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Lecture Notes, ECON 4350

EXPLAINING REAL INVESTMENT:

The neoclassical approach and the supplement by Haavelmo

In all the models studied so far we have seen that capital accumulation plays a significant role, but the explanation differs between the various models. In the classical model it was determined by profits – hence showing the role of institutions (property rights) and income distribution; in the Domar-model investment was mechanically determined from a fixed capital-output ratio; whereas Harrod (not taken up in the lectures) model had an expectation-based investment explanation, leading to bubbles – perhaps the first model with bubbles. In the neoclassical growth model, saving was a fixed fraction of income, and all savings were automatically invested. In Johansen's vintage approach, purchase of new equipment, embodying the newest technology, was based on present-value calculations, taking future scrapping and expectations about prices into account. The decision as to what determines the new capacity or capacity expansion can be seen as one part of the investment story. In these models one vould have assumed that new equipment ordered at some point in time, was delivered with some lag – due to some period of production – a lag that normally would be economically motivated.

We start by asking whether the standard Keynesian investment function $I = h(r)$ can be derived from economic principles. Is the relationship between investment (a flow concept) and the rate of interest a straightforward relationship, or is it difficult to justify it as an *equilibrium theory of investment*?
We then follow a two-way route: One route is emphasising the **demand** side from which we simply and no more derive the demand for capital based on value-maximizing principles. This is the **neoclassical approach**, focusing entirely on demand for capital equipment, based on the fact that capital is a durable input which provides services for its owner for longer period of time than ordinary accounting periods.
From this story we reveal some of the main problems when trying to explain investment: It normally takes time to eliminate the difference between desired amount of capital and actual amount. It takes time to complete or produce say, a ship or a machine. To have a good theory we have to incorporate the supply side in the investment good industry – this was one of Haavelmo's basic and important ideas – to get some background for incorporating the "investment lag" or "period of production". Investment is a flow concept, given as "something per unit of time at some point in time", as opposed to capital which is a pure stock concept. The key is to combine the demand for a stock variable like capital and the supply of investement which is a flow concept. We can visualize the two concepts by looking at

a "bath-tub": At some point in time there is a given volume of water in the tub; given by the stock, $K(t)$, measured in amount of water in m^3. Gross investment, a flow concept, is the inflow of water into the tub per unit of time; as given by $J(t)$, giving the number of litre of water per second. Because there is a small hole in the tub, some water disappears per unit of time; call this "outflow" the depreciation flow, $D(t)$. Then $I(t) = \dot{K}(t) = \frac{dK(t)}{dt} = J(t) - D(t)$ is the net investment or net inflow of water into the tub per unit of time at t. Within this context we can discuss, when ignoring the small hole causing depreciation, that if we start with some initial volume of water at some point in time *t*, and we have, for some reason, a desired amount of water in the tub as given by K^*. Then with $I(\tau)d\tau$ as the flow of water into the tub during a short period of time of length $d\tau$, we have, by definition $K^* - \bar{K} = \int_t^T I(\tau)d\tau$. If we choose the path of intensity or investment $I(\tau)d\tau$, saying that we have a given inflow per unit of time, say $I(\tau)d\tau = id\tau$, then the period during with the gap between desired and actual amount is eliminated, will follow from "solving the integral condition for the constant rate of inflow: $K^* - \bar{K} = \int_t^T I(\tau)d\tau = i\int_t^T d\tau = i(T-t)$. (We have one degree of freedom.)

Hence two questions have come up:

- How to determine the desired amount of capital?
- How long will it take to close the gap between desired and actual amount of capital? Is it possible to formulate an equilibriating mechanism involving the capital price that will include the supply side as well?

How can we then formulate a theory of investment? We expect that behind investment one should find some calculating or rational (sometimes perhaps speculative) behaviour, but this will normally be different from ordinary instantaneous behaviour of the type we find, say in producer theory. The question of how to determine investment must also rely on the fact that capital is a stock that cannot be changed instantaneously – a heavy or sluggish variable – but also that capital provides services for a long period of time. Therefore as we will show, it is difficult to rationalize or justify the simple investment function, found in so-called "standard Keynesian models", $I = h(r)$, with net investment being a declining function of the real rate of interest.

The so-called Keynesian-approach to formulate an "theory of investment" is derived from calculating the net present value of a project, where equipment of some type is purchased in the initial period (discrete time) at $t = 0$, while providing net revenue in future periods; as given by the (here: known) sequence $\{a_t\}_{t=1}^{t=\infty}$. Discounting the future at a constant rate of discount, equal to the real rate of interest, we then have that the present value of a project of initial size k, can be written as:

$V = -k + \sum_{t=1}^{\infty} \frac{a_t}{(1+r)^t}$. The decision rukle says that if V is positive, then the project should be accepted; if V is negative, we would have a better option putting the money into the bank; hence the project should in that case not be implemented. With a higher rate of discount, the value of V is reduced, and one concludes that fewer projects will be accepted or pass the present value-test. Then, one says, investment will decrease as the rate of interest goes up. In fact this is a very fragile suspicious passage. How do we get from the desire to implement a project of a certain size to a flow concept like investment?

We first study the neoclassical approach, where focus is on **demand for capital** derived from pure profit- or value maximising behaviour at the firm level (or at a higher aggregate level). We'll see that we don't necessarily get to an explanation of investment, but if we make sufficiently restrictive and artificial assumptions about the correspondence between the length of the model period and the production period, then we might (in a rather weak sense) have a story to tell. However, the story then relies on the set of very restrictive assumptions. To get a better explanation, we have to incorporate the supply of investment goods along with the demand side. This is Haavelmo's basic idea found e.g. in his "A Study in the Theory of Investment" from 1960. (Haavelmo was permanently occupied with issues related to investment, business cycles and the role of monetary instruments, as revealed in a lot of his publications and lectures given at the University of Oslo. He also prepared notes (a memorandum) for what might have become a textbook in macroeconomics, "Orientering i makroøkonomisk teori" fra 1966; published by Universitetsforlaget i 1969, where a number of very interesting issues related to investment, business cycles and the role of monetary policy are analyzed.[1]

The integration of the supply of investment goods and the demand for capital goods is the proper way of formulating **a theory of investment**. However, several authors avoid the problem taken up by Haavelmo by simply put the whole supply side into a cost of adjustment. This is incomplete and very ad hoc.

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[1] Go to http://www.sv.uio.no/econ/om/aktuelt/haavelmo/publiserte-arbeider/1966-1975.html, where you get access to this text. (You should give yourself the opportunity to read this memorandum; at least section V and VI. Another lecture notes on "Investeringsteori", from 1953-54, by Bjørn Thalberg, should also be paid a visit.)

1. The neoclassical theory – demand for capital
Consider a macro-producer who at some point in time or beginning of some time period wants to determine the future paths for various inputs; among which some (labour) can be changed momentarily, while others, like capital once installed, will provide services for a longer period of time, until the equipment is scrapped.

Let us consider a discrete version of a neoclassical model, with a standard neoclassical production function – with standard properties; strictly increasing and strictly concave:[2]

(1) $X_t = F(L_t, K_t)$ for $t = 1, 2, \ldots.,$

where X_t is gross output available at the end of period t, L_t is labour input in period t, whereas K_t is the amount of capital present **at the end** of period t. (We assume that the services from capital are proportional with capital stock, and the unit of measurement is chosen so that the factor of proportionality is set equal to one.)
At the beginning of period 1 the producer is going to set up a future production plan. Suppose all prices are considered as exogenous, say as given expectations.
Define the current **cash-flow** as the difference between the revenue and expenses in period t as

(2) $R_t = p_t X_t - w_t L_t - q_t J_t$

with $\{p_t, w_t, q_t\}_{t=1}^{t=\infty}$ as a given price sequence, with J_t as the volume of gross investments in period t, and q_t price per unit investment ("new capital equipment") prevailing in period t. Assume that a fixed fraction $\delta \in (0,1)$ of the volume of capital stock present at the beginning of period t depreciates. This implies that capital present at the end of period t must satisfy, with a stock at the beginning of the period being K_{t-1}:

(3) $K_t = J_t + (1-\delta)K_{t-1}$ for $t = 1, 2, \ldots.,$ and K_0 given; $I_t := K_t - K_{t-1} = J_t - \delta K_{t-1}$

(Where the initial capital comes from is not answered; we have trouble explaining how K_0 is determined.)
The objective is to maximise the present value of all future cash-flows, with a discount factor for period t as given by $(1+r)^{-t}$. The choice variables are: Use of

[2] Some of the material in this section can be found in E.Biørn, *Taxation, Technology and the User Cost of Capital*, North-Holland, 1989. See also, only in Norwegian, "Analyse av investeringsatferd: problemer, metoder og resultater", SØS 38, Oslo, 1979, by the same author.

labour in each period and the desired stock of capital at the end of each period so as to maximise

$$V = \sum_{t=1}^{\infty}(1+r)^{-t}R_t = \sum_{t=1}^{\infty}(1+r)^{-t}\left[p_t F(L_t, K_t) - w_t L_t - q_t(K_t - (1-\delta)K_{t-1})\right]$$

We assume that there exists a unique stationary point obeying $\frac{\partial V}{\partial L_t} = 0 = \frac{\partial V}{\partial K_t}$ for every t, which must then be the solution to our problem, where the last FOC is related to **desired stock** at the end of period t. The conditions characterising a maximum are:

$$(4) \quad \frac{\partial V}{\partial L_t} = 0 \Leftrightarrow p_t \frac{\partial F}{\partial L_t} - w_t = 0 \ \mathit{for}\ t = 1,2,.....$$

$$(5) \quad \frac{\partial V}{\partial K_t} = 0 \Leftrightarrow (1+r)^{-t}\left\{p_t \frac{\partial F}{\partial K_t} - q_t\right\} + (1+r)^{-t-1}(1-\delta)q_{t+1} = 0$$

(The capital stock at the end of period t enters **two successive terms** of V.)

The first one of these first-order conditions is standard, as labour input is freely adjustable from period to period, no stock element. The second one is more delicate and can be modified to be written as:

$$(6) \quad p_t \frac{\partial F}{\partial K_t} = q_t - \frac{(1-\delta)q_{t+1}}{(1+r)} = \frac{1}{1+r}\left[rq_t + \delta q_{t+1} - (q_{t+1} - q_t)\right] := c_t \ \mathit{for}\ t = 1,2,....$$

The LHS shows the value productivity of having one more unit of capital equipment installed at the end of period t. The RHS must then show the cost of having one more unit at the beginning of period t+1 or at the end of period t. What is the interpretation of c_t?

Consider the RHS, expressed as $q_t - \frac{(1-\delta)q_{t+1}}{(1+r)}$. The first term q_t shows the unit price of capital at the end of period t when one more unit is purchased. We deduct a term showing the value of the same unit one period later; at the end of the subsequent period, modified by the fraction that disappears during that period, due to depreciation, and then discounted back to the beginning of period. Hence, the RHS shows the cost for the firm to acquire one capital unit at the end of period t, and having the unit in its possession for one period. The RHS is called *the user cost of*

capital. (Note that $c_t = \phi(r,\delta,q_t,q_{t+1})$ with $\frac{\partial \phi}{\partial r} > 0$ and that a higher cost of capital will reduce the desired amount of capital; $\frac{\partial K_t}{\partial c_t} < 0$.)
(In continuous time, we have that the cost of capital can be expressed as $c(t) = q(t)\cdot\left[r+\delta-\frac{\dot{q}(t)}{q(t)}\right]$, where the last term shows the change in instantaneous valuation or the capital gain/loss.)

The user cost of capital can be regarded as a user price or rental price of capital per period and shows what it really costs the firm to have one unit of capital for one period.
The firm's behaviour is *myopic* in the following sense: As seen from (4) and (5), the firm's maximisation problem is solved in each period by considering prices in the same period, as well as the expected capital price in the subsequent period. Why is that the case? Why don't bother with a longer price series? The point is that when investment is reversible – as it is here – the firm can buy a unit of capital equipment, use it and reap the marginal productivity gain of that unit for a "short period" of time, and then sell the portion of the capital unit that has not yet depreciated, possibly at a different price (capital gain). Our problem can now be reformulated in the following direction: For each period the producer maximises *profit* for that period, when the user price is applied as the correct valuation of using capital: $\pi_t = p_t F(L_t, K_t) - w_t L_t - c_t K_t$.

The question now is: What does this model explain about investment behaviour? Consider the FOC for capital and let us for simplicity ignore labour, and let the output price be normalized to one. Suppose also that the capital price and the depreciation rate be constant, but let the real rate of interest be a function of time. Then the (FOC) can be written as $F_K(K(t)) = (r(t)+\delta)q$, where F_K is positive and declining in K. What is going on over time or what kind of dynamics will show up? Differentiate this FOC with respect to time, which yields: $F_{KK}\cdot\dot{K}(t) = q\dot{r}(t)$. Hence, the rate of investment, $\dot{K}(t)$ is determined by the change per unit of time in the real rate of interest, not the real interest rate itself! (Because $F_{KK} < 0$, the rate of investment will increase if the real rate of interest declines sharply.) This relationship is rather different from the Keynesian investment function. We have not invented a theory of investment; the relationship between $\dot{K}(t) := I(t)$ and $\dot{r}(t)$ is a derived realtionship.
If this story should have any power in explaining investment, we must put in some assumption saying that the supply of investment goods will always accommodate the desired increase in capital. The supply side is made a rather "passive" agent accommodating the demand side, without any underlying rational behaviour.

Another interpretation, also an artificial one, is that the length of the period of the model is identical to the production or installation lag or the construction period in the investment goods industry. At the beginning of some period the demand for new capital is determined and the desired increase in the stock is produced and delivered at the end of the production period. (This is an assumption used by Agnar Sandmo in "Investment and the rate of interest", Journal of Political Economy, 1971 , pp.1135-1345.) As long as we always have (5) or (6) satisfied, which has been assumed in the neoclassical approach, and combine this with a construction period equal to the length of the period of the model, then we have nothing more to explain, but the story is not complete!

2. Some of Haavelmo's idea

At some point in time, capital is fixed and can be changed only over a finite period. With exogenous prices in all markets (or simply, producers are price takers), we have, in the continuous case, the following decision rules – showing a knife-edge property:

If $p(t)F_K > c(t)$, then $J(t) = \infty$; determined by the supply-side
If $p(t)F_K < c(t)$, then $J(t) = 0$ or $I(t) = -D(t)$ if no second-hand capital market
If $p(t)F_K = c(t)$, then $J(t) \in [0, \infty)$

The meaning of this is the following: Consider the first line where the marginal value productivity of capital exceeds the user cost. Then the producers should like to have an infinite investment so as to fill momentarily the gap between desired and actual stock of capital. In that case, gross investment will be residually determined from the supply side in the economy, charactereized most likely by full employment – hence the actual investment will follow from what the investment good industry can accomplish. The meaning of the second line should be rather obvious; in that case the producers should like to get rid of capital without destroying it. But in macro, we cannot have a gross investment below zero. The relationships above show one of the main results derived by Haavelmo and is sometimes forgotten in neoclassical model building. *Stocks cannot be changed instantaneously!*[3]
To have a theory, one can tell the following story, where the supply of investment goods is introduced explicitly into the model:[4]

[3] These ideas are elaborated by Haavelmo in a series of lecture notes and paper, or can be found in his book from 1960. Norwegian students interested in this topic should read Bjørn Thalberg "Investeringsteori", Lecture notes from Haavelmo's lectures in 1953-1954, and the highly recommended "textbook" in macroeconomics, written by Haavelmo, called "Orientering i Makro-økonomisk teori", Universitetsforlaget, 1969.

[4] The model is found in chapter 9 in Thalberg (op.cit.). The simplified version presented here is also outlined in some unpublished notes by Michael Hoel and Steinar Strøm.

Suppose we start from an equilibrium position with $p(t)F_K = c(t) = q(t) \cdot \left[r + \delta - \frac{\dot{q}(t)}{q(t)} \right]$ which gives demand for capital as given by $K^*(c(t), w(t))$ when $p = 1$. With standard properties of the production function, desired or capital demanded will be declining in *c*. Suppose that r and w are exogenous variables, and assume static price expectations. Then we have $c = q(r+\delta)$, where δ is a technical parameter. Hence, in our initial equilibrium we have, according to Haavelmo that some equilibriating mechanism (an endogenous variable) must operate so as to **adjust demand for capital to be equal to the supply or available stock at each point in time**:

(8) $K^*(q(r+\delta), w) = \bar{K}$ (given stock available)

In this model the capital price *q* is playing the adjustment or equilibriating role so that (8) always holds. In addition *q* is also the motivating parameter for the investment goods sector (supply of investment goods): The output flow or supply of new investment goods at t, is:

(9) $k(q(t), w) = J(q)$ Supply function for investment goods at t ; with $J'(q) > 0$.

Here $k(q(t), w)$ is a flow concept. By definition of gross and net investment, we have

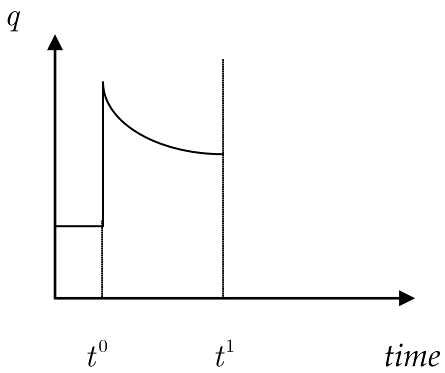
(10) $\dot{K}(t) = J(t) - \delta K(t)$

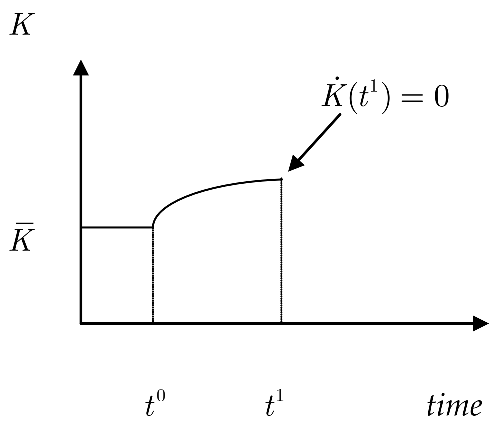
We start out from a stationary equilibrium with (8) satisfied, with an equilibrium price for capital, and with supply of investment goods just equal to the required replacement investment:

(11) $\dot{K}(t) = 0 \; and \; J = \delta K$

Then, for some unexplained reasons (monetary policy), there is a permanent fall in the exogenous rate of interest at $t = t^0$; see the illustrations below. Given the original and initial capital price, we will now have excess demand for capital in the economy; $K^{**} > \bar{K}$. But such a situation shows a state of disequilibrium. To return to a new equilibrium with (8) satisfied also after the fall in the rate of interest, the capital price must increase instantaneously or jump upwards – showing a discontinuous jump. But once we have got a higher capital price, the suppliers of investment goods are triggered or motivated to increase their output flow; hence $\dot{K}$ becomes positive immediately; we have therefore $\dot{K}(t_0^+) > 0 = \dot{K}(t_0^-)$. If we assume capital stock starts to increase immediately, or the new investment goods are placed in the market immediately, the capital price, from the new "high" level, has to decline so as to get the demand side to be willing to purchase and install the new investment goods; or to have (8) satisfied, now with a higher capital stock on the RHS of (8). The only way to get the users of capital to buy the increased supply, is lowering the price. How fast capital will increase will depend on the production conditions (capacity) in the investment goods industry.

In the new stationary equilibrium we have that net investment again will be zero, the economy has reached a state with higher capital stock, a higher capital price than initially, but lower than the level just after the upwards jump. Because of a higher stock of capital at t_1, the depreciation or replacement investment has increased which can only be fulfilled by the suppliers facing a higher capital price than in the initial equilibrium. We can illustrate the situation, where the rate of interest jumps downwards at t^0, and the new stationary equilibrium is attained again at t^1:





The point is that we have to see the interplay between the two sides of the market to learn something about investment, which is very different from demand for capital.

Note that the relation in this model between the initial change in the rate of interest (not the level as such) and investment goes through the change in the capital price. The decline in the interest rate generates an upwards jump in the capital price so as to have the stock equilibrium condition (8) satisfied. Because q also is the motivating factor inducing the investment producers to change their output decisions, a higher capital price will with our assumptions generate an increase in the output (a flow) of the investment good industry. The level of net investment, $\dot{K}(t)$, will now be fully explained by the production conditions on the supply side. To get the buyers to invest, the capital price must now decrease, which again will motivate to suppliers to reduce their output. We have $\dot{K} > 0$, but $\ddot{K} < 0$ as we move towards t^1; see the right-hand side illustration above.