DIVISION OF LABOUR – SIMON REVISITED

A Comment

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This paper is a comment on an earlier work by Artle, Humes Jr. and Varaiya published in this Journal. Its purpose is to show that some of the conclusions about labour migration that are drawn by Simon, Baumol and Artle et al., can be made within a rather simple equilibrium model. Further it is shown that these conclusions may not hold if wage rigidities are present which prevent equalization of wages between sectors.

1. Introduction

R. Artle, C. Humes Jr. and P. Varaiya (AHV) have presented an article in this Journal [Artle et al. (1977)], where they analyze some factors behind labour migration between sectors, in a context of excess demands. However, some of their conclusions can be made more easily accessible within a simpler equilibrium model. This is the main reason for writing this comment. On the other hand, I will also show that it is possible to construct a model which, under certain assumptions, gives totally different conclusions than those obtained by Simon (1947), Baumol (1967) and AHV (1977).

In the following I will sketch the main ideas of an equilibrium model and make a comparison with the conclusions in AHV, Simon and Baumol.

2. The model, definitions and assumptions

2.1. Production and labour

We consider a closed economy consisting of two sectors, each with a production function where only labour is specified as a factor of production. I assume that the capital stock in each sector is constant. The production function in each sector is assumed to be homogeneous of degree one in labour. Further I assume full employment and a Hicks-neutral disembodied

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technical change in each sector. We then have

\[ X_i = A_i N_t e^{\gamma_i}, \quad i = 1,2, \]  

which \( X_i \) is the output in sector \( i \) per unit of time \( t \), \( N_t \) is the quantity of labour employed in sector \( i \) at the point of time \( t \), and \( \gamma_i \) is the rate of technical change in sector \( i \). \( A_i \) is a positive constant taking care of the fixed capital equipment in the sector.

\[ N = N_1 + N_2, \]  

where \( N \) is the total labour force, here assumed to be proportional to the population in the economy.

Further I assume that the population and, because the ratio between labour force and population is assumed to be constant, the labour force are growing with a rate 100n% per unit of time. This means that

\[ \dot{N}/N = n, \]  

where a dot over the variable denotes derivative with respect to time. In addition, there is assumed that the labour force is sufficiently mobile so that the demand for labour in each sector always is fulfilled.

2.2. Demand

I assume that the demand for output or good \( i \) per capita is of the form

\[ X_i/N = B_i (R/N)^{E_{ii}} P_i^{e_{ij}} P_j^{e_{jj}}, \]  

where \( B_i \) is a positive constant, \( R/N \) is per capita income, and \( P_i \) is the price per unit of good \( i \), \( E_{ii} \) is the income elasticity for good \( i \), and \( e_{ij} \) is the price elasticity for good \( i \) with respect to price for good \( j \), given by

\[ E_i = \frac{R}{X_i} \frac{\partial X_i}{\partial R} \quad \text{and} \quad e_{ij} = \frac{P_j}{X_i} \frac{\partial X_i}{\partial P_j}, \quad i,j = 1,2. \]

I assume that none of the goods are inferior, that is \( E_i > 0 \) for \( i = 1,2 \).

From the per capita demand function we can derive the total demand for good \( i \) as

\[ X_i = B_i R^{E_{ii}} N^{1-E_{ii}} P_i^{e_{ii}} P_j^{e_{jj}}, \quad i = 1,2. \]

It must be remarked that there is no preference function from which we can derive such demand functions for all commodities. They must be regarded as
approximations. There is also another point to note in this connection.
Incorporated in these demand functions is a budget constraint of the form
\[ R = P_1X_1 + P_2X_2 = w_1N_1 + w_2N_2. \]
This budget equation is therefore not an independent relation in the model. Further I assume that these demand functions are homogeneous of degree zero in \( R, P_1 \) and \( P_2 \), that means
\[ e_{i1} + e_{i2} + E_i = 0 \quad (i = 1, 2). \]
The demand for labour in each sector can be derived from the conditions for profit maximization, under the assumption of price-taking behaviour in each market. With the special type of production functions, the adjustment gives that the price per unit of good \( i \) is equal to the marginal cost equal to the average cost. That is
\[ P_i = \left( \frac{w_i}{A_i} \right) e^{-\gamma i}, \quad i = 1, 2, \]
where \( w_i \) is the wage rate per unit of time in sector \( i \).

2.3. The wage-fixing

At this point I make a somewhat strange assumption. I introduce an active public authority which has the power to fix the rate of change in the wages for the two sectors. The background for this assumption is that the Norwegian Government has decided to control the growth in incomes (in the model this means the wages which are the only components of income in this economy) for various groups. I assume that
\[ \frac{w_i}{w_i} = g_i, \quad i = 1, 2, \]
where \( g_i \) is a non-negative constant, controlled entirely by the public authority or the Government.

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On the basis of this model it is now possible to derive an expression for the relative rate of change of the labour force in sector 1. Since I have assumed full employment at each point of time, it is easy to derive the same expression for sector 2. In the following passage I will restrict myself to regard \( N_1/N_1 \).

I now introduce the Slutsky equation, which separates the total effect upon the demand for good \( i \) of a change in price \( j \) \( (e_{ij}) \), in a substitution effect \( (\epsilon_{ij}) \) where \( e_{ij} \) is a Slutsky elasticity of demand, and an income effect \( (\alpha_i E_i) \), where \( \alpha_j = P_jX_j / R \quad (j = 1, 2) \). From the 2nd-order conditions for a local maximum of a preference function under a budget constraint, the direct substitution effect \( \epsilon_{ii} \) is negative.
We are now able to derive the central relation in the model, that is the expression for $\dot{N}_1/N_1$. We find

$$\frac{\dot{N}_1}{N_1} = n + \left\{ \alpha_2 \left( \frac{N_1}{N_2} E_1 + E_2 \right) \right\}^{-1} \times \{E_1(\alpha_1 \gamma_1 + \alpha_2 \gamma_2) - \gamma_1 + \epsilon_1[(g_1 - g_2) + (\gamma_2 - \gamma_1)] \}.$$  

(7)

This equation says something about the direction of $N_1(t)$ in a point of time $t$, and which factors that determine this direction. It is the last term in (7) which I find most interesting. The term in front of the brackets can be given the following interpretation: This term says something about gains from migration from the sector with the lower wage rate to that with the higher, and it can be written as

$$\left\{ \alpha_2 \left( \frac{N_1}{N_2} E_1 + E_2 \right) \right\}^{-1} = \left\{ 1 - \alpha_1 E_1 \left( 1 - \frac{w_2}{w_1} \right) \right\}^{-1}.$$  

Suppose that $w_1 - w_2 > 0$ and that a migration from sector 2 to sector 1 arises. This first migration will then generate an increase in income, which through the demand for good 1 will require an additional increase in labour. This increase in the labour force in sector 1 will in turn generate additional income, which creates a need for more labour in sector 1. This process can be expressed by a converging geometrical series the sum of which is exactly the term in front of the brackets. However, there is one quality of the model which must be mentioned. The migration will decrease throughout the process as a result of the fact that the increase in income will decline and not as a result of the wages in the two sectors equalizing. Such a mechanism does not exist in the model.

Let us then go back to (7) and look at some of the conclusions which can be drawn. Starting with Simon's (1947) theorem, that if two sectors have the same rate of technical progress, then labour migrates towards the more income elastic sector. We make the following assumptions:

$$\gamma_1 = \gamma_2 = \gamma > 0,$$

$$w_1 - w_2,$$

$$g_1 = g_2.$$

By inserting these conditions into (7), we find

$$\frac{\dot{N}_1}{N_1} = n + \gamma(E_1 - 1).$$  

(8)
If $E_1 > 1$, then $E_2 < 1$ for all combinations of $\{x_1, x_2\}$ where both are positive and less than one. Then we have that $\hat{N}_1/N_1$ will be greater, the greater $E_1$ is, given that $E_1 > 1$.

This means that Simon's theorem is shown to be fulfilled in this model.

Baumol (1967) has shown the following proposition (also shown in AHV's paper):

In a model of unbalanced productivity there is a tendency for the outputs of the 'nonprogressive' sector whose demands are not highly inelastic to decline and perhaps, ultimately, to vanish.

AHV have shown the following theorem (Theorem 4.2), corresponding to Baumol's proposition:

In the case of unbalanced growth, labour migrates towards the progressive (non-progressive) sector if demand for its output is elastic (inelastic) to its own price, i.e., $\varepsilon_{ii} < -1$ ($\varepsilon_{ii} > -1$).

The definition of a progressive sector is a sector with a positive rate of technical progress, while a non-progressive sector is a sector with zero technical progress. Unbalanced growth should indicate this difference.

By putting

$$\gamma_1 = \gamma > 0,$$
$$\gamma_2 = 0,$$
$$w_1 = w_2,$$
$$g_1 = g_2,$$

we will have from (7)

$$\hat{N}_1/N_1 = n + \gamma(x_1 E_1 - 1 - \varepsilon_{11})$$
$$= n + \gamma(-\varepsilon_{11} - 1),$$

(9)

where the Slutsky equation has been used. Labour will migrate towards the progressive sector if and only if $\varepsilon_{11} < -1$.

This means that the proposition of Baumol and Theorem 4.2 of AHV both seem to be fulfilled in this model under certain assumptions.

So far I have assumed both $w_1 = w_2$ and $g_1 = g_2$. My reason for doing this has been to demonstrate that the model, under these assumptions, gives the same conclusions as in Simon, Baumol and AHV. If we now drop either one of these two or both of them, the conclusions will have to be somewhat modified.
Let us go back to (7) and examine the same problems discussed above, but now under the following assumptions:

\[ \gamma_1 = \gamma_2 = \gamma > 0. \]  
(Simon's problem)

\[ w_1 \neq w_2, \]

\[ g_1 \neq g_2. \]

From (7) we then have

\[
\frac{N_1}{N_2} = n + \left\{ \alpha_2 \left( \frac{N_1}{N_2} E_1 + E_2 \right) \right\}^{-1}
\times \{ \gamma(E_1 - 1) + \varepsilon_{11}(g_1 - g_2) \}.
\]  \hspace{1cm} (10)

If we assume \( E_1 > E_2 \), then we must have \( E_1 > 1 \). We see from (10) that the first term within the brackets is positive. Since \( \varepsilon_{11} < 0 \), the sign of the last term is dependent on whether \( g_1 \geq g_2 \). If \( g_1 \leq g_2 \), that is, if the wage rate in sector 1 grows relatively slower or equally than the wage rate in sector 2, then the labour migrates towards the more income elastic sector. If, on the other hand, \( g_1 > g_2 \), it is then possible that the labour migrates to the less income elastic sector. This will happen when the absolute value of the last term, \( |\varepsilon_{11}(g_1 - g_2)| \), is larger than \( \gamma(E_1 - 1) \). This might be the case if \( g_1 \gg g_2 \) and/or \( |\varepsilon_{11}| \) is large. Therefore it is possible that Simon's result will not be fulfilled.

This, rather strange, result is a consequence of the model, and the way the supply of labour enters the model.

Let us return to the problem of Baumol, stated in AHV's Theorem 4.2. The assumptions made here are

\[ \gamma_1 = \gamma > 0, \]

\[ \gamma_2 = 0, \]

\[ w_1 \neq w_2, \]  
for simplicity: \( w_1 > w_2, \)

\[ g_1 \neq g_2. \]

From (7) we have

\[
\frac{N_1}{N_1} = n + \left\{ \alpha_2 \left( \frac{N_1}{N_2} E_1 + E_2 \right) \right\}^{-1}
\times \{ \alpha_1 E_1 \gamma - \gamma \varepsilon_{11} + \varepsilon_{11}(g_1 - g_2) \}
\]
where we have used the Slutsky equation. In the case of unbalanced growth, it is not sufficient that $e_{11} < -1$ to guarantee that labour migrates towards the progressive sector. As in the modified Simon problem, we might have that labour migrates towards the non-progressive sector if $e_{11} < -1$. This will happen if $g_1 > g_2$ and/or when the absolute value of $e_{11}$ is large. With my special assumptions about the labour market, we might reach a conclusion about migration that goes in opposite direction of that of Baumol and AHV. In the case $g_1 \leq g_2$, the Baumol proposition and Theorem 4.2 in AHV's article, will be fulfilled.

3. Conclusions

I had two reasons to make this note. The first is that some of the conclusions drawn in AHV's article, might be obtained by using a more simple model, like the one I have presented. On the other hand, I would show that it is possible to construct a model which, under certain assumptions, might give totally different conclusions about labour migration than those obtained by Simon, Baumol and AHV.

The model cannot give answers to all the questions raised by AHV, it is too simple for that, but I think that this paper has demonstrated that it is possible to study the problem of labour migration in a rather simple context.

Appendix

The derivation of formulae (7)

From (1) and (4) we have

$$A_i N_i e_{it} = B_i R^{E_{1i}} N^{1-i} - E_{i1} P_{1i}^{e_{1i}} P_{2i}^{e_{2i}}.$$

By taking logarithms on both sides and differentiate with respect to time, we get

$$\frac{\dot{N}_i}{N_i} + \gamma_i = E_i \frac{\dot{R}}{R} + (1 - E_i) \frac{\dot{N}}{N} + e_{i1} \frac{\dot{P}_1}{P_1} + e_{i2} \frac{\dot{P}_2}{P_2}. \quad (A.1)$$

From the budget constraint we will find an expression of $\dot{R}/R$. 

$$= n + \left\{ \alpha_2 \left( \frac{N_i}{N_2} E_1 + E_2 \right) \right\}^{-1} \times \{ \gamma(-e_{11} - 1) + \varepsilon_{11}(g_1 - g_2) \}.$$
\[ R = w_1 N_1 + w_2 N_2 \Rightarrow \]

\[ \frac{\dot{R}}{R} = \frac{w_1 N_1}{R} \left[ \frac{\dot{N}_1}{N_1} + \frac{w_1}{w_1} \right] + \frac{w_2 N_2}{R} \left[ \frac{\dot{N}_2}{N_2} + \frac{w_2}{w_2} \right] \]

\[ = \alpha_1 \left( \frac{\dot{N}_1}{N_1} + g_1 \right) + \alpha_2 \left( \frac{\dot{N}_2}{N_2} + g_2 \right), \quad (A.2) \]

where we have used (6), the fact that \( p_i x_i = w_i n_i \) and the definition of \( \alpha_j \).

From (5) and (6) we have

\[ \frac{\dot{p}_i}{p_i} = \frac{\dot{w}_i}{w_i} - \gamma_i = \gamma_i. \quad (A.3) \]

Since \( N = N_1 + N_2 \) and \( \dot{N}/N = n \), we will have that

\[ \frac{\dot{N}_2}{N_2} = n + \frac{N_1}{N_2} \left( n - \frac{\dot{N}_1}{N_1} \right). \quad (A.4) \]

By inserting (A.2), (A.3) and (A.4) into (A.1) and by putting \( i = 1 \), we then have

\[ \frac{\dot{N}_1}{N_1} = E_1 \left\{ \alpha_1 \left( \frac{\dot{N}_1}{N_1} + g_1 \right) + \alpha_2 \left( \frac{\dot{N}_2}{N_2} + g_2 \right) \right\} \]

\[ + (1 - E_1) n - \gamma_1 + e_{11}(g_1 - \gamma_1) + e_{12}(g_2 - \gamma_2) \]

\[ = E_1 \alpha_1 \left( \frac{\dot{N}_1}{N_1} + g_1 \right) + \alpha_2 E_1 \left[ n + \frac{N_1}{N_2} \left( n - \frac{\dot{N}_1}{N_1} \right) + g_2 \right] \]

\[ + (1 - E_1) n - \gamma_1 + e_{11}(g_1 - \gamma_1) + e_{12}(g_2 - \gamma_2) \]

\[ \Rightarrow \]

\[ \frac{\dot{N}_1}{N_1} \left( 1 - \alpha_1 E_1 + \alpha_2 E_1 \frac{N_1}{N_2} \right) = \left( \alpha_2 E_1 + \alpha_2 E_1 \frac{N_1}{N_2} + 1 - E_1 \right) n \]

\[ + \alpha_1 E_1 g_1 + \alpha_2 E_1 g_2 - \gamma_1 \]

\[ + e_{11}(g_1 - \gamma_1) + e_{12}(g_2 - \gamma_2). \quad (A.5) \]
Since $\alpha_2 = 1 - \alpha_1$, by definition, the term in front of $n$ in (A.5) can be written as

$$\alpha_2 E_1 + \alpha_2 E_1 \frac{N_1}{N_2} + 1 - E_1 = (1 - \alpha_1) E_1 + \alpha_2 E_1 \frac{N_1}{N_2} + 1 - E_1 = 1 - \alpha_1 E_1 + \alpha_2 E_1 \frac{N_1}{N_2}.$$  

From (A.5) we then get

$$\frac{\dot{N}_1}{N_1} = n + \left\{ 1 - \alpha_1 E_1 + \alpha_2 E_1 \frac{N_1}{N_2} \right\}^{-1} \times [E_1 (\alpha_1 g_1 + \alpha_2 g_2) - \gamma_1 + \varepsilon_{11} (g_1 - \gamma_1) + \varepsilon_{12} (g_2 - \gamma_2)].$$

By inserting the Slutsky equation and by naming the term in front of the brackets $G$, we have

$$\frac{\dot{N}_1}{N_1} = n + G [E_1 (\alpha_1 g_1 + \alpha_2 g_2) - \gamma_1 + (\varepsilon_{11} - \alpha_1 E_1) (g_1 - \gamma_1)] + (\varepsilon_{12} - \alpha_2 E_1) (g_2 - \gamma_2)].$$

Making use of the fact that the demand functions are homogeneous of degree zero in $R$, $P_1$, and $P_2$, which means that $\varepsilon_{i1} + \varepsilon_{i2} = 0$ ($i = 1, 2$), we have (7),

$$\frac{\dot{N}_1}{N_1} = n + G [E_1 (\alpha_1 g_1 + \alpha_2 g_2) - \gamma_1 + \varepsilon_{i1} [(g_1 - g_2) + (\gamma_2 - \gamma_1)]]].$$

Regarding the term $G$, this can be written as

$$G = \left( 1 - \alpha_1 E_1 + \alpha_2 E_1 \frac{N_1}{N_2} \right)^{-1} = \left( \alpha_2 E_2 + \alpha_2 E_1 \frac{N_1}{N_2} \right)^{-1},$$

since $\alpha_1 E_1 + \alpha_2 E_2 = 1$.

The term $G$ expresses something about gains from migration from the sector with the lower wage rate to that with the higher.

In the text the following proposition was made:

$$\left\{ a_2 \left( \frac{N_1}{N_2} F_1 + F_2 \right) \right\}^{-1} = \left\{ 1 - \alpha_1 E_1 \left( 1 - \frac{w_2}{w_1} \right) \right\}^{-1}.$$
I will now demonstrate this equality,

\[
G = \left( \frac{w_2 N_1}{R} + \alpha_2 E_2 \right)^{-1} \\
= \left( \frac{w_2 w_1 N_1}{R} \alpha_1 E_1 \right)^{-1} \\
= \left( 1 - \alpha_1 E_1 + \frac{w_2}{w_1} \alpha_1 E_1 \right)^{-1} \\
= \left[ 1 - \alpha_1 E_1 \left( 1 - \frac{w_2}{w_1} \right) \right]^{-1}.
\]

Again I have used the identity \( \alpha_1 E_1 + \alpha_2 E_2 = 1 \).

References


Simon, H., 1947, Effects of increased productivity upon the ratio of urban to rural population, Econometrica XV, Jan., 31–42.

Vislie, J., 1977, A note on the distribution of labour between sectors (in Norwegian), Memorandum from the Institute of Economics, University of Oslo, Jan.