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October 2011,
Lecture notes, ECON 4350

ENDOGENOUS GROWTH: Schumpeter's "process of creative destruction"

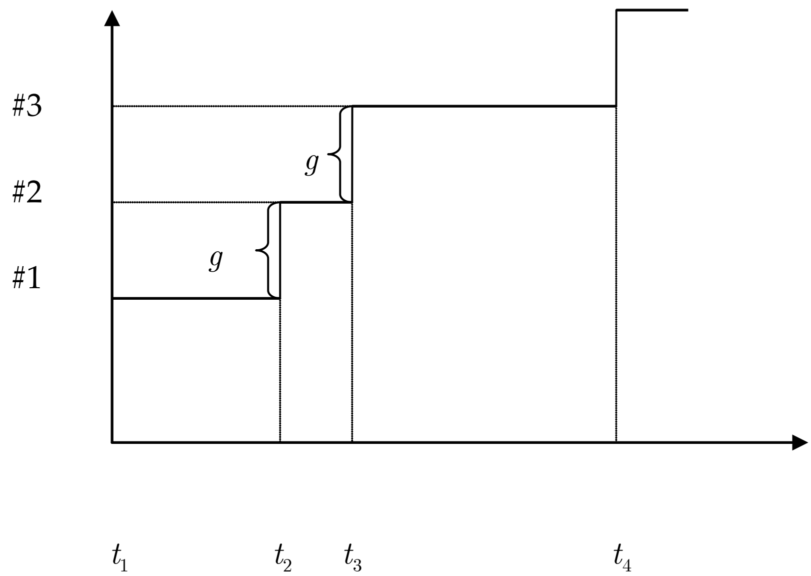
Joseph Schumpeter made early contributions with a permanent influence on our understanding of the role of R&D; process and product innovations, inventions, imitations, product development etc., issues we believe affect the technological development to a large extent.
The modern growth models building on Schumpeter's insight are highly endogenous in the sense that the growth rates emerging from these models are derived from basic decision principles.
We will present a simple version of Schumpeter's idea of "the process of creative destruction", based on the work by Aghion & Howitt "A Model of Growth Through Creative Destruction", Econometrica 1992 (March); pp. 323-351.

As in all models of economic growth following the traditional neoclassical set-up, the objective has been to understand the technological progress as an important engine to economic growth. Some models within the area of "endogenous economic growth" relied heavily on learning by doing, technological spillovers/externalities and increasing returns (external to the firms) due to the non-rivalry of knowledge. During the production and installation of new machines, one learns more how to operate them etc. The increased knowledge was seen as an unintended by-product of capital accumulation, and was not taken into account when the agents made their investment decisions. This "learning by investing" generated an externality, as more knowledge accrues to the agents free of charge, in a public good-fashion. The increased knowledge will be diffused perfectly in the economy; no property right is needed as this activity is not a deliberate part of the decision-making. (One way the diffusion can take place is through the mobility of key personal between firms within the same industry.)

The AK-model was an example or a special case where we impose some learning or technological spillovers. Knowledge is included in "human capital" that is treated as an ordinary capital object. We were able to derive, under some restrictive assumptions, a macro production function with overall increasing returns to scale and increasing marginal productivity of capital, with a growth rate that was increasing over time.

Within the area called "Schumpeterian growth" focus is more on how knowledge is created and used. Focus is on innovation, the incentives to innovate and the implications which are related to "the process of creative destruction"; also found in the putty-clay model of Leif Johansen with scrapping and investment in new plants. Schumpeter emphasised the role of competition in this process. The model has perhaps more obvious policy implications than some of the other models we have looked upon; special emphasis on industrial policy or support like subsidies to promote innovation at a desired speed, and also the role of a good educational system. One focus is on the relationship between growth and competition – a claim has been that there is an inverse relationship: the more competitive environment, innovation would be discouraged; hence growth would be reduced. Such an inverse relationship has been difficult to detect from data. In the simplified version of the creative-destruction-model we consider, we'll detect such an inverse relationship.

In the version of "creative destruction" we consider here, industrial innovations improve the quality of products: lower-quality products are driven out by higher-quality products (Gresham's law which here is a theory of obsolescence) and progress creates gains and losses, through the process of creative destruction. More effective production processes replace old and less effective ones, but that also implies some destruction. This can be envisaged as we move up a "quality-ladder" as progress is made. Each new innovation can be seen as an upward step on the ladder; each point in time we move upwards is random, but once an innovation takes place, the upwards jump in quality is of some given size. The process can be illustrated as:



Innovation #1 is introduced at point in time t_1. This innovation will set the best standard in some production process, until the next innovation occurs, at some uncertain future point in time; at t_2. (Hence the length of the period between two innovations $t_2 - t_1$ is a random variable.) Once an innovation is introduced, the quality is raised by a fixed factor *g*. The newest method will replace the old one, and will dominate until the next innovation takes place.

The expected growth rate of the economy depends on the economy-wide amount of research, brought into a dynamic model with forward-looking agents.
When a firm contemplates undertaking some research activity, the payoff from this activity is the period over which the researcher can enjoy being a monopolist, by having a monopoly rent in the next period (enforced by some patent laws). The temporary monopoly period has a random duration or length, until the next innovation occurs, rendering the knowledge underlying the present monopoly rent obsolete. The likelihood of getting one's own method or idea obsolete opr overthrown, depends on research effort exerted by outside firms', competing to become the next innovator, capturing the subsequent monopoly rent. This is the process of creative destruction, making the expected present value of the rents negatively affected by future research activity or research productivity (modelled as the Poisson arrival rate of the next innovation). Anticipating more research in the future will discourage current research. The incentive or carrot to allocate resources on research for a firm is the expected monopoly profit that the firm might capture. The likelihood from destroying someone's monopoly power, for then to be in a temporary monopoly-position, is the driving force inducing or encouraging a creative activity like innovations.

Consider the the following model:
We have economy with L persons (skilled labour), each person having one unit of labour, supplied inelastically; hence L is the labour supply at each point in time. This labour can be used either in research or in ordinary manufacturing.
Output of some consumption good is generated by using an intermediate good as the sole input. The output of this final good per unit of time is given by the production function $y = Ax^a$, where *x* is the (most productive) intermediate good used as input (neglecting physical capital), with $0 < a < 1$. A is a parameter indicating the productivity or quality of the intermediate input.

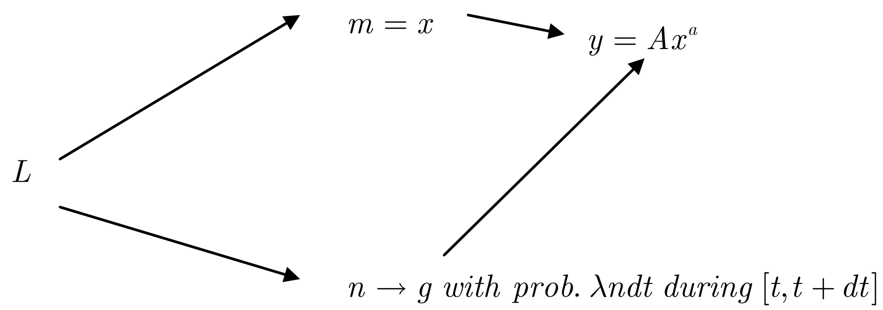
We have a labour market for skilled labour. In producing the intermediate good, only labour is used, in an amount *m*, in a one-to-one-manner; i.e. $x = h(m) = m$; a linear production technology. The amount of people entering research is *n*; hence we have:

$$(L) \qquad L = m + n$$

i) The R&D-stage
R&D is modelled as a process of improving the quality or productivity of the intermediate input. Through innovative activities there is a flow of new inventions manifested as better-quality varieties of the intermediate good used as input that will replace older varieties. This process is modelled as increasing the technological parameter A in the production function by the constant factor $g > 1$ from one

innovation to the next. Each innovation will therefore improve the quality of the intermediate input from A to gA, with *$g > 1$ measuring the size of the innovation*.

Each researcher allocating one unit of labour to R&D will enter a lottery (a stochastic R&D-process). Suppose that during a short interval of time dt, either one or no innovation will occur. Each innovation will give an increase in A equal to $g > 1$. With a mean arrival rate $\lambda > 0$, which can be seen as the probability per unit of time of having an innovation, the probability that an innovation will occur in the short time interval of length dt, is λdt. Hence the probability that an innovation will not occur is $1 - \lambda dt$.
(As an approximation: Let Z denote the number of innovations taking place during some fixed time interval, distributed according to Poisson, with
$P_r(Z = 1 \; on \; [0, dt]) = \frac{(\lambda dt)}{1!} e^{-\lambda dt} \approx \lambda dt \cdot (1 - \lambda dt + \frac{1}{2}(\lambda dt)^2 - ...) \approx \lambda dt$.)
When n persons allocate their labour time on research, each having the same arrival rate λ, then the probability that an innovation occurs in the time interval of length dt, will be $\lambda n dt$. We can consider λ as a measure of **the productivity of the research technology**. With probability $\lambda n dt$ there will be an innovation or a new intermediate input, measured by a jump g during some short interval of time. After innovation #t, the input is x_t, with a higher productivity than the previous one; $A_t = gA_{t-1}$.



The innovation means that one discovers an improved version (or a higher quality) of the intermediate input. We assume that each successful innovator has a time-limited monopoly power; lasting until the next innovation occurs, by some other firm. (This period might be considered as the period over which one has an effective patent until a new innovation takes over.)

A remark on the stochastic research process
Suppose that some random event Z is governed by a Poisson process, with a known arrival rate $u > 0$. This means that during some given time period, of length T, the

number of innovations that is expected to take place per unit of time is u. We know that $\Pr(Z = z) = \frac{(uT)^z}{z!} e^{-uT}$, with $EZ = uT = VarZ$. Then we know also that the length of time you have to wait for Z to occur will be exponentially distributed with parameter u which is then the probability per unit of time that the event will occur or the "flow probability" of the event.
Let τ be the random date of innovation. The probability for the innovation to occur before T is then given by $P_r(\tau \leq T) := F(T)$. Then for the innovation *not* to occur prior to or before *T+dt* consists of two disjoint and independent outcomes: no innovation in [0,T] **and** no innovation in [T,T+dt]. The last probability is $1 - udt$, because *udt* is the probability for the event to occur in the small interval of length *dt*. If $Q(T) = 1 - F(T)$ is the probability for **no** innovation before T, we have, when multiplying the two probabilities:

$$Q(T + dt) = Q(T) \cdot (1 - udt) \Rightarrow \frac{Q(T + dt) - Q(T)}{dt} = -uQ(T).$$

Let $dt \to 0$, and assume differentiability, then we get:

$$Q'(T) = -uQ(T) \Leftrightarrow \frac{Q'(T)}{Q(T)} = -u. \ Use \ \frac{d}{dt}(\ln Q(T)) = \frac{Q'(T)}{Q(T)}$$, to give:

$\ln Q(T) = c - uT \Rightarrow Q(T) = e^{c-uT}$. Because $Q(0) = 1$, we have $Q(T) = e^{-ut}$. Hence: The probability for the event to occur before τ is: $F(\tau) = 1 - Q(\tau) = 1 - e^{-u\tau}$, with probability density $F'(\tau) = f(\tau) = ue^{-u\tau}$. The waiting time in a Poisson process with parameter u is exponentially distributed, with expected waiting time as given by $\frac{1}{u}$.

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At some stage during the process of innovations, when *t* is the number of innovations that so far have been realized, the amount of research undertaken by an innovator, must obey some equilibrium, free entry or arbitrage condition. The expected gain from going for innovation #t+1 for some person is $\lambda V_{t+1} - w_t$, where w_t is the wage under these circumstances, after innovation #t has occurred. (This is the wage that is paid in the manufacturing industry after innovation #t has occurred.) In equilibrium we therefore have:

$$(A) \qquad w_t = \lambda V_{t+1}$$

At the margin, a person is indifferent between allocating his labour between the two activities. The return from working in the manufacturing sector, producing the intermediate good for the final-good-sector, must in equilibrium be equal to the expected return from doing research, which, with some probability, makes me a monopolist for some uncertain period of time. Hence, the current wage has to be balanced against the discounted expected return from using labour in research to become the innovator *#(t+1)*. In equilibrium this wage must be equal to the discounted expected marginal gain from obtaining the prize (or monopoly rent) V_{t+1}; multiplied by λ which is the probability per unit of time for a successful innovation. Each innovation creates a monopoly for the innovator when selling x_{t+1} to the final good sector, as a temporary monopolist.

An innovator balances his or her marginal return in the two competing activities. The two options must have equal expected marginal return. If I win the race, I will be a monopolist until a new innovation destroys my monopoly position.

How can we calculate the value of innovation *#t+1*? This depends on the length of the period over which I can reap the profit flow π_{t+1} from this innovation. This length is random and affected by the outside (or subsequent) firms trying to become the new innovator:

For a *fixed* monopoly period $[0,\tau]$, and some time-invariant discount rate *r*, the present discounted value of the profit flows is given by

$$v_{t+1}(\tau)=\int_0^{\tau}\pi_{t+1}e^{-rt}dt=\pi_{t+1}\cdot\frac{1-e^{-r\tau}}{r}.$$

The point in time when a new innovation takes place, τ, is not fixed but a random variable, being exponentially distributed (λn_{t+1}); i.e. with density $\lambda n_{t+1}e^{-\lambda n_{t+1}\tau}$. (To any feasible outcome of the length of the monopoly period; $\tau\in(0,\infty)$, we attach a positive density, which is depending on the subsequent choice of research effort by outside firms. Future innovations undertaken by outside firms will destroy my monopoly position.)

We then have when V_{t+1} is the expected present discounted value of being innovator #(t+1), having a patent so as to operate as a temporary monopolist:

$$V_{t+1}=\int_0^\infty v_{t+1}(\tau)\lambda n_{t+1}e^{-\lambda n_{t+1}\tau}d\tau=\frac{\lambda n_{t+1}\pi_{t+1}}{r}\int_0^\infty (1-e^{-r\tau})e^{-\lambda n_{t+1}\tau}d\tau$$
$$=\frac{\lambda n_{t+1}\pi_{t+1}}{r}\cdot\left\{\left[-\frac{1}{\lambda n_{t+1}}e^{-\lambda n_{t+1}\tau}\right]_0^\infty-\int_0^\infty e^{-(r+\lambda n_{t+1})\tau}d\tau\right\}$$
$$=\frac{\lambda n_{t+1}\pi_{t+1}}{r}\cdot\left\{\left[-\frac{1}{\lambda n_{t+1}}e^{-\lambda n_{t+1}\tau}\right]_0^\infty+\left[\frac{1}{r+\lambda n_{t+1}}e^{-(r+\lambda n_{t+1})\tau}\right]_0^\infty\right\}$$
$$=\frac{\lambda n_{t+1}\pi_{t+1}}{r}\cdot\left(0+\frac{1}{\lambda n_{t+1}}\right)+\frac{\lambda n_{t+1}\pi_{t+1}}{r}\cdot\left(0-\frac{1}{r+\lambda n_{t+1}}\right)=\frac{\pi_{t+1}}{r}\cdot\left[1-\frac{\lambda n_{t+1}}{r+\lambda n_{t+1}}\right]$$
$$=\frac{\pi_{t+1}}{r}\cdot\frac{r}{r+\lambda n_{t+1}}=\frac{\pi_{t+1}}{r+\lambda n_{t+1}}$$

From which we get:

$$(r+\lambda n_{t+1})\cdot V_{t+1}=\pi_{t+1}\Leftrightarrow\underbrace{rV_{t+1}=\pi_{t+1}-\lambda n_{t+1}V_{t+1}}_{(*)}\Leftrightarrow\underbrace{V_{t+1}=\frac{\pi_{t+1}}{r}-\frac{\lambda n_{t+1}V_{t+1}}{r}}_{(**)}$$

The second part, (*), says that at each point in time, as long as the patent is active, the expected return of having the patent (like a return on some asset or capital object) is equal the current monopoly profit flow, minus the expected capital loss from being overthrown by a new innovator. The new innovation occurs with probability λn_{t+1}; or with probability λn_{t+1} the innovator #(t+1) will be replaced by a subsequent innovation; #(t+2); so $\lambda n_{t+1}V_{t+1}$ is the expected capital loss per unit time of being overthrown by #(t+2). The term (**) shows that the present expected value of the monopoly position is the annuity of an infinite stream of the monopoly profit minus the annuity of the capital loss due to a new innovation.

The equality $V_{t+1}=\frac{\pi_{t+1}}{r+\lambda n_{t+1}}$ shows the expected present value of having a monopoly position, where the discount factor $\frac{1}{r+\lambda n_{t+1}}$ is being reduced due to competition from subsequent innovations. The future gets a lower weight. The denominator in $\frac{1}{r+\lambda n_{t+1}}$ can be interpreted as an obsolescence-adjusted interest rate, and will show the impact of creative destruction. The more research is expected to follow the innovation #(t+1), the shorter is the likely duration of the monopoly period, which will reduce the value of innovation. The creative destruction element is captured by replacing an old monopolist by a new one, destroying the rent that motivated the previous innovator or previous creation.

ii) The stage after innovation #t.
The monopolist with innovation #t, will sell the input x_t to the competitive final-good or consumption good industry as a monopolist, facing a demand function derived from the decision-making problem in the competitive industry. We then have: The final good sector is competitive; with a price set equal to one. If paying p per unit of *x* used as input in the manufacturing sector, where *p* is measured in units of the final good, they will choose *x* so as to: $Max\{Ax^a - px\}$; with first-order condition $aAx^{a-1} = p$, which is the inverse demand function facing the supplier of the intermediate good, as given by value marginal productivity.

The supplier of this intermediate good or input is the temprary or preliminary winner of the innovation contest; with a random length of the monopoly period, selling his or her output used as input in the production of the final good, as monopolist. If wage per unit of labour after innovation #t, is w_t, then innovator #t, with a temporary monopoly, will solve the following problem: He or she can sell the innovation or intermediate good as a monopolist, at price $p_t(x) = aA_t x^{a-1}$ as long as the monopoly position is not overthrown. When using $m = x$, we have :

$$\text{Max}_x \ \{\pi_t = p_t(x)\cdot x - w_t x = aA_t x^a - w_t x\}.$$

We then get: $w_t = a^2 A_t x_t^{a-1} = ap_t \Rightarrow w_t x_t = a^2 A_t x_t^a$

$x_t = Arg\max_x \{aA_t x^a - w_t x\} = \left[\frac{a^2}{\frac{w_t}{A_t}}\right]^{\frac{1}{1-a}} := \left[\frac{a^2}{\omega_t}\right]^{\frac{1}{1-a}}$ where $\omega := \frac{w}{A}$ is the "productivity-adjusted wage", and with $\pi_t = \left(\frac{1}{a} - 1\right)\cdot w_t x_t$. Both *x* and π will be declining in ω.

We have $x_t := X_t(\omega_t) = \left[\frac{a^2}{\omega_t}\right]^{\frac{1}{1-a}}$, with $\frac{dX_t}{d\omega_t} < 0$.

Let $\pi_t = A_t\left(\frac{1}{a} - 1\right)\cdot \omega_t X_t(\omega_t)$. We insert and get:

$\Pi(\omega_t) = A_t(\frac{1}{a} - 1)a^{\frac{2}{1-a}}\omega_t^{1-\frac{1}{1-a}} = A_t(\frac{1}{a} - 1)a^{\frac{2}{1-a}}\omega_t^{-\frac{a}{1-a}} := A_t\tilde{\Pi}(\omega_t)$. We easily see that $\frac{d\tilde{\Pi}_t}{d\omega_t} < 0$, where $X_t(\omega_t)$ is labour used in the production of the intermediate good as function of the productivity-adjusted wage.

iii) Equilibrium

We now have two equations that characterise the equilibrium, when using that $A_{t+1} = gA_t$:

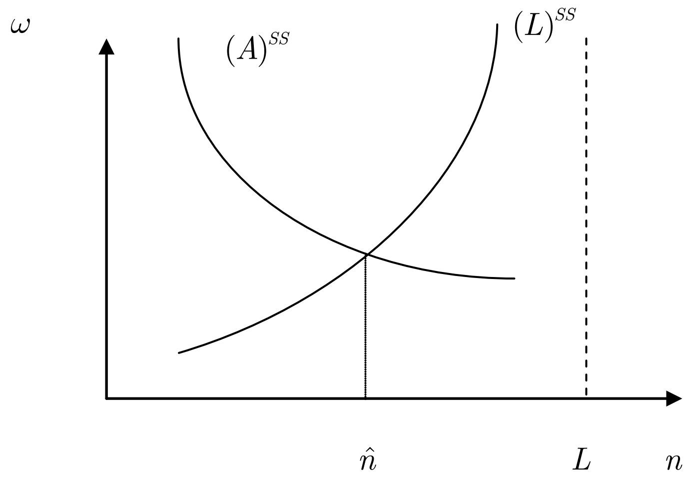
$$(L) \qquad L = X_t(\omega_t) + n_t$$

$$(A) \quad w_t = \lambda \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \Leftrightarrow \frac{w_t}{A_t} := \omega_t = \frac{1}{A_t}\lambda \frac{\pi_{t+1}}{r + \lambda n_{t+1}} = \frac{1}{A_t}\lambda \frac{A_{t+1}\tilde{\Pi}(\omega_{t+1})}{r + \lambda n_{t+1}} = \lambda \frac{g\tilde{\Pi}(\omega_{t+1})}{r + \lambda n_{t+1}}$$

Consider a steady-state or balanced growth equilibrium; with $\omega_t = \omega$ and $n_t = \hat{n}\ \forall t$:

$$(A)^{SS} \quad \omega = \frac{\lambda g \tilde{\Pi}(\omega)}{r + \lambda \hat{n}} = \frac{\lambda g \frac{1-a}{a} \omega X(\omega)}{r + \lambda \hat{n}}$$

$$(L)^{SS} \qquad L = \hat{n} + X(\omega) \Leftrightarrow X(\omega) = L - \hat{n}$$



Then: $(A)^{SS} \Rightarrow \quad 1 = \dfrac{\lambda g \frac{1-a}{a}(L - \hat{n})}{r + \lambda \hat{n}} \Rightarrow \hat{n} = n(g, L, a, r, \lambda) = \dfrac{g\frac{1-a}{a}L - \frac{r}{\lambda}}{1 + g\frac{1-a}{a}} > 0$

Assume that the higher is $\hat{n}$, the higher is the expected growth rate in this economy – when growth itself is stochastic.

From $\hat{n} = n(g, L, a, r, \lambda) = \dfrac{g\frac{1-a}{a}L - \frac{r}{\lambda}}{1 + g\frac{1-a}{a}}$, we can derive a relation between expected growth and the parameters of the model:

- A higher discount rate r will reduce growth. The value of future monopoly rent will then go down; hence the incentive to do research is weakened.
- A higher pool of skilled labour will increase expected growth, as the wage or opportunity cost of doing research will go down; and also the future stream of profit will be higher. This calls for institutional arrangements with good educational system providing sound knowledge about growth-promoting subjects; like mathematics, physics, chemistry, biology, medicine.
- A more productive research technology (higher λ) will have a positive impact on expected growth. A higher λ will increase the role of creative destruction (lower value of the patent), which, cet. par., should have a negative impact on expected growth. However, in in opposite direction, there is the impact on the marginal cost of production. The latter effect dominates.
- An increase in the size of the innovation (*g* goes up), will increase the next period's monopoly profit; stronger incentive to do research. (We have $\frac{\hat{n}}{L} = \frac{g\frac{1-a}{a} - \frac{r}{\lambda L}}{1 + g\frac{1-a}{a}} = \frac{1 - \frac{ra}{g(1-a)\lambda L}}{1 + \frac{a}{g(1-a)}}$ will be increasing in g.) Require a good patent system and intellectual property rights.
- The absolute value of elasticity of demand in the final good sector using the intermediate good sold by the monopolist, is $Elx : p = \frac{1}{1-a}$. If higher competition means a more elastic demand curve facing the monopolist, then a more competitive environment should be associated with *a* approaching one. We have from $p_t(x) = aA_t x^{a-1}$, that $x = (\frac{p}{aA})^{\frac{1}{a-1}} = (\frac{aA}{p})^{\frac{1}{1-a}}$, so that $px = (aA)^{\frac{1}{1-a}} p^{1+\frac{1}{a-1}} = kp^{\frac{a}{a-1}} = kp^{-\frac{a}{1-a}}$. The closer *a* is one, the more elastic is demand, or the higher is the degree of competition. ($\pi = A(\frac{1}{a} -)\omega x$ becomes smaller as *a* goes to one.) Product market competition is bad for R&D, and hence for growth. More product market competition will reduce future monopoly rents that can be reaped by a successful innovator; hence the incentive to undertake R&D is weakened.