclusions from this by means of fig. 2, in which N and K are measured along the two axes.

Consider first equation (20). Clearly $(1/N)(dN/dt)$ is constant for a constant ratio K/N. For a certain ratio K/N we will have $G(bK/N) = 0$, and accordingly $(1/N)(dN/dt) = 0$ (compare (6)). We can thus draw a beam OA from the origin in the diagram along which $(1/N)(dN/dt) = 0$. Furthermore, since both h and G are increasing functions, we will have $(1/N)(dN/dt) > 0$ above this line and $(1/N)(dN/dt) < 0$ below.

Next consider (19). If $f(K)/K$ were constant as in the previous section, $(1/K)(dK/dt)$ would be constant for a constant ratio K/N. We could then draw another beam OB through the origin along which $(1/K)(dK/dt) = 0$. Now $(1/K)(dK/dt)$ would be negative above this line and positive below. Evidently growth in both N and K would be possible if OB is above OA, and there would be a beam between OB and OA which would correspond to the steady growth path of the previous section. The condition that OB should be above OA is the same as condition (13).

However, we are now assuming that $f(K)/K$ will decrease with K, at least for values of K above a certain level. By implicit differentiation in the equation obtained by setting (19) equal to zero it is easy to verify that for points with such values of K the curve for $(1/K)(dK/dt) = 0$ will be less steep than a beam from the origin to the same point—i.e.

$$0 < (dK/dN) < K/N.$$

In fig. 2 we have drawn a curve OC for $(1/K)(dK/dt) = 0$ which first follows OB, representing constant returns to scale up to some value of K, and then (from Q) falls below OB as we encounter decreasing returns. $(1/K)(dK/dt)$ will be negative above OC and positive below OC.

In the diagram we have now four areas delimited by OA and OC, with different combinations of signs of $(1/N)(dN/dt)$ and $(1/K)(dK/dt)$. These are indicated by $\dot{N}$ and $\dot{K}$ being > 0 or < 0. P is evidently a stable equilibrium point, representing a stationary state. From any initial situation in the area $OQPO$ we will have continuous growth in both K and N, but the growth will slow down as we approach P. Asymptotically when $t \to \infty$ K and N will approach the values $\bar{K}$ and $\bar{N}$ which are obtained by setting both (19) and (20) equal to zero. These correspond to

$$\frac{f(\bar{K})}{\bar{K}} = bw^* \tag{21}$$

$$G\left(\frac{b\bar{K}}{\bar{N}}\right) = 0. \tag{22}$$

The first condition gives $\bar{K}$, which inserted in (22) gives $\bar{N}$.

Also from starting points outside the region $OQPO$ we would have convergence towards the same stationary state. From an economic point of view, situations above the curve for $\dot{K} = 0$ in the diagram are, however, implausible.

Condition (21) means that average capital productivity is just sufficient to pay the normal wage rate to the labour employed. Evidently $w = w^*$ in this stationary state.

During the growth process, i.e. when we are moving in the interior of $OQPO$, the wage rate will be above w^*. But it inevitably approaches w^* when we move towards P.

All this confirms the logic of the classical vision of an inevitable stationary state when there are decreasing returns to scale.

For completeness it should be mentioned that we may of course have a production function which does not obey the law of constant returns to scale, but nevertheless gives a curve OC which does not cross through OA. This is true when average productivity $f(K)/K$ decreases with K, but in such a way that it never reaches a value as low as bw^*.

Employing (10) instead of (9) will not change the reasoning about the approach to a stationary state in any essential way. It will, however, result in a stationary state at a lower level for N and K if $\alpha(\pi/K)$ reaches zero for a profit rate above zero. The same would be true if we introduced some marginalistic considerations into capitalists' investment decisions instead of the simple mechanism that they automatically invest the whole or a fraction of profits. As the formulae stand above, capitalists go on investing out of their profits even when this causes profits to be reduced. I think Ricardo is on some occasions reasoning *hypothetically* on the basis of such an assumption, but he hardly believed it to be realistic.

5. NEUTRAL TECHNICAL PROGRESS

In classical writings one can find some suggestions about technical progress postponing stagnation, perhaps for an infinite future. We shall consider two types of technical progress; first, in the present section, neutral technical progress, and thereafter labour-saving technical progress.

By neutral technical progress we shall simply mean positive shifts in the production function (1) which leave the labour/capital ratio b in (2) unaffected.

In the case of constant returns to scale a once-and-for-all technical change of this type would be represented by a positive shift in the constant a in (1′). It is seen from fig. 1 that this would shift the steady growth path to one with both a higher wage rate $\bar{w}$ and a higher growth rate r. The stability property of the steady growth path ensures that the actual development would gradually approach this new steady growth path after the shift in the productivity coefficient a.

Now consider technical progress going on continually according to

$$X(t) = a(t)K(t), \quad \dot{a} = \frac{da(t)}{dt} > 0, \tag{23}$$

where a is now an increasing function of time.

In this case we can reduce the system to the following two equations

$$\frac{1}{K}\frac{dK}{dt} = a - b\left[w^* + G\left(\frac{bK}{N}\right)\right] \tag{24}$$

$$\frac{1}{N}\frac{dN}{dt} = h\left(G\left(\frac{bK}{N}\right)\right). \tag{25}$$

Now steady growth in the sense defined in section 3 is no longer possible. This follows directly from (23). From the reasoning about the once-and-for-all change in productivity, it seems intuitively obvious that the growth rate of the economy and the wage rate will now be increasing in the long run.

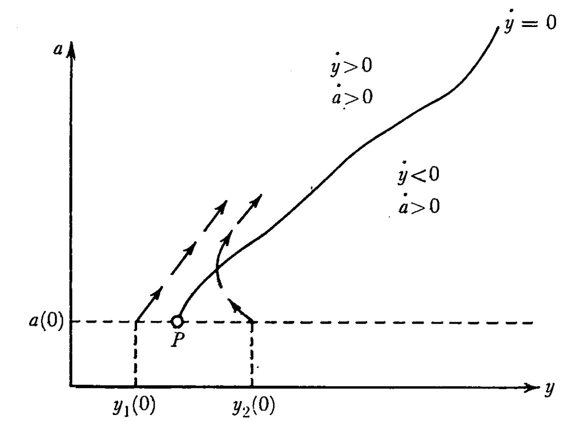

Fig. 3.

This can be more clearly seen by transforming (24) and (25) into a differential equation for capital per head, say y:

$$\frac{1}{y}\frac{dy}{dt} = a - [bw^* + bG(by) + h\{G(by)\}], \quad \text{where } y = K/N. \tag{26}$$

The development generated by (26) can be studied by means of fig. 3. In this diagram y is measured along the horizontal axis and a along the vertical axes. Any initial situation $a(0)$, $y(0)$ is represented by a point in this diagram. Since $a(t)$ is increasing with t we are only interested in the part of the diagram lying above $a(0)$, the initial value of the productivity coefficient.

Now for any value of y there is a certain value of a which would make $\dot{y} = 0$, obtained by setting the expression in (26) equal to zero. This value of a would increase with y; it is represented by the curve starting from P in the diagram. Above this curve $\dot{y} > 0$, and below it $\dot{y} < 0$.

Now assume that $\dot{a} > 0$ everywhere in the diagram and that $a(t)$ increases beyond any limit with increasing t. We can then distinguish two cases. First assume that $y(0) = y_1(0)$ to the left of P. Then clearly y would be increasing with t right from the beginning. Next assume that $y(0) = y_2(0)$ to the right of P. Then y will first decrease; but the point must inevitably pass through the curve for $\dot{y} = 0$, and from then y will increase. (Also for the special case where $y(0)$ corresponds to the point P, one would clearly move into the region of $\dot{y} > 0$.) In the long run therefore y will increase, whatever initial value we start from. When y increases, the wage rate will also increase since we have

$$w = w^* + G(by) \quad \text{with } G' > 0.$$

With the favourable combination of assumptions which we have now been considering conditions for the workers could thus improve continually.

A more interesting question is perhaps whether technical progress can, in the long run, offset the effects of decreasing returns to scale so as to prevent the eventual approach of wages to the normal wage level in this case. The answer will of course depend on the more precise assumptions made. Let us consider the case of a logarithmic linear production function with a technical progress term, i.e.

$$X = aK^{\mu}e^{\epsilon t}, \tag{27}$$

where μ is a constant, $0 < \mu < 1$, and ϵ is a positive constant.

By insertions we reduce the system to three equations in K, N and w:

$$\frac{1}{K}\frac{dK}{dt} = aK^{\mu-1}e^{\epsilon t} - bw, \tag{28}$$

$$\frac{1}{N}\frac{dN}{dt} = h(w - w^*), \tag{29}$$

$$w = w^* + G\left(b\frac{K}{N}\right). \tag{30}$$

We cannot in this case reduce the number of equations by introducing capital per head, y, as we did in (26) since K appears separately on the right hand side of (28).

Is steady growth now possible? Try again

$$\frac{1}{K}\frac{dK}{dt} = \frac{1}{N}\frac{dN}{dt} = r, \quad w = \overline{w}, \tag{31}$$

r and $\overline{w}$ being constant.

For this to satisfy (28) it is evident that the growth rate r must be such that the term $K^{\mu-1}e^{\epsilon t}$ remains constant through time, which requires

$$r = \frac{\epsilon}{1-\mu}. \tag{32}$$

The wage rate on the steady growth path is then, from (29),

$$\bar{w} = w^* + h^{-1}(r) = w^* + h^{-1}\left(\frac{\epsilon}{1-\mu}\right), \tag{33}$$

provided that the function h allows this solution, i.e. that there is a wage rate that makes population grow at the rate $\epsilon/(1-\mu)$.

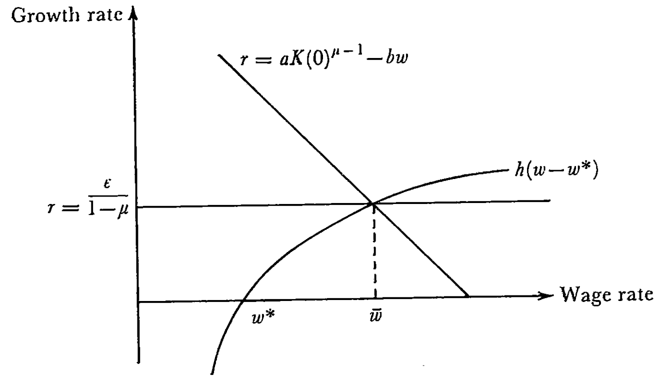

Fig. 4.

This steady growth requires a certain proportion between K and N as is seen from (30) when $\bar{w}$ is inserted from (33). Furthermore, it also requires that the economy starts at a certain level at $t = 0$. It is seen from equation (28) that $K(0)$ must satisfy

$$aK(0)^{\mu-1} - b\bar{w} = r, \tag{34}$$

where r and $\bar{w}$ are given by (32) and (33).

The whole situation is depicted in fig. 4, which corresponds to fig. 1 in the simpler situation with constant returns and no technical progress.

As is seen a solution for r and $\bar{w}$ will exist only if the function $h(w-w^*)$ reaches the level $\epsilon/(1-\mu)$ for some value of w.

The value of $K(0)$ is determined by the requirement that the straight line representing (34) shall pass through the solution point for r and $\bar{w}$. Such a value will always exist.

In contrast to the steady growth path studied in §3, we now have a steady growth path with a growth rate determined completely by parameters characterizing production conditions (provided that this is not so high that population cannot follow up). The wage rate on this steady growth path will be higher the faster is the growth, i.e. the higher is the rate of technical progress and the less marked is the decrease in returns.

Will this growth represent a stable growth path? I think a full mathematical treatment would prove that it does. Here we shall however be content with a more intuitive reasoning.

We assume initially that $K(0)/N(0)$ is lower than it would be on the steady growth path, which implies $w(0) < \bar{w}$. Furthermore we assume that $K(0)$ is smaller than it should be according to (34). Then it is seen from (28) that K would start growing at a higher rate, and from (29) that N would start growing at a lower rate than on the steady growth path. This would through (30) make for w to increase, which would again make $(1/K)(dK/dt)$ and $(1/N)(dN/dt)$ approach each other.

Alternatively, let us start with a ratio $K(0)/N(0)$ equal to what it should be on the steady growth path, and accordingly $w(0) = \bar{w}$, but with a lower initial capital stock than what we should have, according to (34), for steady growth. Then initially N would grow at a rate r, while K would grow at a higher rate. It is clear from (28) however that the growth rate for K would have to decline again, since with K growing at a rate higher than $r = \epsilon/(1-\mu)$, the term $K^{\mu-1}e^{\epsilon t}$ in (28) will decline. Also the movements of w would of course influence the development of K, and it would help to pull up the size of the population so as to approach the 'right' proportion again between K and N.

Such reasoning for different initial conditions seems to indicate that the steady growth path is indeed stable in the sense that we would approach such a growth path from an arbitrary initial situation.

The same conclusion can alternatively be obtained by trying to let K/N increase or decrease in the long run, both attempts leading to contradictions in the system (28–30).

The above discussion assumes that population can grow at a rate $\epsilon/(1-\mu)$. Let us briefly consider the case when $\epsilon/(1-\mu)$ is so high that population can not follow up. We then reasonably assume that population will in the long run grow at a maximal rate, say ν, so that

$$N(t) = N(0)\,e^{\nu t}. \tag{35}$$

Introducing now $y = K/N$ as we did in connection with (26), we get from (28) and (30):

$$\frac{1}{y}\frac{dy}{dt} = a[N(0)]^{\mu-1}y^{\mu-1}e^{[\epsilon+\nu(\mu-1)]t} - b[w^* + G(by)] - \nu. \tag{36}$$

In a similar way as for (26), it can be seen from (36) that y, and then also the wage rate, will increase in the long run on the assumptions now made.

6. LABOUR-SAVING TECHNICAL PROGRESS

This section is inspired by Ricardo's Chapter XXXI, '*On Machinery*'. The main problem raised there is whether 'such an application of machinery...as should have the effect of saving labour' could be 'injurious to the interests of the class of labourers.' And, as is well known, Ricardo's answer was affirmative.

From the standpoint of modern theory one would be inclined to investigate this question on the basis of a growth model with a shifting two-factor production function. However, I think we remain closer to classical assumptions by simply introducing the possibilities of 'labour saving' as shifts in the labour/capital ratio.

We shall do this within the framework of the model as used in § 3, i.e. we shall assume constant returns to scale. The interactions between labour-saving progress and the problems treated in §§ 4 and 5 would hardly give any further insight.

Let us first consider a once-and-for-all change in technology which can immediately be introduced in all production. In this we follow Ricardo: 'To elucidate the principle, I have been supposing that improved machinery is *suddenly* discovered and extensively used...'

We assume then that the economy has for a time been growing along the steady growth path, as explained in § 3, with technical coefficients a and b. Then suddenly the coefficients change to a' and b', with $b' < b$ so that the new technology implies less labour per unit of capital. We may have $a' > a$, $a' = a$ or $a' < a$; that means the introduction of new labour-saving technology may increase, leave unchanged or decrease the amount of production per unit of capital.

On the growth path before the switch in technology, the wage rate is $\overline{w}$, given by formula (14). With the sudden switch the wage rate must necessarily fall since the momentary wage rate is determined by (2) and (3), and N and K need time to change. Thus the immediate effect of the labour-saving switch in technology is detrimental to the wage-workers.

Because of the stability property of the steady growth path, the wage rate will however start moving towards a new level which will in general be different from $\overline{w}$. This new level will be

$$\overline{w}' = \frac{a' + \lambda w^*}{\lambda + b'}. \qquad (37)$$

(For simplicity we use the linearized form (7) of (5).)

This new level is higher than the wage rate on the previous steady growth path if and only if the following condition is fulfilled:

$$\lambda[w^*(b-b') + (a'-a)] + ba' - ab' > 0. \qquad (38)$$

This is always fulfilled if $a' \geqq a$. However, a more general and more interesting conclusion can be given.

Consider the profitability of the new technique. Being on the old steady growth path and considering the question of introducing the new technique (a', b') it is natural to evaluate the profitability of this technique by means of the current wage rate $\overline{w}$. Evaluated in this way the expected profit rate by the new technique will be

$$m' = a' - b'\overline{w} = a' - b'\frac{a + \lambda w^*}{\lambda + b}. \qquad (39)$$

Consider now the condition for this expected profit rate by the new technique to be higher than the profit rate on the old steady growth path, i.e. the condition for

$$a' - b'\overline{w} > a - b\overline{w}. \qquad (40)$$

This condition turns out to be equivalent to (38). Thus we have the conclusion: A labour-saving switch in technology causes an immediate fall in the wage rate, but in the long run the wage rate will increase towards a higher level than before the switch if the new technology is profitable. The effect is of course that, in spite of reduced demand for labour per unit of capital, the *rate of increase* of demand for labour will be higher after the switch because of faster accumulation.†

These results hold for the case of a once-and-for-all change in technology. If we now consider a never-ceasing flow of technological changes, the economy will evidently be chasing, but never reaching, a steadily moving target. The total effect of the technological changes on the wage level at a certain point of time, will then consist of two different tendencies: the positive long-term effects of labour-saving changes in a distant past, and the negative short-run effects of changes in the more immediate past. An interesting question is then: Is it possible that a continuous flow of labour-saving changes can produce a flow of negative short-run effects which will always dominate the positive long-run effects, i.e. that the latter will never catch up with the former?

We shall consider this question by assuming that $b = b(t)$ is a decreasing function of time, i.e.

$$\frac{db(t)}{dt} = \dot{b}(t) < 0, \qquad (41)$$

while a remains constant. In this case we are sure that new technology will always appear more profitable than old technology, and consequently be preferred by capitalists.

† It should be noted that if the technology (a', b') is more profitable than (a, b) evaluated by means of the wage rate $\overline{w}$, it is also more profitable when evaluated by means of the wage rate $\overline{w}'$ on the new steady growth path, i.e. $a' - b'\overline{w}' > a - b\overline{w}'$.

It is convenient in this case to consider again the differential equation governing the development of capital per head, $y = K/N$. The equation now appears as

$$\frac{1}{y}\frac{dy(t)}{dt} = a - [w^* + G(b(t)\,y(t))]b(t) - h[G(b(t)\,y(t))]. \qquad (42)$$

We can study the properties of this equation in fig. 5 where y is measured along the horizontal axis and b along the vertical axis.

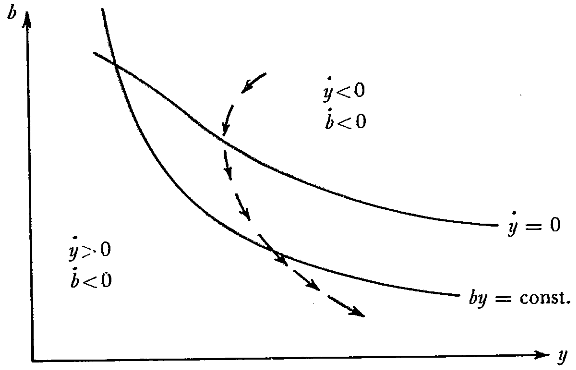

Fig. 5.

In this diagram we have drawn a curve representing b, y—points which give $\dot{y} = 0$. It is obtained by setting (42) equal to zero. This curve must evidently be falling with increasing y. From the way b and y enter the formula it is, furthermore, clear that the product by is increasing along this curve, which means that the curve in any point is less steep than the hyperbola $by =$ const. through the point.

Above this curve $\dot{y} < 0$, and below it $\dot{y} > 0$.

From any starting point $y(0), b(0)$, the economy will evidently enter upon a path of increasing y—after a temporary decline in y if $y(0), b(0)$ is above the curve for $\dot{y} = 0$.

What happens now to the wage rate? The wage rate is

$$w(t) = w^* + G(b(t)\,y(t)),$$

which means that it increases/decreases with increases/decreases of the product $b(t)\,y(t)$. The question then is whether the path followed by the y, b-point in the diagram will cut through hyperbolas $by =$ const. from above or from below. Apparently, it is possible that $\dot{b}$ is everywhere so big in absolute value that the y, b-point follows a path which cuts through these hyperbolas from above. Then $w(t)$ will be a decreasing function of time. This is the case indicated in the diagram.

Logically such a development could go on forever. It is thus theoretically possible that a continuous flow of labour-saving technological changes can produce a declining wage rate even in the long run.

Some of the arguments of this section could, I think, by changing the interpretation of the model, be related to the recent debate on choice of technique in economically underdeveloped countries. Furthermore, I think the arguments about the effects of a continuous flow of labour-saving technological changes could be related to some of Marx's ideas about the laws of development under capitalism. To pursue these matters would, however, be beyond the purpose of the present exposition.